



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

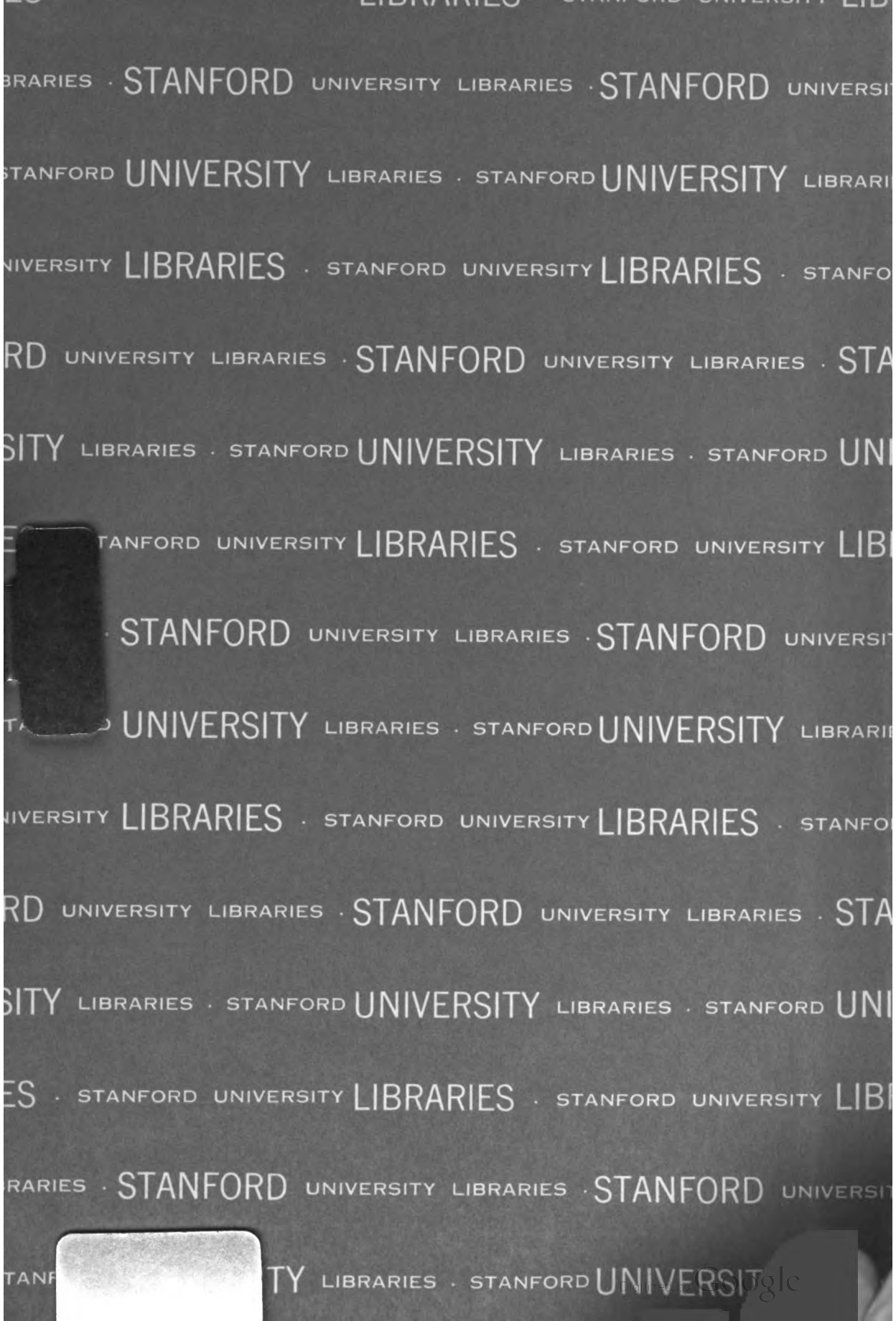
About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

Stanford University Libraries



3 6105 000 642 329





THE PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS*

CONDUCTED BY

THE
AMERICAN PHYSICAL SOCIETY

BOARD OF EDITORS

F. BEDELL, Managing Editor

G. K. BURGESS

HENRY CREW

E. L. NICHOLS

A. D. COLE

L. V. KING

C. M. SPARROW

A. C. LUNN

H. S. UHLER

W. F. G. SWANN

VOL. XVI., SERIES II.

THE PHYSICAL REVIEW

LANCASTER, PA., AND ITHACA, N. Y.

1920

•

**PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.**

CONTENTS OF VOL. XVI., SECOND SERIES

JULY, 1920

Light Absorption and Fluorescence. E. C. C. BALY.....	1
Note on the Adsorption of Nitrogen and Oxygen by Charcoal. ROBERT E. WILSON....	8
On Electromagnetic Momentum. H. A. WILSON.....	17
Quantum Emission Phenomena in Radiation. DAVID L. WEBSTER.....	31
Ionization Potentials of Argon, Nitrogen, Carbon Monoxide, Helium, Hydrogen and Mercury and Iodine Vapors. CLIFTON G. FOUND.....	41
New Facts about Surface-Friction. CH. TERZAGHI.....	54
Speeds in Signaling by the Use of Light. W. E. FORSYTHE.....	62
Phenomena in Oxide-coated Filament Electron Tubes. H. D. ARNOLD.....	70
Note on the End Correction in the Determination of Gas Viscosity by the Capillary Tube Method. WILLARD J. FISHER.....	83

AUGUST, 1920

The Existence of Homogeneous Groups of Large Ions. OSWALD BLACKWOOD.....	85
Electrical Discharges from Pointed Conductors. JOHN ZELENY.....	102
Some Physical Properties of Nickel-Iron Alloys. L. R. INGERSOLL AND OTHERS.....	126
The Selective Reflection of Heat Waves by Linear Resonators. E. C. WENTE.....	133
The Crystal Structure of Sodium Nitrate. RALPH W. G. WYCKOFF.....	149
A Note on the Correction of Contact Difference of Potential Developed in Compton's Modification of the Quadrant Electrometer. OTTO STUHLMAN, JR.....	158
Note on the Electromagnetic Force between Two Atoms. G. A. SCHOTT.....	162

SEPTEMBER, 1920

Charcoal Activation. H. HORTON SHELTON.....	165
The Effect of Temperature upon the Transmission of Infra-Red Radiation through Various Glasses. GEORGE ROSENGARTEN.....	173
The Theory of Linear-Sinoidal Oscillations. HENRY G. CORDES.....	179
The K-Characteristic Absorption Frequencies for the Chemical Elements Magnesium to Chromium. HUGO FRICKE.....	202
The Audion Oscillator. R. A. HEISING.....	216
Application of Electric Currents in the Bunsen Flame. C. W. HEAPS.....	238

OCTOBER, 1920

The Tones from Bells. ARTHUR TABER JONES.....	247
The Valency of Photo-electrons and the Photo-electric Properties of Some Insulators. M. J. KELLY.....	260
The Detecting Efficiency of the Electron Tube Amplifier. E. O. HULBURT AND G. BREIT.....	274
The Minimum Arcing Voltage in Helium. K. T. COMPTON, E. G. LILLY, P. S. OLMSTEAD	282
Electrification by Impact. HAROLD P. RICHARDS.....	290
A New Design of Precision X-Ray Spectrometer. C. D. COOKSEY.....	305
The Performance of Conical Horns. G. W. STEWART.....	313
The High Frequency Spectra of Lead Isotopes. C. D. COOKSEY AND D. COOKSEY.....	327

On the Variation of the Factor h in the Equation $\frac{1}{2}mv^2 = h\nu$. FERNANDO SANFORD.....	337
A Study of Residual Ionization in a Gas with Reference to Temperature Effects. C. H. KUNSMAN.....	349
American Physical Society.....	362
Minutes of the One hundred and Fourth Meeting; A Catenary Loaded at one Point, <i>S. H. Anderson</i> ; Gibbs Thermodynamical Models, <i>W. P. Boynton</i> ; Transparency to Total Heat Radiation, <i>S. L. Brown and C. E. Normand</i> ; A Study of the Residual Ionization in a Gas with Reference to Temperature Effects, <i>C. H. Kunsman</i> ; The Continuous Spectrum of Hydrogen in the Schumann Region, <i>R. P. Lewis</i> ; Multiple Reflections from the Interior of a Ring, <i>A. A. Knowlton and G. A. Wall</i> ; Velocity of Sound from a Moving Source, <i>R. B. Abbott and J. W. Cook</i> ; Voltage Wave Analysis with Indicating Instruments, <i>Leslie F. Curtis</i> ; The Dielectric Constant of Silk, <i>F. J. Rogers</i> ; The Mathematical Structure of X-Ray Spectra, <i>R. T. Birge</i> ; The Spectroscopic Committee of the Division of Physical Sciences of the National Research Council, <i>Charles E. St. John</i> .	

NOVEMBER, 1920

Arcing Voltages in Mercury Vapor as a Function of the Temperature of the Cathode. T. C. HEBB.....	375
The Calculation of Detecting and Amplifying Properties of an Electron Tube from its Static Characteristics. G. BREIT.....	387
The Detecting Efficiency of the Single Electron Tube. E. O. HULBURT AND G. BREIT.....	408
Variation with Pressure of the Residual Ionization due to the Penetrating Radiation. K. MELVINA DOWNEY.....	420
The Kinetic Theory of Magnetism. WARREN WEAVER.....	438
A Photographic Method of Finding the Instantaneous Velocity of Spark Waves. ARTHUR L. FOLEY.....	449
Is the Atom the Ultimate Magnetic Particle? ARTHUR H. COMPTON AND OSWALD ROGNLEY.....	464
On the Free Oscillations of Spheroids. RAJENDRA NATH GHOSH.....	477
Soft X-Rays; A Note of Interpretation. H. M. DADOURIAN.....	481
Velocity of Sound from a Moving Source. R. B. ABBOTT AND J. W. COOK.....	486
Proceedings of the American Physical Society.....	493
The Spectra of Compound Gases Flowing in Vacuum Tubes, <i>W. H. Bair</i> .	

DECEMBER, 1920

On K. S. Magnet Steel. KOTARO HONDA AND SHOZO SAITO.....	495
The Effect of Fluorescence and Dissociation on the Ionizing Potential of Iodine Vapor. H. D. SMYTH AND K. T. COMPTON.....	501
The Absorption of Sound by Rigid Walls. PAUL E. SABINE.....	514
On the Theory of Powell's Bands and the Group-Velocity in Dispersive Media. NIHAL KARAN SEITH.....	519
On the X-Ray Spectra of Tungsten. WILLIAM DUANE AND R. A. PATTERSON.....	526
On the Effective Capacity and Resistance of a Condenser for High Frequency Currents. F. C. BLAKE.....	540
The Action of Roentgen and Gamma Radiations upon the Electrical Conductivity of Selenium Crystals. A. M. MCMAHON.....	558
The Effect of Temperature upon the Infra-red Absorption of Certain Glasses. G. E. GRANTHAM.....	565
Errata.....	575
Index.....	576

THE PHYSICAL REVIEW.

LIGHT ABSORPTION AND FLUORESCENCE.

By E. C. C. BALY.

SYNOPSIS.

Quantitative Relationships between the Vibration Frequencies of Molecules as Shown by Absorption, Fluorescence, or Phosphorescence Bands; Comparison of Theoretical Predictions with Experimental Results.—In an earlier paper the author predicted the frequencies of the absorption bands of naphthalene in the infra-red from the ultra-violet absorption and fluorescence. The later experimental data of Stang did not confirm the predicted values, and this fact caused the proposer of the theory to present in the present paper an explanation of the discrepancy and to give a generalization of the original theory.

The lack of agreement was due to two causes: (a) the theory was dependent upon the alleged unsatisfactory nature of the conceptions of Bjerrum and of Krüger, which relate respectively to molecular rotational velocities and to molecular precessional velocities, and (b) the experimental data upon which the hypotheses were based were incomplete. From a critical study of the frequencies exhibited by sulphur dioxide and by water vapor, the following *general law of molecular frequencies* is deduced, namely: the frequencies truly characteristic of a molecule are multiples of the least common multiple of the frequencies of the atoms it contains, and the subgroups are due to intra-molecular frequencies, that is, the least common multiples of groups of atoms within the molecule.

Fluorescence of Naphthalene.—The subgroup series in the fluorescence band of this substance has a constant frequency difference of 1.4136×10^{14} , which is now shown to be an intramolecular frequency instead of a true molecular frequency, as assumed in the earlier paper. This conclusion is justified by the chemical constitution of the naphthalene molecule and the excellent agreement between the predicted frequencies and the experimental data obtained by Coblenz for ethylene.

Fluorescence of Uranyl Salts.—In further confirmation of the hypothesis of intra-molecular frequencies, the author discusses (with tabulated data) the experimental results of Nichols and Merritt on the fluorescence bands of uranyl ammonium chloride and of uranyl nitrate trihydrate.

IN a recent paper in the PHYSICAL REVIEW,¹ Stang has published some very interesting and important measurements of the infra-red absorption bands exhibited by naphthalene and a few of its derivatives. One of the objects was to test the validity of my theory of the quantitative

¹ PHYS. REV., 9, p. 542, 1917.

relationship between the various vibration frequencies possessed by molecules and evidenced by them as absorption, fluorescence, or phosphorescence bands.¹ I ventured to foretell the frequencies of the absorption bands of naphthalene in the infra-red from the ultra-violet fluorescence and absorption, and at first sight it might be a little disappointing to find that the values obtained by Stang do not agree with my calculations.

On the other hand since the date of my paper it has been found possible to obtain much more information as to the relationships which exist between the various frequencies shown by a given compound. This has been rendered possible by a study of the absorption spectrum of sulphur dioxide, for which very accurate determinations of the vibration frequencies in the infra-red and ultra-violet have been made. It may be noted here that my prediction of the infra-red spectrum of naphthalene was based entirely on the Bjerrum conception of molecular rotational frequencies and there is now little doubt that this theory together with the Krüger² conception of molecular precessional velocities is not capable of explaining the relationships that have been observed.

In order to find an explanation of the apparent discrepancy in the case of naphthalene the complete relationship found in sulphur dioxide may be very briefly detailed.³ Every single frequency that has been observed with this gas can be expressed in terms of three fundamental frequencies, 2.4531×10^{11} , 8.19×10^{11} , and 1.296×10^{12} . Of these 2.4531×10^{11} is characteristic of oxygen, and 8.19×10^{11} and 1.296×10^{12} are characteristic of sulphur, since the infra-red bands of oxygen can be expressed in terms of the first, whilst those of sulphur and hydrogen sulphide can be expressed in terms of the last two. I have already shown in previous papers that the least common multiple principle forms the basis of molecular frequencies, and the least common multiple of the three fundamental frequencies given above is 2.89299×10^{12} . This number multiplied by 10, 12, 14, 18, 26, and 33 gives the exact central frequencies of all the absorption bands which have yet been observed for sulphur dioxide in the infra-red. Then again of these absorption bands the one with the central frequency $2.89299 \times 14 \times 10^{12}$ has the greatest intensity, and this central frequency multiplied by 25 gives the exact central frequency of the less refrangible absorption band in the ultra-violet, the center of the more refrangible band not having yet been observed.

Now it is well known that the central frequency of an absorption band is a true molecular frequency since it is the only frequency which persists

¹ *Astrophys. Journal*, 42, p. 4, 1915.

² *Ann. der Physik*, 50, p. 346, 51, p. 450, 1916.

³ Baly and Garrett, *Phil. Mag.*, 31, p. 152, 1916.

when the substance is cooled to a very low temperature. The important conclusion therefore is reached that the frequencies truly characteristic of a molecule are multiples of the least common multiple of the frequencies of the atoms it contains.

Again the ultra-violet absorption band of sulphur dioxide consists of a series of subgroups symmetrically arranged round a central subgroup, and further each subgroup consists of a central line with a series of lines symmetrically arranged round it. Considering first the central lines of the subgroups it has been found that these form a series with constant frequency difference, these constant frequency differences being the least common multiples of two only of the atomic frequencies. Thus the central lines of the subgroups in the less refrangible ultra-violet band of sulphur dioxide form a series with constant frequency difference of 6.69696×10^{12} which is the least common multiple of 8.19×10^{11} and 2.4531×10^{11} . In the more refrangible band the constant frequency difference between the sub-groups is 1.05972×10^{13} , which is the least common multiple of 8.19×10^{11} and 1.296×10^{12} . Lastly the individual lines within each sub-group also form a series with constant frequency difference, and in this case the constant frequency difference is equal to one of the atomic frequencies. Thus in each subgroup of the less refrangible ultra-violet band the component lines form a series with the constant frequency difference 8.19×10^{11} .

It may be concluded from this that whilst the true molecular frequencies are multiples of the least common multiple of the frequencies of the atoms it contains, the subgroups are due to intra-molecular frequencies, that is the least common multiples of groups of atoms within the molecule. In case this be thought too important a deduction to make from one set of observations it may be stated that exactly the same relationship has been found in the case of the frequencies exhibited by water vapor. Obviously if 2.4531×10^{11} is a characteristic frequency of oxygen it must form one of the bases of the molecular frequency system of water. Then we have the two intramolecular frequencies of water, 7.5×10^{11} and 1.7301×10^{12} , which have been called the molecular rotational velocities of the water molecule. These intramolecular frequencies will be the least common multiples of two out of the three fundamental atomic frequencies active in the water molecule, one of these being 2.4531×10^{11} . The other two atomic frequencies are at once found to be 1.0635×10^{11} and 2.1159×10^{11} . The least common multiple of all three is 6.1326×10^{12} and this number multiplied by 8 gives the exact central frequency of the great absorption band of water at 6.115μ , and when multiplied by 16 the exact central frequency of the

band at 3.0575μ . Further, by assuming an exactly analogous relationship with sulphur dioxide I have calculated the individual lines in the absorption band at 6.115μ and these agree extraordinarily closely with the actual measurements made by Sleator.¹

Apart from the somewhat obvious conflict between these relationships now established and the Bjerrum and Krüger theories, they have a very material significance as regards the infra-red absorption of naphthalene. The subgroup series in the fluorescence band of naphthalene shows a constant frequency difference of 1.4136×10^{13} ($1/\lambda = 47.12$). In my original paper I assumed on the basis of the Bjerrum theory that this was the molecular rotational frequency and that naphthalene should exhibit absorption bands in the infra-red with central frequencies of $1.4136 \times 10^{13} \times 1, 2, 3, 4, 5$, etc., or wave-lengths $21.22 \mu, 10.61 \mu, 7.07 \mu, 5.31 \mu, 4.24 \mu$, etc. The experience now gained from sulphur dioxide and water shows that this frequency is not a molecular frequency at all, but an intra-molecular frequency due to a group of atoms within the naphthalene molecule, and need not necessarily appear in the infra-red absorption spectrum of naphthalene.

Now if this frequency, 1.4136×10^{13} , is an intra-molecular frequency of naphthalene it must clearly be a true molecular frequency of a group of atoms in the naphthalene molecule. There are two very obvious groups in this molecule, namely the benzene grouping and the olefine linking. It is therefore to be expected that either benzene or the olefines should show as true molecular frequency the intra-molecular frequency of naphthalene, 1.4136×10^{13} . In other words one or the other should show absorption bands with centers at $21.22 \mu, 10.61 \mu, 7.07 \mu, 5.31 \mu, 4.24 \mu$, and further, since the intensity of the effect in naphthalene decreases with the size of the multiple of 1.4136×10^{13} , the intensity of these bands should be greatest at 21.22μ and should decrease with decreasing wave-length in the series.

It is well known that benzene does not show any of these bands but the infra-red absorption spectrum of ethylene has been observed by Coblentz between 13μ and 3μ .² He finds absorption bands with centers at $10.5 \mu, 6.98 \mu, 5.30 \mu$, and 4.32μ , *these being the only bands observed between these limits*. The agreement is very striking and moreover with the thickness of ethylene used the amount of light absorbed was 98, 81, 47, and 14 per cent. respectively. The sixth member of the series was not detected owing doubtless to its weakness and also to the presence of the band at 3.28μ which does not belong to this series. Coblentz'

¹ Astrophys. Journal, 48, p. 125, 1918.

² Carnegie Inst. Publ., No. 35, 1905. Attention may be drawn to the remarkable similarity between the infra-red absorption spectra of the members of a homologous series.

measurements did not extend beyond $13\ \mu$ and consequently he did not observe the very strong band which must exist at $21.22\ \mu$.

It may fairly be claimed that this establishes the reality of these intramolecular frequencies. Possibly also I may be excused for the error I made in assuming the complete validity of the Bjerrum theory which led me to believe that 1.4136×10^{13} was a true molecular frequency of naphthalene.

Reference may also be made here to the very interesting results obtained by Nichols and Merritt¹ upon the fluorescence spectra of uranyl

TABLE I.

	1/ λ	Diff.		1/ λ	Diff.
1st group.....	1469.7	20.2 = 1×20.2	5th group.....	1789.1	31.4 = 2×15.7
	1489.9	center		1802.8	17.7 = 1×17.7
	1507.1	17.2 = 1×17.2		1820.5	center
	1521.8	31.9 = 2×15.95		1839.4	18.9 = 1×18.9
Average difference, 17.32				1856.0	35.5 = 2×17.75
					Average difference, 17.25
2d group.....	1538.2	35.2 = 2×17.6			
	1553.3	20.1 = 1×20.1	Central group..	1871.2	32.8 = 2×16.4
	1573.4	center		1886.1	17.9 = 1×17.9
	1588.9	15.5 = 1×15.5		1904.0	center
	1605.1	31.7 = 2×15.85		1922.6	18.6 = 1×18.6
Average difference, 17.25				1939.2	35.2 = 2×17.6
					Average difference, 17.42
3d group.....	1621.5	34.4 = 2×17.2	7th group.....	1955.3	30.9 = 2×15.45
	1638.9	17.0 = 1×17.0		1970.1	16.1 = 1×16.1
	1655.9	center		1986.2	center
	1672.8	16.9 = 1×16.9		2005.7	19.5 = 1×19.5
	1689.4	33.5 = 2×16.75		2022.6	36.4 = 2×18.2
Average difference, 16.97					Average difference, 17.15
4th group.....	1705.9	32.8 = 2×16.4	8th group.....	2038.3	32.7 = 2×16.35
	1719.9	18.8 = 1×18.8		2053.3	17.7 = 1×17.7
	1738.7	center		2071.0	center
	1755.4	16.7 = 1×16.7		2089.0	18.0 = 1×18.0
	1772.2	33.5 = 2×16.75		2105.9	34.9 = 2×17.45
Average difference, 16.97				2121.5	50.5 = 3×16.83
					Average difference, 17.09
					Mean of all the subdifferences, 17.16

salts which would seem to conform in a very remarkable way to the general theory of absorption and fluorescence. The diagram given by these authors for the fluorescence bands of uranyl ammonium chloride represents an almost perfect example of the symmetrical distribution of the

¹ *Phys. Rev.*, 6, p. 360, 1915.

TABLE II.

TABLE 11.

	1/A	Diff.		1/A	Diff.
1st group.....	1629.2	14.7 = 2 × 7.35	4th group.....	1858.7	43.3 = 6 × 7.2
	1637.2	6.7 = 1 × 6.7		1865.1	37.9 = 6 × 7.6
	1643.9	center		1873.0	30.0 = 4 × 7.5
				1877.4	25.6 = 3 × 8.5
				1889.0	14.0 = 2 × 7.0
				1895.3	7.7 = 1 × 7.7
				1903.0	center
	1667.2	23.3 = 3 × 7.77		1908.9	5.9 = 1 × 5.9
				1915.9	12.9 = 2 × 6.45
	1677.0	33.1 = 5 × 6.62		1923.3	20.3 = 3 × 6.8
				1930.1	27.1 = 4 × 6.8
				1938.8	35.8 = 5 × 7.18
Average difference, 7.07			Average difference, 7.24		
2d group.....	1686.1	43.1 = 6 × 7.2	5th group.....	1945.7	44.2 = 6 × 7.37
	1699.8	29.4 = 4 × 7.35		1952.9	37.0 = 5 × 7.4
	1704.2	25.0 = 3 × 8.3		1959.1	30.8 = 4 × 7.7
	1715.2	14.0 = 2 × 7.0		1965.3	24.6 = 3 × 8.2
	1722.8	6.4 = 1 × 6.4		1976.4	13.5 = 2 × 6.75
	1729.2	center		1982.3	7.6 = 1 × 7.6
				1989.9	center
	1741.8	12.6 = 2 × 6.3		1995.9	6.0 = 1 × 6.0
	1748.6	18.4 = 3 × 6.1		2002.1	12.2 = 2 × 6.1
	1755.0	25.8 = 4 × 6.45		2009.8	19.9 = 3 × 6.6
	1765.0	35.8 = 5 × 7.16		2017.4	27.5 = 4 × 6.9
				2025.0	35.1 = 5 × 7.0
Average difference, 7.02			Average difference, 7.18		
3d group.....	1772.0	44.1 = 6 × 7.35	6th group.....	2033.2	43.1 = 6 × 7.2
	1778.7	37.4 = 5 × 7.48		2046.8	29.5 = 4 × 7.4
	1785.9	30.2 = 4 × 7.55		2051.0	25.3 = 3 × 8.4
	1791.8	24.3 = 3 × 8.1		2064.3	12.0 = 2 × 6.0
	1802.1	14.0 = 2 × 7.0		2070.3	6.3 = 1 × 6.3
	1808.7	7.4 = 1 × 7.4		2076.3	center
	1816.1	center		2083.8	7.5 = 1 × 7.5
	1821.3	5.2 = 1 × 5.2		2089.7	13.4 = 2 × 6.7
	1828.1	12.0 = 2 × 6.0			
	1835.3	19.2 = 3 × 6.4		2103.5	27.2 = 4 × 6.8
	1842.4	26.3 = 4 × 6.6		2112.7	36.4 = 5 × 7.3
	1851.5	35.4 = 5 × 7.1			
Average difference, 7.10			Average difference, 7.17		
Mean of the subdifferences, 7.13					

subsidiary intramolecular frequencies round the central and most strongly marked molecular frequency. According to my theory this band consists of a central group with five groups on the red side and two groups on the blue side. Each group at ordinary temperatures consists of five subgroups symmetrically arranged round a central subgroup, and at lower temperatures these subgroups are still further resolved. This

arrangement is very similar indeed to that already described for sulphur dioxide. Two constant wave number differences are present, namely 83.20 between the centers of the groups and a secondary difference of 17.1 between the subgroups of each group. This secondary difference is shown in the following table and is calculated in each instance from the central band of the group, the average for each group being calculated from the total number of times the interval occurs.

The central wave number of the band or the molecular wave number is 1904 and this therefore should be a multiple of the wave number of an important band in the infra-red spectrum of uranyl ammonium chloride.

A second fact of importance is the change in the wave number difference in the absorption band between the heads of the groups, this difference being about 71 instead of 83.2 in the fluorescence band. This is exactly similar to the case of sulphur dioxide as stated above, and it shows that the effective intramolecular or atomic frequencies are different according to the condition of the molecular force field of the salt.

The grouping is still better shown in the fluorescence band of uranyl nitrate trihydrate which has more recently been examined.¹ This band may be arranged in six groups, the maximum number of subgroups in each group being 12. The secondary differences are shown in Table II.

In the case of the above group the principal intramolecular wave number is 86.8 while the secondary wave number is 7.13. It is interesting to note that the principal intramolecular wave number for the absorption band of uranyl nitrate trihydrate is just 10 times the secondary wave number for the fluorescence band.

THE UNIVERSITY,
LIVERPOOL.

¹ Nichols and Merritt, *Phys. Rev.*, 9, p. 113, 1917.

NOTE ON THE ADSORPTION OF NITROGEN AND OXYGEN
BY CHARCOAL.

BY ROBERT E. WILSON.

SYNOPSIS.

Discussion of Previously Published Results on the Adsorption of Oxygen, Nitrogen, and their Mixtures in Activated Charcoal at Liquid Air Temperatures.—This paper is based on experimental data recently set forth in this REVIEW, "Studies in the Adsorption of Gases by Charcoal, II.," by Lemon and Blodgett.¹

Definite Relationship between Mols of Oxygen, Nitrogen, or Mixtures Adsorbed.—Attention is called to the surprising fact, hitherto not mentioned, that the number of mols of oxygen adsorbed to any given final pressure is almost exactly 1.30 times the number of mols of nitrogen adsorbed to the same final pressure. The pressure vs. volume adsorbed curve for pure oxygen and for mixtures of oxygen and nitrogen can, therefore, be very accurately calculated from the results obtained with nitrogen alone. Furthermore, it is also possible to reproduce all the curves representing the rate of adsorption from the results obtained with nitrogen alone.

Bearing of Foregoing Relationship on Theory of Adsorption; Nature of Forces by which Gas Molecules are Held.—By a mathematical treatment of the foregoing data, it can be demonstrated that neither Langmuir's one layer adsorption theory (which undoubtedly applies to that portion of the gas held with the greatest tenacity) nor the "capillary condensation theory" (which applies to loosely held liquid in capillaries of moderate size) are applicable to the intermediate range of pressures covered by Lemon and Blodgett's data. Their results can, however, be explained on the basis of the following hypotheses: (1) the ratio of the molecular volume of adsorbed nitrogen to that of adsorbed oxygen is the same as for the free liquids (1.31 at $-192^{\circ}\text{C}.$); (2) the gases are held in layers several molecules deep primarily by the attractive force of the charcoal surface; (3) the stray field around oxygen and nitrogen molecules is substantially the same at any given distance.

SOME very interesting data have recently been published in this journal by Lemon and others, on the adsorption of various gases by charcoal. In the second of that series of articles Lemon and Blodgett¹ describe the results of a series of experiments on the adsorption of oxygen and nitrogen and mixtures of the two by charcoal. In these experiments various amounts of oxygen or nitrogen or their mixtures were suddenly admitted to a chamber containing an activated and previously evacuated specimen of cocoanut-shell charcoal at liquid air temperature. The rate of decrease in pressure and the final equilibrium pressure attained by each mixture was recorded and presented in tabular or graphic form in the article. On the basis of the results therein detailed it was con-

¹ PHYSICAL REVIEW, XIV., 5, November, 1919.

cluded that oxygen and nitrogen are not adsorbed independently, but no attempt was made to point out any definite relationship between the adsorption of the two gases and their mixtures.

A close study of the data therein presented brings to light, however, an extremely interesting fact which seems to have considerable bearing on the question of the mechanism of adsorption. Although at first sight there appears to be no simple relationship between the results obtained on the adsorption of oxygen and of nitrogen, as a matter of fact if the simple assumption is made that one mol of nitrogen is in every way equivalent to 1.30 mols of oxygen, it is possible to calculate not only the results obtained with pure oxygen, but also those for various mixtures of the two gases, by using merely the data obtained with pure nitrogen.

Since the observed points on the three curves do not exactly correspond,

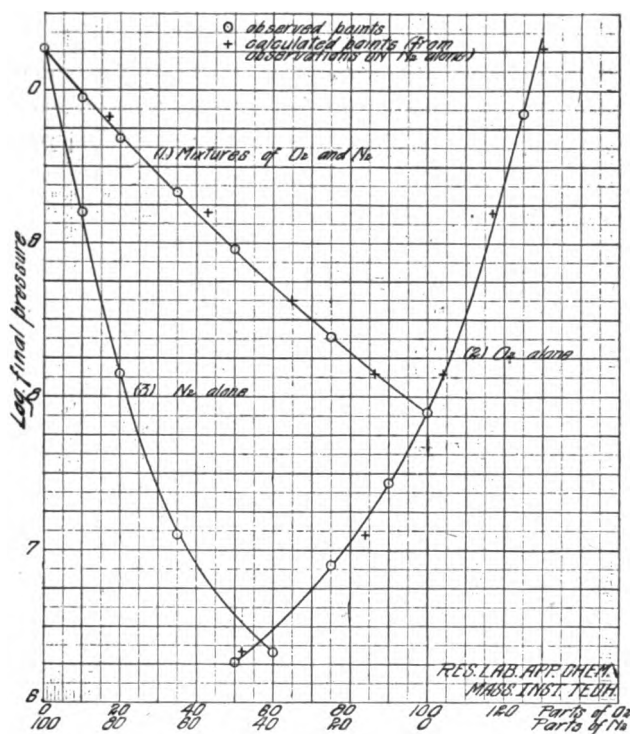


Fig. 1.

Agreement between calculated and observed values of final pressures of oxygen and mixtures of oxygen and nitrogen absorbed in charcoal.

the best way to test the above assumption is to calculate from one nitrogen observation to a point which should lie on the smooth curve connecting the observed points for oxygen or the mixtures. These smooth curves

are obtained by plotting the log of the final pressure against the parts of gas adsorbed (see Fig. 1 corresponding to Fig. 5 of the original article).

The method of making these calculations is as follows: If one mol of nitrogen is exactly equivalent to 1.30 mols of oxygen, then 130 parts O_2 should give the same log final p as that given by 100 parts N_2 (0.270). Similarly the observed pressures for 80 parts N_2 should exactly correspond to $1.3 \times 80 = 104$ parts O_2 . By calculating the five observed nitrogen points over to their oxygen equivalent in this manner, the five crosses on the oxygen curve were obtained. As will be observed, these agree remarkably closely with the smooth curve drawn through the circles which represent the observed points on the oxygen curve. In a similar manner, it is possible to calculate points on the mixture curve, using only the observed points for nitrogen; and it will be observed that again the agreement between the calculated points and the observed curve is excellent.

Another way to show the agreement between the calculated and observed values is to calculate values corresponding to each observed point for oxygen or the mixtures from the corresponding points on the smooth curve connecting the nitrogen observations. The agreement thus obtained is shown in Table I.

TABLE I.

Parts O_2 .	Parts N_2 .	Should = Parts N_2 .	Log p Calculated.	Log p Observed.	Difference.
125	0	96.2	9.86	9.85	-0.01
100	0	76.8	7.88	7.90	+0.02
90	0	69.2	7.38	7.43	+0.05
75	0	57.7	6.84	6.89	+0.05
50	0	38.5	6.30	6.26	-0.04
75	25	82.7	8.39	8.38	-0.01
50	50	88.5	8.98	8.97	-0.01
35	65	91.9	9.36	9.33	-0.03
20	80	95.4	9.74	9.69	-0.05
10	90	97.7	9.98	9.95	-0.03

Small deviations, of course, occur in plotting a smooth curve through any series of points, so that the "calculated" values themselves cannot be considered especially precise even if the theory were exact.

It is thus possible from a single series of observations on different amounts of nitrogen to calculate very accurately the corresponding curves for oxygen, and for mixtures of oxygen and nitrogen, merely by assuming that one mol of nitrogen is exactly equivalent to 1.30 mols of

oxygen. Furthermore, even the curves for the rate of adsorption may be reproduced very closely by making the same assumption. For example, the rate curve for 80 parts N_2 should, by the above method of calculation, lie between the curve for 100 parts O_2 ($= 76.8$ parts N_2) and that for 75 parts O_2 and 25 parts N_2 ($= 82.7$ parts N_2); and as a matter of fact this is found to be the case when the three curves (taken from Figs. 2 and 4 of original article) are plotted on the same coördinates

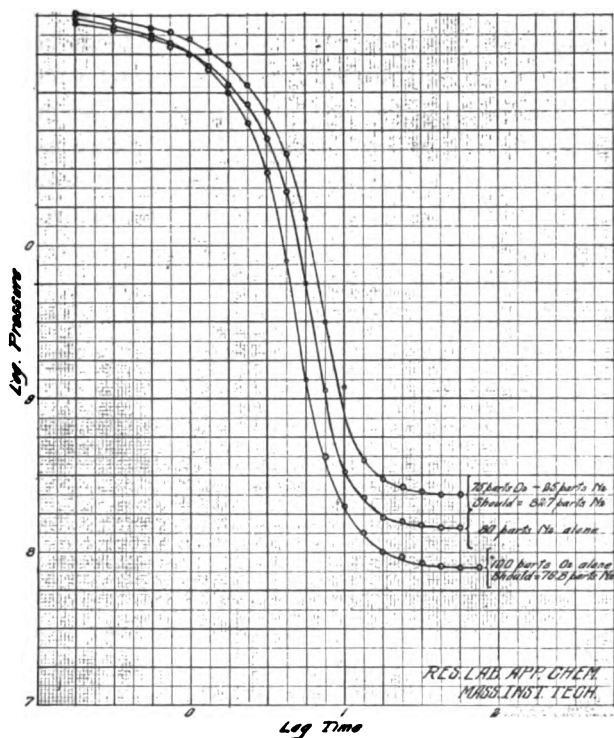


Fig. 2.

Correspondence between rates of adsorption of pure gases and mixtures.

as in Fig. 2 of this article. By similar interpolation, it is possible to approximate any other rate curve for the results for nitrogen alone.

This remarkable parallelism between the adsorption of oxygen and nitrogen, extending as it does over a variation in final pressures of nearly ten thousand-fold, should certainly throw some light on the mechanism of the adsorption of these two gases. As in any case where adsorption takes place inside capillaries of unknown size and number, there is a wide variety of assumptions which can be made to explain any set of known facts providing a sufficient number of variables are introduced. It is the purpose of this note, however, to call attention to an extremely

simple assumption which seems to explain adequately all the foregoing facts.

An important question which should be considered at the outset is whether the gases are adsorbed in films one layer deep, as Langmuir has demonstrated to be the case in certain examples of adsorption, or whether they are several layers deep, possibly completely filling the smaller capillaries. In this particular case both the amount of gas adsorbed and the pressure which it exerts definitely support the latter view. Hulett and others have shown that well-activated charcoal holds very considerable amounts of oxygen and nitrogen even in a good vacuum at high temperatures. This firmly held gas would certainly seem to be that which is adsorbed in a layer one molecule deep. Since its pressure is inappreciable even at temperatures of 200–300° C., it would certainly not be measurable at liquid air temperatures. Furthermore, as the temperature is lowered and approaches the condensing point of the gases, the amounts of gas adsorbed increase greatly—certainly to more than ten-fold those held at the higher temperatures. It would appear, therefore, that only a small fraction of the gas adsorbed in the experiments described by Lemon and Blodgett could be held in the layer one molecule deep, and that the measurably high vapor pressures recorded were produced by the last portions of the gas which were held but loosely in layers possibly 8 or 10 molecules distant from the charcoal surface.

As a first step, then, it seems reasonable to assume that the above mentioned definite ratio between the number of mols of the different gases adsorbed to any given final pressure, might correspond to the relative molecular volumes occupied by the adsorbed gases, which at liquid air temperatures would presumably approximate the liquid state. The best data on the densities of liquid oxygen and nitrogen seem to be those of Baly and Donnan¹ who give the following figures for the density of liquid oxygen as a function of the absolute temperature in the vicinity of its boiling point. (The figures are rounded.)

$$d\text{O}_2 = 1.249 - 0.0048(T - 68),$$

$$d\text{N}_2 = 0.854 - 0.0048(T - 68).$$

Assuming liquid air temperature to be -192°C ., this gives the densities as 1.187 and 0.792 respectively, which corresponds to relative *molecular* volumes of 1.310 for N_2 to 1 for O_2 . It will be observed that this is remarkably close to the ratio of 1.30 determined from the adsorption measurements. Certainly the agreement seems too close to be considered a coincidence, in view of the relationship which might reasonably be expected to exist between these two quantities.

¹ J. Chem. Soc., 81, 911.

These simple and apparently reasonable results do not, however, fit in well with the customary explanation of the absorption of gases in capillaries—the “capillary condensation” theory. This theory assumes that adsorption in a porous material such as charcoal is almost entirely (except for the first layer of molecules) due to the known fact that the vapor pressure of a liquid inside a fine capillary, the walls of which are wet by the liquid, decreases greatly with the decrease in the diameter of the capillary. This law may be expressed quantitatively as follows:¹

$$\ln \frac{p}{P} = - \frac{2\lambda}{RT\rho r}$$

or in more convenient form for calculations:

$$\log_{10} \frac{P}{p} = \frac{2.085\lambda V}{10^3 T d}.$$

Where	Values at -192°C.	
	For O_2 .	For N_2 .
P = vapor pressure of free liquid	25.2 cm. ²	114 cm. ¹
p = “ “ of absorbed liquid		
λ = surface tension in dynes/cm.	15.73 ³	8.27 ³
V = molal volume of liquid	26.95	35.30
T = absolute temp. in $^{\circ}\text{C.}$	81.1	81.1
d = diameter of capillaries		

Using these figures, if the vapor pressure lowering in the case under consideration is due to capillary condensation the pressure of condensed oxygen should be:

$$\log p\text{O}_2 = 1.401 - \frac{10.90}{10^3 d}$$

and for nitrogen

$$\log p\text{N}_2 = 2.056 - \frac{7.50}{10^3 d},$$

where d is the diameter of the *largest* capillaries in which the condensed liquid is held (since this part of the liquid is responsible for the observed final pressures).

From the foregoing it should be possible to calculate the diameter of the capillaries which would correspond to different final pressures of oxygen or nitrogen, and then find, by referring to Fig. 1, the number of

¹ For derivation see R. v. Helmholtz, Wied. Ann., 27, 508, 1886.

² Baly, Phil. Mag., 49, 517, 1900.

³ Values for -193°C. by Baly and Donnan, Trans. Chem. Soc., 81, 907, 1902.

parts of oxygen or nitrogen which were actually adsorbed between these pressure limits. Table II. summarizes the results of such calculations.

TABLE II.

Diameter of Capillaries Centimeters.	Calc. log p of O_2 .	Calc. log p of N_2 .	Parts of O_2 Condensed.	Parts of N_2 Condensed.	Ratio.
$0-2 \times 10^{-8} \dots$	$-\infty$ to -4.05	$-\infty$ to -1.69	35	82	0.23
$2-3 \times 10^{-8} \dots$	-4.05 to -2.23	-1.69 to $-.44$	62	12	5.2
$3-4 \times 10^{-8} \dots$	-2.23 to -1.33	$-.44$ to $+.18$	15	5	3.0

In other words, a given set of capillaries ($0 - 2 \times 10^{-8}$ cm.) appear to take up more than four times as much oxygen as nitrogen, but the next largest group of capillaries ($2 - 3 \times 10^{-8}$) condense over five times as much nitrogen as oxygen! Furthermore, practically all of the gas adsorbed would appear to be held in capillaries whose diameters lie between the narrow limits of 1 and 5×10^{-8} cm. Since these absurd results follow necessarily from the mathematical application of the capillary condensation theory, it must be concluded that the theory is not applicable for such small capillaries, although its validity seems to be established for the larger capillaries where the lowering of vapor pressure is only very slight.

Since neither the one-layer adsorption theory nor the capillary condensation theory appears to apply to the intermediate range of pressures covered by Lemon and Blodgett's work, it seems desirable to develop some new conceptions to explain the surprising results pointed out previously in this paper. If, for example, it be assumed that the primary reason for the low vapor pressure is the attraction of the charcoal surface for the gases, the vapor pressure of the adsorbed oxygen and nitrogen would then depend only upon their distance from the charcoal surfaces and upon their own stray fields of force. If the *stray fields* of force of the two kinds of molecules are *equal* at a given distance, and the *relative volumes* of the condensed gases are as 1 to 1.30, as seems probable from the density data, then it would follow as a necessary conclusion that a given sample of charcoal at any definite temperature and any final pressure would hold 1.30 times as many mols of oxygen as of nitrogen. Since this is in precise accord with the observed facts, and the assumptions are not only simple but justified by other known facts, this would seem to be a very satisfactory working hypothesis in the light of the information at present available.

It may at first sight seem unlikely that oxygen and nitrogen should have equal stray fields at any given distance from their center, in view

of the fact that their boiling points differ by about 12° C. This divergence could, however, be entirely accounted for by the fact that the molecules of oxygen come closer together in the liquid state and hence the attractive forces of these molecules for one another would be somewhat greater, even though the stray field at any given distance were exactly the same as that for nitrogen.

It must, of course, be admitted that certain other more complicated assumptions might be made to explain the same facts; for example, the capillary condensation theory, or indeed almost any theory, could be explained by assuming that the ratio of the molal volumes of the adsorbed gases varied widely from the 1.30 which seems most reasonable, coupled with certain very arbitrary assumptions as to the gradation in the sizes of the pores in the particular sample of charcoal used. The writer has followed several of these more complicated hypotheses through to their logical conclusion without finding one of enough value to justify its detailed presentation as a possible substitute for the foregoing theory, at least in the light of the data at present available.

In conclusion, although Langmuir's theory of one layer adsorption

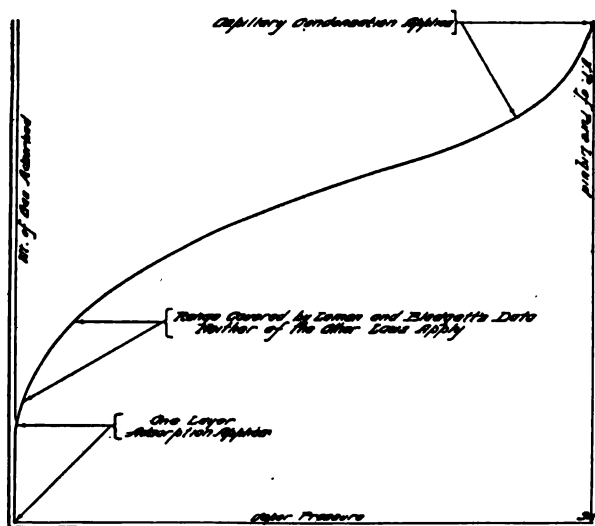


Fig. 3.

appears to be well established for the small amounts of gas which are held with extreme tenacity on a solid surface, and although the capillary condensation theory seems to adequately explain the slightly depressed vapor pressures in capillaries of moderate size, there is a wide intermediate range of adsorption covered by Lemon and Blodgett's work in which

neither theory seems to be satisfactory. The range over which different types of adsorption appear to hold are shown roughly in Fig. 3, which represents a typical adsorption curve for charcoal over the whole range of volumes and pressures.¹

To explain the results obtained in this intermediate region, the following hypotheses are therefore suggested:

1. The ratio between the molecular volumes of adsorbed oxygen and nitrogen is very close to the ratio between the molecular volumes of the free liquids at the same temperature (about 1 to 1.30).
2. The stray field around oxygen molecules is substantially the same as that around nitrogen molecules at any given distance.
3. The gases are held primarily by the attractive force of the charcoal surface, in layers more than one molecule deep, and not by capillary condensation.

These hypotheses adequately explain the known facts, and require no assumptions whatever as to the gradation in the sizes of the pores in the charcoal, or as to the functional relationship between the distance from the charcoal surface and its attractive force.

It would be extremely interesting to compare the similar results obtained by varying the gases, the charcoal, or the temperature from that prevailing in these experiments. Data obtained along these lines should serve either to definitely verify or discredit the foregoing hypotheses.

RESEARCH LABORATORY OF APPLIED CHEMISTRY,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

¹ See article by the writer and others in *J. Ind. Eng. Chem.*, 11, 420, 1919.

ON ELECTROMAGNETIC MOMENTUM.

BY H. A. WILSON.

SYNOPSIS.

Fundamental relations involving electromagnetic momentum.—A simple way of deducing the force per unit volume acting in the electromagnetic field is given which makes clear the nature of the force in terms of elementary ideas. The well-known relation between the *flux of energy* and the momentum in the field suggests that all momentum is due to energy flux. Starting with this assumption a simple relation is obtained between the momentum, energy and *velocity of the center of mass* of any system. This relation when applied to a single particle leads to the same relations between the longitudinal and transverse *masses* and the velocity as have been deduced from the principle of *relativity*. Several particular examples are discussed.

Relations of matter and ether to stresses.—It is argued that since the electromagnetic field has momentum, energy and is subject to *gravitational attraction* it therefore possesses all the essential properties of matter and that therefore the *stresses* in the field should be regarded as acting through the field and on it in the same way that stresses act in ordinary matter. When there is a resultant force on a space containing matter the force is supposed to act on the matter and not on the ether in the space and the motion of matter and the stresses in it are not supposed to act on the ether or to tend to set it in motion. We should therefore regard the stresses in the electromagnetic field as acting on the field and not on the ether. There is therefore no more reason to suppose that the ether is set in motion in an electromagnetic field than there is for supposing that it is set in motion by the stresses in or the motion of ordinary matter.

The transmission of energy and momentum in the field.—This is considered from the point of view indicated. The theory is similar to that given by E. Cunningham who however regards the motion in the field as motion of the ether. Cunningham also supposes that the velocity is always equal to the velocity of light. I have not adopted this assumption but instead I suppose that the direction of the velocity of the field coincides with the direction of the momentum in the field. In this way the *flux of energy* in the field is explained as partly due to the working of the stresses in the field and partly to *convection of energy* with the moving field. Several examples are discussed. Finally some results are mentioned which follow from the idea that energy is subject to *gravitational attraction*.

THE hypothesis that momentum can exist in an electromagnetic field or in the ether was first put forward by J. J. Thomson in 1893 and was afterwards developed by Poincaré and Abraham. It now forms part of the generally accepted electromagnetic theory.

If F denotes the total force on the electricity inside a closed surface S then it may be shown that

$$F = \int_S (\text{Maxwell's Stresses}) dS - \frac{1}{c} \int \frac{d}{dt} [d \cdot h] d\sigma,$$

where σ denotes the volume enclosed by S ; c is the velocity of light; d denotes the electric and h the magnetic intensity. Thus it appears that the resultant of the stresses acting across the surface S is greater than the total force acting on the electricity inside S by an amount

$$\frac{1}{c} \int_{\sigma} \frac{d}{dt} [d \cdot h] d\sigma.$$

This integral has therefore been said to represent a mechanical force acting on the ether inside S which it is natural to suppose is made up of forces $(d/dt)[d \cdot h]/c$ per unit volume. The nature of this mechanical force may be made clear in the following way.

According to Maxwell's theory a displacement current ($\dot{d}$) produces the same magnetic field as an equal conduction current. The force on a current in a magnetic field is undoubtedly due to the interaction of the magnetic field of the current with the external field so that we should expect a force on a displacement current equal to that on a conduction current. We should therefore expect a force equal to $[\dot{d} \cdot h]/c$ per unit volume on the displacement current $\dot{d}$ when in a magnetic field of strength h since the force on a conduction current of density i is $[i \cdot h]/c$ per unit volume.

A varying magnetic field may be said to produce a magnetic displacement current $\dot{h}$ which produces an electric field opposite in direction but otherwise precisely analogous to the magnetic field due to a current. We should therefore expect a force on a magnetic displacement current when in an electric field of amount $[d \cdot \dot{h}]/c$ per unit volume.

The total force per unit volume is therefore

$$[\dot{d} \cdot h]/c + [d \cdot \dot{h}]/c = \frac{1}{c} \frac{d}{dt} [d \cdot h]$$

which agrees with the usual result. Since force is equal to momentum communicated per unit time the existence of this force $(1/c)(d/dt)[d \cdot h]$ has therefore led to the view that there is in the electromagnetic field or in the ether an amount of mechanical momentum equal to $[d \cdot h]/c$ per unit volume.

J. J. Thomson supposes this momentum to be the momentum of moving tubes of electric force which on his theory give rise to the magnetic field.

It appears that in the electromagnetic field there is momentum $[d \cdot h]/c$ associated with a flux of energy $c[d \cdot h]$. A similar result¹ has been deduced from the principle of relativity namely that the convection of energy by a moving body implies an amount of momentum equal to the product of the energy and velocity divided by c^2 .

¹ Relativity and the Electron Theory, E. Cunningham, p. 80.

It seems probable therefore that a flux of energy is always associated with an amount of momentum equal to the flux divided by c^2 . It is natural therefore to regard this momentum as due to inertia of the energy and to regard all momentum as due to a flux of energy. Consider any material system moving in any manner with reference to coördinates x, y, z . Let E denote the energy density at any point in the system and let the flux of energy per unit area per unit time be denoted by F . Then if $\bar{x}$ denotes the x coördinate of the center of mass of the energy in the system we have

$$\bar{x} = \frac{\int x E dS}{\int E dS},$$

where dS denotes an element of volume. Differentiating this with respect to the time t and writing $\mathcal{E}$ for $\int E dS$ this gives

$$\mathcal{E} \dot{\bar{x}} = \int x \dot{E} dS.$$

Now $\dot{E} = -\text{div } F$, so that

$$\mathcal{E} \dot{\bar{x}} = - \int x \text{div } F dS.$$

If the integration is extended over a volume so large that F is zero over the surface enclosing it this gives

$$\mathcal{E} \dot{\bar{x}} = \int F_x dS,$$

where F_x denotes the x component of F . If now we suppose that a flux of energy F_x in the x direction is associated with x momentum per unit volume equal to F_x/c^2 we get

$$M_x = \frac{1}{c^2} \int F_x dS,$$

where M_x denotes the total momentum of the system, due to energy flux, in the x direction. Hence

$$\mathcal{E} \dot{\bar{x}} = M_x c^2.$$

Thus it appears that the total momentum of the system due to energy flux is equal to the total energy in the system multiplied by the velocity of the center of mass of the energy in the system and divided by c^2 . If all momentum is due to energy flux then the center of mass of the energy will coincide with the center of mass of the system.

As an example consider a condenser consisting of two large parallel plates kept at a fixed distance s apart by means of insulating blocks. Let the electric intensity between the plates be d and let the condenser be moving with a velocity v in a direction perpendicular to the planes of its plates.

The energy density between the plates is $d^2/2$ and the flux of energy between the plates is zero since there is no magnetic field. The plates attract each other with a force $d^2/2$ per unit area so that there will be a pressure in the blocks separating the plates equal to $Ad^2/2$, where A is the area of each plate. There is therefore a flux of energy in the blocks equal to $Ad^2v/2$ so that if we take v to be along the x direction

$$\int F_x dS = \frac{Ad^2vs}{2}.$$

The momentum due to the flux of energy is therefore

$$Ad^2vs/2c^2,$$

which is equal to the electrical energy $Asd^2/2$ multiplied by v/c^2 .

Charging the condenser therefore increases its momentum in a direction perpendicular to its planes although there is no flow of electromagnetic energy in this case. We may observe that between the plates there is a tension $d^2/2$ and a density of energy $d^2/2$ which produces a flow due to convection $vd^2/2$ so that the total energy flux is zero whatever the value of v in agreement with the value of $c[d \cdot h]$ in this case.

Now consider the case when the condenser moves in a direction parallel to the planes of its plates. When v is very small compared with c we get a magnetic field between the plates equal to vd/c and perpendicular to v and d . There is therefore a flow of energy between the plates equal to $c[d \cdot h] = vd^2$ which gives for the electromagnetic momentum $Asvd^2/c^2$. There is also a tension in the plates in the direction of v equal to $\frac{1}{2}sd^2(1 - v^2/c^2)$ which gives a flow of energy per unit length in the plates equal to $\frac{1}{2}vsd^2(1 - v^2/c^2)$ in the opposite direction to v . The momentum corresponding to this is $-\frac{1}{2}Asvd^2(1 - v^2/c^2)/c^2$ so that the total momentum is

$$\frac{Asvd^2}{c^2} - \frac{1}{2} \frac{Asvd^2}{c^2} (1 - v^2/c^2) = \frac{Asvd^2}{2c^2} (1 + v^2/c^2).$$

The electromagnetic energy is equal to

$$\mathcal{E} = \frac{As}{2} (d^2 + h^2) = \frac{Asd^2}{2} \left(1 + \frac{v^2}{c^2} \right),$$

so that

$$\frac{\mathcal{E}v}{c^2} = \frac{Asvd^2}{2c^2} (1 + v^2/c^2),$$

which agrees with the value just found for the momentum as was to be expected.

As another example consider the Lorentz electron, which when at rest is a thin hollow sphere of electricity of radius a and charge e . When

moving with velocity v in the x direction it contracts into a spheroid of revolution with axes

$$a\sqrt{1 - v^2/c^2}, \quad a \text{ and } a.$$

The electromagnetic momentum of the Lorentz electron calculated by integrating $[d \cdot h]/c$ is equal to

$$\frac{e^2}{6\pi ac^2} \beta(1 - \beta^2)^{-\frac{1}{2}}$$

where $\beta = v/c$. The sum of the electric and magnetic energies is equal to

$$\frac{e^2}{\pi a} \frac{3 + \beta^2}{24(1 - \beta^2)^{\frac{1}{2}}}.$$

Thus the electromagnetic momentum is not equal to the electromagnetic energy multiplied by v/c^2 .

In order to maintain the electricity in equilibrium it is necessary, as was shown by Poincaré, to suppose that it is pulled inwards by an internal stress the x component of which must be equal to $e^2/32\pi^2 a^4$. The work done by this stress when v is increased from zero to c is equal to the stress multiplied by the volume $\frac{4}{3}\pi a^3$ of the sphere so that we may suppose that there is energy inside the sphere of amount $e^2/32\pi^2 a^4$ per unit volume.

The flux of energy inside the sphere is therefore zero since there is a tension in the direction of motion equal to the energy density. The internal energy therefore does not contribute anything to the momentum. The total energy is therefore

$$\frac{e^2}{\pi a} \frac{3 + \beta^2}{24(1 - \beta^2)^{\frac{1}{2}}} + \frac{e^2}{32\pi^2 a^4} \frac{4}{3} \pi a^3 (1 - \beta^2)^{\frac{1}{2}} = \frac{e^2}{6\pi a(1 - \beta^2)^{\frac{1}{2}}}.$$

Multiplying this by $v/c^2 = \beta/c$ we get

$$\frac{e^2 \beta}{6\pi ac} (1 - \beta^2)^{-\frac{1}{2}},$$

which is equal to the momentum. It appears that when the energy inside the sphere is taken into account the momentum of the Lorentz electron is equal to its total energy multiplied by v/c^2 as it should be.

If a force F acts on a particle moving in the direction of the force then $F\delta t = \delta M$ and $Fv\delta t = \delta \mathcal{E}$ so that $v\delta M = \delta \mathcal{E}$ which with $M = \mathcal{E}v/c^2$ gives $\mathcal{E} = \mathcal{E}_0(1 - v^2/c^2)^{-\frac{1}{2}}$ where $\mathcal{E}_0$ is the energy of the particle when $v = 0$. This result is a well-known deduction from the principle of relativity. The momentum of the particle due to energy flux is therefore given by

$$M = \mathcal{E}_0 v c^{-2} (1 - v^2/c^2)^{-\frac{1}{2}}.$$

The transverse mass m' of the particle due to energy flux is given by

$$m' = M/v = \mathcal{E}_0 c^{-2} (1 - v^2/c^2)^{-\frac{1}{2}}$$

and the longitudinal mass m'' due to energy flux by

$$m'' = dM/dv = \mathcal{E}_0 c^{-2} (1 - v^2/c^2)^{-\frac{3}{2}}.$$

The ratio of the transverse mass m' to the longitudinal mass m'' is therefore

$$\frac{m'}{m''} = 1 - v^2/c^2.$$

These results will be true for a particle of any kind. They will therefore be true for any of the different types of electrons which have been proposed. It follows that the measurements of e/m for electrons for different values of v , made by Kaufmann, Bucherer and others do not really enable us to decide in favor of any particular type. The results obtained agree with the view that momentum is due to energy flux but if this view is correct then the variation of m with v is the same for any sort of particle whether composed of electricity or not.

If all momentum is due to energy flux then we should expect all energy to be subject to gravitational attraction. This is confirmed by the recent discovery of the deflection of light by the gravitational field of the sun.

The force represented by $(1/c)(d/dt)[d \cdot h]$ has usually been regarded as a force acting on the ether. Thus H. A. Lorentz says that in consequence of the existence of this force we must either regard the ether as having so great a density that its motion due to this force is negligible or else we must regard Maxwell's stresses as imaginary ones, merely auxiliary mathematical quantities not representing real stresses.¹

In ordinary mechanics we deal with stresses acting in material bodies which are supposed to move through the ether without setting it in motion. For example when power is transmitted from one pulley to another by means of a belt we have a flux of energy along the belt which is partly a convection of kinetic energy equal to $\frac{1}{2}\rho v^2 \times v$ where ρ is the density and v the velocity of the material of the belt and partly a transmission of energy due to work done by the tension T in the belt equal to $-Tv$ per unit area of cross-section of the belt.

Now the ultimate particles of the matter composing the belt are believed to be some sort of ethereal modifications which move through the unmodified ether without setting it in motion presumably by a process of growing in front and fading away behind. Thus in ordinary

¹ Theory of Electrons, p. 31.

mechanics forces act on ethereal modifications and energy is transmitted by stresses existing in such modifications when they move through the ether. When there is a resultant force on a portion of space we do not regard this force as tending to set the ether in the space in motion but only as tending to produce a motion of the ethereal modification or matter there present.

The electromagnetic field is like matter in that it is believed to be some sort of modification of the ether. Like matter it can move through the ether and possesses energy and momentum and is subject to gravitational attraction. It seems therefore that we ought to regard the force $(1/c)(d/dt)[d \cdot h]$ as acting not on the ether but on the ethereal modification present that is on the electromagnetic field. If we adopt this view then we may regard the ether as always at rest and the Maxwell's stresses as acting through the field so that when these stresses give a resultant force on a portion of space then this force tends to produce a motion of the ethereal modifications present in the portion of space whether the modifications present consist of matter, electricity or only an electromagnetic field. This way of regarding the matter is consistent with our conceptions of the nature of matter and the transmission of energy through ordinary matter in motion. Force, as we know it, is something which tends to change the motion through the ether of an ethereal modification without setting the ether itself in motion. The motion of the ethereal modification is really a process of change of state and not motion at all but for convenience may be referred to as a motion through the ether. A force does work on the moving matter although there is really only change of state and no motion. This involves no contradiction because in the definition of work nothing is said about the nature of the process by which the matter acted on by the force moves.

If the observed effects of force are really only changes of state in a stagnant ether then there is no reason to suppose that the ether can ever be set in motion. We may still speak of matter as moving through the ether provided we understand that the expression merely refers to the change of position and not to the nature of the process by which the change takes place.

If we regard the force $(1/c)(d/dt)[d \cdot h]$ as acting on the electromagnetic field then we must regard the momentum $[d \cdot h]/c$ as due to a motion in the field. We have seen that this momentum can be regarded as due to the flux of energy so that it is natural to conclude that the flux of energy is the motion in the field to which the momentum is due.

Now the flux of energy may be made up of two parts, a convection of the energy present per unit volume through the ether with a velocity v

that is $\frac{1}{2}(d^2 + h^2)v$ and a flux due to work done by stress. The velocity v may be taken to be in the direction of the momentum and the energy flux that is along the direction of the vector $[d \cdot h]$. The velocity v is the velocity with which the field moves through the ether. The process by which this motion takes place being the same as the process by which matter moves through the ether. Presumably a process of growing in front and fading away behind by changes of state in a stagnant ether. In the case of light waves moving with velocity c such a process is merely wave motion in the usual sense. It may be objected that we ought not on this view to regard any energy as moving by a process of convection but it must be understood that the process of convection is of the same nature as the convection of energy by a moving particle which also moves by a process of change of state. Thus what we call convection is really very much of the same nature as wave motion but it is convenient to express what happens in the electromagnetic field in terms of ideas derived from experience with matter in bulk. The statement that some of the energy in the field moves by a process of convection is intended to be understood as meaning convection like that by a moving particle which moves by changes of state in a stagnant ether.

In the same way the flux of momentum may be due partly to convection of momentum and partly to stress in the field. The Maxwell stresses of course represent the total flow of momentum.

At a point in an electromagnetic field suppose the momentum $[d \cdot h]/c$ is directed along the x axis and let d be along the y axis and let the angle between d and h be θ so that $h_y = h \cos \theta$ and $h_x = h \sin \theta$.

Consider a small plane area α and let the direction cosines of its normal be l, m, n . Let X_n', Y_n', Z_n' be the components of the stresses acting across α . Then if q be written for $[d \cdot h]/c$ the flux of energy along the normal to α is lqc^2 so that we have

$$-X_n'v + Elv = lqc^2, \quad (1)$$

where v denotes the velocity of the field which is along x and E denotes the energy density $\frac{1}{2}(d^2 + h^2)$.

The flux of x momentum across α will be given by

$$X_n' - qlv = X_n, \quad (2)$$

where X_n denotes the x component of the Maxwell's stress across α .

The fluxes of y and z momentum will be given by $Y_n' = Y_n$ and $Z_n' = Z_n$ simply because all the momentum in the field is x momentum so that there is no convection of y or z momentum.

Now when d is along y and $h_x = 0$ as we are supposing to be the case

we have

$$X_n = -\frac{1}{2}l(d^2 + h^2) = -lE.$$

Substituting this value of X_n in (2) we get

$$X_n' = qlv - lE.$$

Hence (1) becomes

$$2Ev - qv^2 = qc^2,$$

which gives

$$v = \frac{E}{q} \pm \sqrt{\frac{E^2}{q^2} - c^2}.$$

One root is less than c and the other greater. Presumably the one less than c should be taken.

A calculation similar in principle to the above has been published previously by E. Cunningham who however regards the velocity v as that of the ether, whereas I prefer to regard it as the velocity of the electromagnetic field through the ether.

In the case of light waves we have $E = d^2$ and $q = d^2/c$ so that $v = c$. This means that in light waves the field moves along through the ether with the velocity c as was to be expected. Cunningham however concludes that in light waves the ether moves along with the light with the velocity c of light so that the waves are at rest relative to the ether.

Cunningham also does not assume that the velocity is along the direction of $[d \cdot h]$ but he assumes that the resultant velocity of the ether is always equal to c . It seems to me to be much more reasonable to assume that the velocity is along the direction of $[d \cdot h]$. For all the momentum is in this direction and we should naturally expect the velocity of the field to be in the same direction as the momentum.

We have

$$\begin{aligned} X_n' &= qlv + X_n \\ &= l(qv - E) \\ &= -l\sqrt{E^2 - q^2c^2}, \end{aligned}$$

so that there is a pressure in the x direction. In the case of light waves travelling along x if we take d along y as before then h will be along z so that $\theta = 90^\circ$, $E = d^2$, $q = d^2/c$ and $h = d$. Hence

$$\begin{aligned} X_n' &= -l\sqrt{E^2 - q^2c^2} = 0, \\ Y_n' &= Y_n = 0, \\ Z_n' &= Z_n = 0. \end{aligned}$$

Thus in light there is no stress, as was shown by Cunningham, but simply the field moving through the ether carrying its energy and momentum with it.

This result at first sight appears to be inconsistent with the wave theory of light but the paradox is only apparent. The motion of the field through the ether must be supposed to take place by a process of growing in front and fading away behind of some form of modification of the ether. The modification being the presence of electric and magnetic fields. This is clearly the same thing as is usually meant by the wave theory. The calculation given above shows that the flux of energy in the electromagnetic field can be explained on mechanical principles as due to convection and the working of stresses in the moving field. By mechanical principles we mean the ordinary mechanical processes which are supposed to take place in moving matter. The electromagnetic theory is more fundamental than our ideas of mechanical processes so that when electromagnetic theory is interpreted mechanically we have to use the ideas of convection and motion through the ether derived from experience with matter in bulk instead of the more fundamental idea of wave motion or motion by a process of change of state in a stagnant medium.

The equation

$$v = \frac{E}{q} \pm \sqrt{\frac{E^2}{q^2} - c^2}$$

in the case when d and h are at right angles reduces to either $v = ch/d$ or $v = cd/h$. The proper value to take is presumably the one which is less than c .

When d and h are at right angles and h less than d the relation between v , h and d is the same as in the theory of J. J. Thomson, in which h is supposed to be due to the motion of the electric tubes of force. We may remark that J. J. Thomson's theory is especially successful when applied to cases in which d and h are at right angles and h less than d .

It appears that the flow of energy in an electromagnetic field can be explained on mechanical principles. The energy is transmitted partly by convection due to motion of the field through the ether and partly by the working of stresses in the moving field. The ether remains at rest and in fact does not enter into consideration at all. The momentum in the field is regarded as associated with the energy flux and the velocity of the field is along the direction of the momentum and the energy flux.

It is interesting to consider the flux of energy along a tube of energy flow in a steady field. Such a tube may be bounded by two neighboring equipotential surfaces of electrostatic potential and two equipotential surfaces of magnetic potential. If we consider a short length of such a tube then it will, in general, be curved. The flow of energy along it

and the associated momentum therefore change in direction as the field moves along the tube. There must therefore be a force on the tube at right angles to its length sufficient to produce the change in the direction of the momentum. The Maxwell stresses in a steady field give no resultant force on any element of volume free from charge. The stresses perpendicular to the tube are the Maxwell stresses but along the tube the true stress X' is less than the Maxwell stress X . We have seen that $X' = qv + X = qv - E$. The pressure in the direction of the tube of energy flow is therefore less than the Maxwell pressure by qv . There will therefore be a transverse force on the tube equal to qv/R per unit volume, where R is the radius of curvature of the tube. This force will be directed towards the center of curvature. This force is of just the right amount required to keep the flux of energy along the tube for it is equal to the momentum times the velocity divided by R just as the force required to keep a particle of mass m on a circular path is mv^2/R .

Consider again the case of a moving charged parallel plate condenser. First suppose it is moving in a direction perpendicular to its plates with velocity V . The energy flux between the plates and the velocity of the field are both zero. Thus the plate ahead continually generates new electric field behind it as it moves along and the plate behind continually absorbs the energy from the field. The energy flows from the back plate to the front plate through the insulating supports. The velocity of the field relative to the plates is $-V$ so that the field goes into one plate and comes out of the other.

If the condenser moves parallel to its plates with velocity V then $h = Vd/c$ so that the velocity v of the field is equal to

$$v = \frac{E}{q} - \sqrt{\frac{E^2}{q^2} - c^2} = V.$$

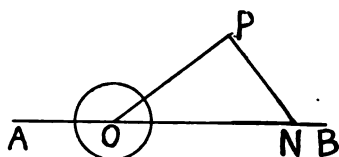


Fig. 1.

Thus the field moves along with the plates which seems a reasonable conclusion.

Now consider the case of a charged sphere moving with a small velocity V along a straight line AB . Let the center of the sphere be at O (Fig. 1)

at a point P if $OP = r$ and $\hat{POB} = \theta$ we have

$$d = \frac{e}{4\pi r^2}, \quad h = \frac{eV \sin \theta}{4\pi r^2 c},$$

so that the velocity of the field v is given by $v = ch/d = V \sin \theta$ and is directed along PN . Thus if ON represents V then PN represents v .

The velocity of the field relative to the sphere will be got by adding $-V$ to v so that it will be represented by PO . That is, the field to an observer on the sphere will appear to be coming in towards the center of the sphere with a velocity $V \cos \theta$. If θ is greater than $\pi/2$ the relative velocity is along the radius away from the sphere. Thus the field is absorbed by the sphere in front of it and emitted behind. The velocity of the field relative to the sphere at the surface of the sphere is equal and opposite to the velocity of the surface along the normal.

Next consider the case of a condenser consisting of two long concentric cylinders with radii a and b . Let the charge on the inside cylinder be e per cm. of length along the axis and suppose that the condenser is moving with a velocity V in the direction of its axis. There is then a convection current eV carried by the inner cylinder and a current $-eV$ carried by the outer cylinder. The magnetic intensity between the cylinders will be equal to $eV/2\pi rc$ at a distance r from the axis. The radial electric intensity will be equal to $e/2\pi r$. The velocity of the field is therefore, since d and h are perpendicular to each other and $h < d$, given by

$$v = c \times \frac{eV}{2\pi rc} \times \frac{2\pi r}{e} = V,$$

so that the field moves along with the condenser as was to be expected. The velocity v is equal to C/e , where $C = eV$ denotes the current. If ϕ denotes the potential difference between the cylinders then

$$e = 2\pi\phi/\log b/a$$

so that

$$v = \frac{C}{2\pi\phi} \log b/a.$$

In the case of two concentric cylindrical conductors with a current C flowing along the inner one and a current $-C$ along the outer one the magnetic and electric fields between the cylinders are the same as in the case of the moving condensers so the velocity of the field will be the same. The flow of energy between the cylinders is in either case equal to $C\phi$ so that the momentum in the field is equal to $C\phi/c^2$ per unit length along the axis. If V is small so that the magnetic energy can be neglected compared with the electric then since the electrostatic energy per cm.

is equal to $\frac{1}{2}e\phi$ the flow of energy due to convection is equal to $\frac{1}{2}e\phi V$ or $\frac{1}{2}C\phi$ so that half the energy is convected and half transmitted by the working of the stress in the field.

In the case of the condenser there is a backward flow of energy in the cylinders equal to $\frac{1}{2}e\phi V$ due to the tension along them so that the total flux of energy along the axis is $\frac{1}{2}e\phi V$ as it evidently must be.

In the case of the concentric conductors at rest there is no backward flow and the total flow is $C\phi$. In this case the velocity of the center of mass of the energy is evidently $lC\phi/\frac{1}{2}e\phi l = 2C/e$ where l is the length of the conductors so that the momentum due to energy flux is given by

$$\frac{E\bar{v}}{c^2} = \frac{2C}{e} \times \frac{l\frac{1}{2}e\phi}{c^2} = \frac{C\phi l}{c^2}$$

or $C\phi/c^2$ per unit length as before. In the case of the condenser the momentum is only $\frac{1}{2}C\phi l/c^2$ because half the energy flows back along the cylinders.

When the current is first started in the conductors they will receive an impulse in the negative direction due to the generation of this momentum in the field. This impulse is due to the field existing at the end where the current is started before it gets to the other end. The pressure due to the field therefore acts at one end for a short time before it is balanced by the equal pressure at the other end.

An analogous case in mechanics can be easily imagined. Suppose we have a long closed box with a machine gun in the box at one end. If the gun starts firing along the box the box will receive an impulse which will last until the bullets begin to strike the other end. The box will therefore move backwards if it is free to move as long as the gun continues to fire and will be stopped shortly after the gun stops firing.

Consider now the case of a condenser consisting of two long concentric cylinders when a uniform magnetic field is generated parallel to the axis. The momentum per unit volume between the cylinders in this case is $he/2\pi rc$ so that the total angular momentum in the field per unit length along the axis is $(b^2 - a^2)he/2c$ where b and a are the radii of the cylinders. While the field is being generated the cylinders are acted on by a couple due to the induced electric intensity which gives them angular momentum equal but opposite to that generated in the field.

This case offers a possibility of testing the theory of electromagnetic momentum experimentally. A cylindrical condenser with its axis vertical could be suspended by a fibre so that it could rotate. The outside cylinder could be closed at each end and the inner one given a static charge. The condenser could be suspended inside a solenoid to produce

the magnetic field. The condenser should be deflected when the magnetic field is produced. If however the force $(1/c)(d/dt)[d \cdot h]$ is transmitted through the field to the charges on the cylinders instead of generating momentum in the field then no deflection would be obtained. The effect to be expected in such an experiment is very small but might possibly be detected with a well designed apparatus.

If we regard all momentum as due to energy flux then we might expect all energy to be subject to gravitational attraction. This idea has been confirmed by the recent discovery of the deflection of light when passing by the sun. The momentum M in the light is finite so that the transverse mass which is equal at any velocity to M/v is also finite but the longitudinal mass is equal to $M(1 - v^2/c^2)^{-1}$ and so should be infinite for light. This means that a force acting on light in the direction of motion should not increase the velocity of the light but should merely increase the momentum and energy in the same proportion so that $v = Mc^2/\mathcal{E}$ remains constant. We should therefore expect the gravitational force of the sun to change the direction of light but not its velocity. Of course Einstein's theory gives a change of velocity but it is based on the idea that the gravitational field distorts space.

If energy is subject to gravitation then various effects must exist which are unfortunately probably too small to be detected. For example suppose electrical energy is transmitted from a battery through a wire of small resistance to a higher level where it is converted into heat in a resistance R . Let C denote the current and E the electromotive force of the battery. Then we have

$$EC = C^2R + C^2Rgh/c^2,$$

where g denotes the acceleration due to gravity and h the difference of level. Thus the resistance will apparently be $R(1 + gh/c^2)$ instead of R . If $h = 10,000$ meters the increase in R is about one in 10^{12} .

Again in a circuit composed of two metals if one junction is at a higher level than the other we should expect a small electromotive force even when the two junctions are at the same temperature. This electromotive force should be approximately equal to Pgh/c^2 where P denotes the Peltier coefficient.

The work done against gravity in transmitting 10,000 kilowatts up 10,000 meters is about 100 ergs per second.

THE RICE INSTITUTE,
HOUSTON, TEXAS,
February, 1920.

QUANTUM EMISSION PHENOMENA IN RADIATION.

BY DAVID L. WEBSTER.

SYNOPSIS.

Comparison of X-rays and Light; a Summary.—This covers the best known cases of excitation of line spectra by electron impact, and the form of the Bohr theory, that is required by them.

Comparison of Emission and Absorption.—The above phenomena are compared with corresponding absorption phenomena, with especial reference to the accumulation of energy for photoelectrons by absorption.

Deductions.—Theories of the type of Bohr's appears inconsistent with these facts. The phenomena suggest that the law of the conservation of energy, as applied to atomic oscillators, holds only statistically. A set of postulates to replace it for individual oscillators is outlined.

COMPARISON OF X-RAYS AND LIGHT.

THE purpose of this paper is to compare the quantum phenomena in X-rays and light and to draw conclusions from them on the laws governing the emitting mechanism. Let us consider first the simpler

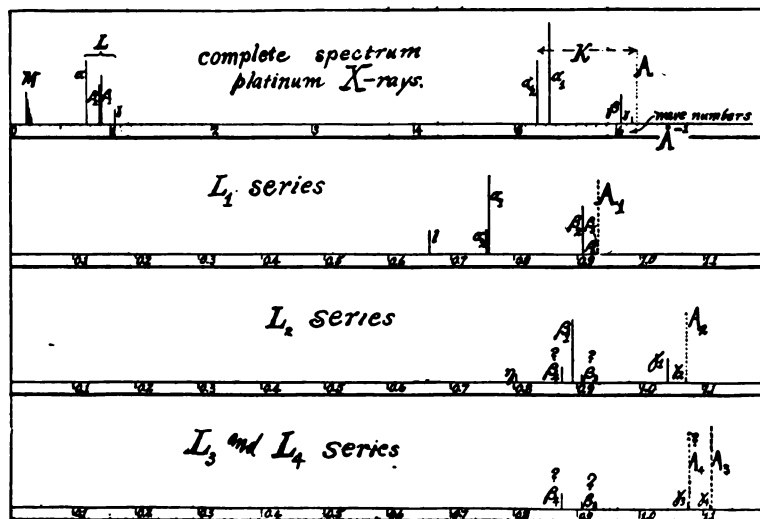


Fig. 1.

case of X-rays. In Fig. 1, we have the spectrum of platinum, on a scale of wave numbers, showing the K, L and M series. In the K series we

have the strong α lines, the weaker β and γ lines and the absorption limit A . This point A is especially important, because waves of a higher frequency than A are absorbed by the element much more strongly than those of lower frequency, and also the higher frequency waves give rise to a fluorescence consisting of the K series emission lines. If now we subject a piece of platinum to cathode rays, we should find no K series emission from it unless the energy of a single cathode ray is as large as a quantum of the frequency A . (This has not been tested directly for platinum, but has been for other elements,¹ and we may be sure they all act the same.) For any higher value of the cathode ray energy, we

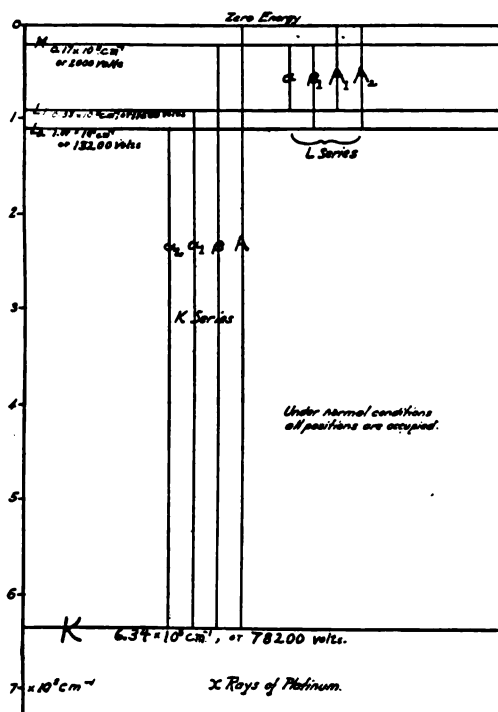


Fig. 2a.

have all the K series lines emitted with constant intensity ratios under all conditions. Evidently the mechanism used in the production of K series rays by cathode rays is such as to demand the same energy quantum as that used in fluorescence, and presumably it is the same mechanism.²

In the L series we have a similar state of affairs, except that the series must be divided into three or four sub-series, each with its own absorption

¹ For rhodium see D. L. Webster, *PHYS. REV.*, 7, 599-613, June, 1916. For molybdenum and palladium see B. A. Wooten, *PHYS. REV.*, 13, 71-86, Jan., 1919.

² D. L. Webster, *l.c.*

limit and corresponding critical potential. The exact status of the series marked L_3 and L_4 in the diagram is not settled, but L_1 and L_2 are definitely known to behave exactly like the K series as shown by evidence described elsewhere.¹ In every case the energy required is represented by many thousands of volts.

In light, however, conditions are different. For example, in sodium vapor, studied by Tate and Foote,² if the energy of the bombarding electrons is gradually increased there is no radiation till we reach 2.1

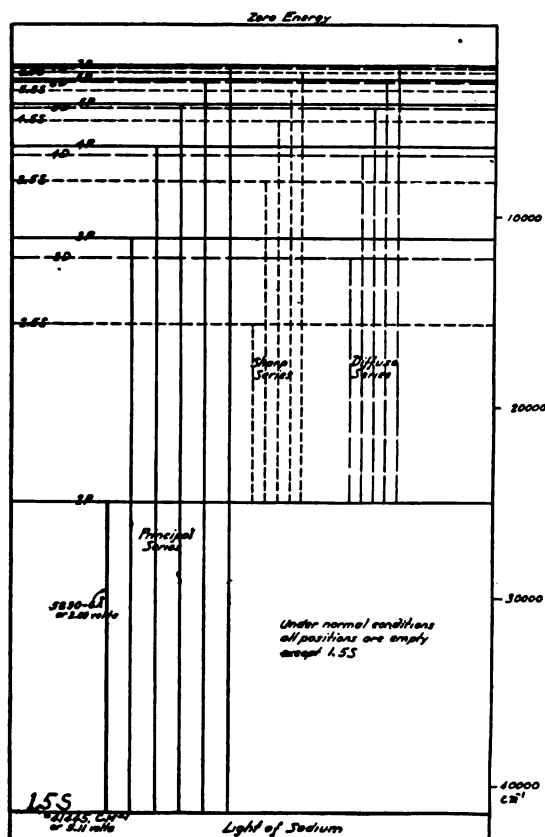


Fig. 2b.

volts, when the well-known D lines appear, alone, as a so-called "single-line spectrum." Now 2.1 volts corresponds exactly to the quantum of their frequency. But they are the first lines of a series, like the $K \alpha$ lines; and the limit of the series, corresponding to the point A , is at a

¹ D. L. Webster and H. Clark, Proc. Nat. Acad., 3, 181-5, March, 1917, and D. L. Webster, *ibid.*, Jan., 1920.

² J. T. Tate and P. D. Foote, Phil. Mag., 36, 64-75, July, 1918.

frequency more than twice as high. The D lines are the lines $1.5S-2P$, and the limit, $1.5S$, corresponds to a voltage of 5.1 volts. When this voltage is reached, we get the whole series $1.5S-mP$, and with it the subordinate series, some of whose lines have lower frequencies than the D lines. The important difference from the X-ray case is the existence of the so-called "single-line spectrum," at voltages between 2.1 and 5.1 volts. Such a phenomenon is unknown in X-rays.

Now this difference between the two cases is readily explained by the diagram in Fig. 2. According to Kossel's¹ modification of the Bohr theory, the K series is produced by removing an electron from the innermost ring, called the K ring, and allowing one to fall into its place from another ring, the L , M or N . While I think there is ample evidence against the idea that the Bohr rings actually exist, it may be that the stable positions that do exist will act in much the same way. I have therefore drawn the diagram giving the K position at the bottom, indicating that a potential of 78,200 volts is required to lift the electron from it to the surface of the atom, where its energy would be zero. The energy required to do this is the quantum of the absorption limit, A , of the K series. Similarly the energies required to lift an electron from the L_1 and L_2 positions are the quanta of the absorption limits A_1 and A_2 , respectively; and the $K\alpha$ lines are produced by removing an electron from the K position to infinity and replacing it by one falling from L_1 or L_2 . By such considerations Bohr² predicted the fact that the critical potential for emission of the K series would be that which gives an electron a quantum of the frequency A , and Kossel, in 1914, predicted the relation $\nu_{K\alpha} = \nu_{KA} - \nu_{LA}$ which he tested as accurately as he could, and he also predicted some relations between emission line frequencies. An essential point for this explanation of the critical potential by Bohr and Kossel is that the electron removed from the K position *must* go to infinity, and cannot be allowed to come to rest in the L or M positions. That is, *in the normal atom all positions involved in X-ray processes are full.*

In light, on the other hand, we have a set of stable positions, shown in Fig. 2 as $1.5S$, $2P$, $2.5S$, $3P$, $3D$, etc., and the process of exciting the single line spectrum is evidently to lift an electron from $1.5S$ to $2P$ and let it fall back. If the electron is lifted to infinity, or zero energy, we shall excite the whole system, including the subordinate series. As van der Bijl³ has suggested, the existence of the "single-line spectrum" means simply that *in the normal atom no position is filled above $1.5S$.*

¹ W. Kossel, Verh. d. D. Phys. Ges., Nov. 30, 1914.

² N. Bohr, Phil. Mag., 26, 1-25, July, 1913.

³ H. J. van der Bijl, Phys. Rev., 10, 546-556, Nov., 1917. Van der Bijl does not call it the $1.5S$ ring, but describes it in terms equivalent to that.

In mercury, which has been studied by Frank and Hertz, McLennan and Henderson, Davis and Goucher and others, we have a more complex state of affairs indicated in Fig. 3. The lines $1.5S-2p_1$ and $1.5S-2P$ both appear at the potentials given by the quantum law, applied to their

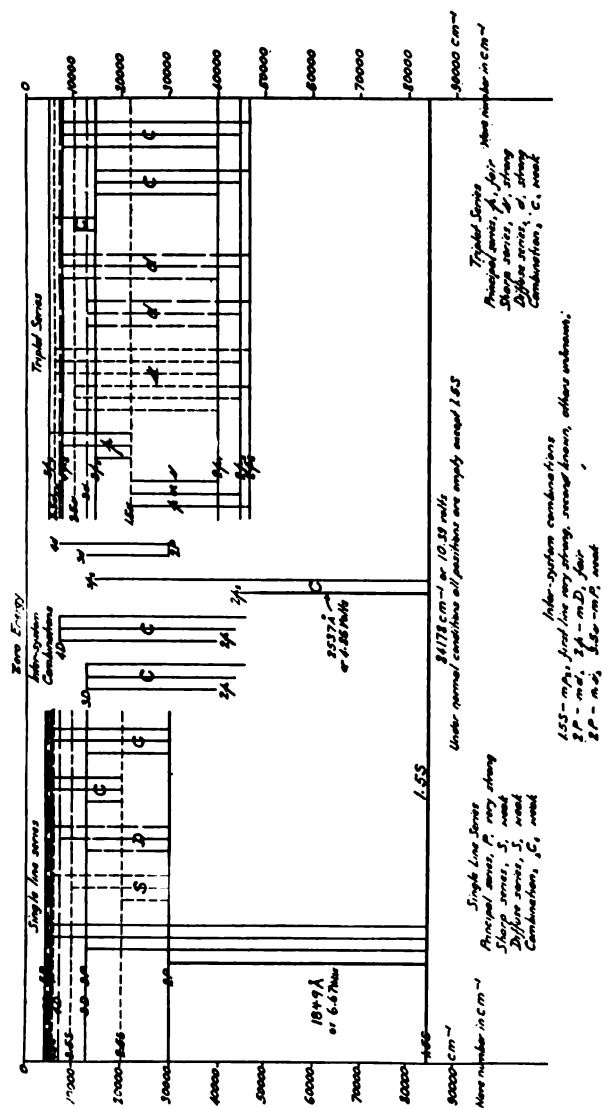


Fig. 3.

respective frequencies, and the whole spectrum appears at the potential given by the quantum of $1.5S$. Again the explanation is van der Bijl's hypothesis that $1.5S$ is the only position that is normally full. In calcium also, effects exactly like those in mercury have recently been

observed by Mohler, Foote and Stimson,¹ and the explanation is again van der Bijl's hypothesis. These phenomena in light show clearly the existence of empty positions of stability, and if such empty positions were present in the X-ray mechanism they ought to show in the same way, by a "single-line spectrum" in each X-ray series. The fact that such a single line spectrum fails to occur is therefore strong evidence against any theory of X-rays that assumes empty positions of stability in the normal atom below the 1.5S position, as several of the recent theories do.

COMPARISON OF EMISSION AND ABSORPTION.

So far the theory is good, and these elements are typical of the groups whose quantum emission phenomena are best known. When we apply the theory to absorption, referring again to Fig. 2, we see to some extent why it is possible to have sodium vapor absorb the *D* lines, and lift its electron to 2*P*, and why it is impossible for any element to show selective absorption for any of its X-ray emission lines. Instead, it must absorb rays above the frequency *A*, since a quantum of such a frequency is required to remove the electron to a place where it can stay. But then the question arises, why can not the same process occur in light, and why does not sodium vapor absorb not only the *D* lines but also all frequencies above 1.5*S*, and give a fluorescent spectrum under such conditions similar to X-ray fluorescence? Applied to absorption, this type of theory is most unsatisfactory.

A more fundamental difficulty appears when we consider the nature of the absorption process. The electron must in some way collect energy enough from the X-rays to appear as a photoelectron. This, according to Barkla,² means the amount $h(\nu + \nu_A)$, where ν is the frequency of the rays absorbed and ν_A is that of the absorption limit. This conclusion is drawn from the observed fractions of the total X-ray energy that appear in photoelectrons and in fluorescent X-rays and the fact that the photoelectrons all appear to have the same energy, $h\nu$. While the evidence on this latter point does not seem quite conclusive, it is significant that the energy $h(\nu + \nu_A)$ is exactly the amount suggested by the Bohr theory, although Barkla's evidence was not drawn at all from this theory. I say "suggested," rather than "required," because it is not certain that the electron could not be helped out of the atom by another electron falling in from an outer stable position as the first one goes out, thus neutralizing the force that would otherwise restrain the one that leaves.

¹ F. L. Mohler, P. D. Foote and H. F. Stimson, Bull. Bur. Stan., 1920. Abstract in *PHYS. REV.*, 14, 534, Dec., 1919.

² C. G. Barkla, Proc. Roy. Soc., A.92, 501-4, Aug., 1916.

However that may be, the electron leaves with energy of the order of $h\nu$ and must collect from somewhere a quantity of this order or perhaps $h(\nu + \nu_A)$. Millikan¹ has recently described experiments leading to conclusions of the same general type for photoelectrons produced by light.

Now where does the electron get this energy? If in light or X-rays the energy is all concentrated in a point of the wave front, the electron might get at once as much as the energy $h\nu$ of the primary cathode ray, but no more. But energy is the ability to do work; and to produce coördinated motions of electrons in different parts of a crystal, a train of waves of X-rays must have this ability at points widely scattered over the wave front and perpendicular to it. And in a reflected or scattered train of waves the whole energy is very much less than the quantum emitted from the primary radiator, according to any reasonable theory of these phenomena, so that the amount of energy coming within reach of any one atom exposed to this beam must be correspondingly reduced. In light, as shown by the diffraction pattern at the focus of a high-power microscope objective, the ability to do work must be distributed over practically a whole hemisphere of the wave front if not more, thus indicating a distribution of energy in light even wider than any we have definite evidence of in X-rays. It would be most unreasonable to assume that X-ray waves are not constructed like those of light, and probably the energy is distributed as widely in one case as in the other. If now the X-rays have passed through a slit, the wave train thus limited contains but a small fraction of a quantum, and if they are then reflected from a crystal it contains even less. The same is true of light subjected to such treatment. But in either case the photoelectron has as much energy when produced by the weakest beam as when produced by the strongest. And in X-rays photoelectrons can be obtained easily, as shown by ionization methods, when the heads of the successive wave trains are as far apart, on the average, as many millions of wave-lengths. One might perhaps attempt to explain the accumulation of energy by the coincidence of a large number of wave trains. But it is well known that this would not lead to a photoelectric current proportional to the intensity of the rays; and so it is evident that the absorption of energy by the photoelectron is not dependent upon coincidence. If not, then it must be by a process of gradual accumulation. According to the electromagnetic theory it would be very gradual indeed and, even in a strong monochromatic beam a resonating electron should take many weeks to acquire the energy $h\nu$. But aside from such theoretical considerations, the practical reasons given here seem sufficient to show the gradual nature of the process.

¹ R. A. Millikan, Am. Phys. Soc. meeting, Dec. 31, 1919.

Now let us suppose an electron from the *K* ring is absorbing X-rays and has lifted itself half way to the position of zero energy, and then is struck by a cathode ray. One would expect that if the cathode ray has only energy enough to lift the electron the rest of the way to the surface, it will produce the *K* series lines. But no. It must have a whole quantum of the critical frequency and no less. The same difficulty appears also in light, as one may see by reference to the phenomena described above.

In such a theory as I have just described there seems to be no way to distinguish the energy stored by absorption from the energy acquired from a cathode ray, and prevent them from being combined as suggested above. In a theory of heat radiation that I proposed in 1915,¹ based on Parson's magneton theory, the distinction might be made, because the energy accumulated by absorption was stored in a rotary form not readily affected by the impact of a cathode ray. But even here, there is some difficulty. The X-ray absorption spectrum is continuous, and it is hard to see how we can avoid the assumption that the natural absorption frequency of an electron is different at different times. Certainly it is not fixed by Bohr's energy considerations, because a larger quantum than $h\nu_A$ could not be collected continuously: the electron would escape as soon as it has the energy $h\nu_A$. Some other determining factor must be present. In light also, in solids and liquids, especially metals with free electrons, there must be continual changes and re-adjustments of the electron's frequency.

If now such a change of frequency occurs when an electron has nearly finished collecting a quantum, and the change is to a lower frequency, for which the energy already stored is more than a quantum, one would expect photoelectric action even if no rays happen to be falling on the electron when the change occurs. In this or some such way, one would expect some evidence of the stored energy to appear. Thus the magneton hypothesis, while distinctly better than the other, is still unsatisfactory, because evidence of stored energy fails to appear. The stored energy causes trouble, and perhaps more trouble than it is worth.

After all, what is it worth? As Poincaré² has said, every time we deal with a new type of phenomenon, we find or invent a new quantity that we can call energy, and define it so as to make the total energy of the system a constant. As both he and Ritz³ have said, the law of the conservation of energy is not a law, but a postulate. Some time, a phe-

¹ D. L. Webster, *Proc. Amer. Acad.*, 50, 131-145, Jan., 1915; see also *PHYS. REV.*, 8, 66-9, July, 1916.

² H. Poincaré, *Science and Hypothesis*, Paris, 1901, Chapter VIII.

³ W. Ritz, *Ann. Chim. Phys.*, XIII, 145-275, or *Collected Works*, Paris, 1911, p. 345.

nomenon may arise where this postulate is not advantageous in its most complete form. It seems to me that this time has come. Explanations of phenomena are important, but a postulate is not.

To preserve our explanations of refraction and other classical phenomena of physical optics, let us assume as usual that the oscillator is governed by an equation of the form

$$m\ddot{x} + g\dot{x} + fx = eE_x,$$

x being the displacement of the electron from its equilibrium position and E_x the external electric force in the x direction. As Ritz suggested, a vibration controlled by a magnetic field or some similar agency is more probable than the elastic force fx , but as this is the usual form of dispersion equation, we may use it here. Any magnetic or similar vibration could be substituted without changing our present conclusions. This equation gives absorption at a rate $g\dot{x}^2$, the mean value of which through a whole period is $\frac{g}{m} U$ where U is the energy of the oscillation. Now, instead of assuming this absorbed energy to be stored somewhere, and reëmitted some time later, *let us assume it to be simply annihilated.* Then let us assume that the electron may at any time start an emitting oscillation of large amplitude, continuing uniformly until a whole quantum is radiated, or else emit a photoelectron with a quantum of energy. And let us assume that the probability of starting such an oscillation or photoelectron during a time dt is

$$\frac{g}{m} \frac{U}{Q} dt,$$

where Q is the "accumulated energy" defined above, which may be $h\nu$ or may be something larger. Further let us assume that transfers of energy either to or from cathode rays or other colliding particles can take place, but by quanta only.

That these hypotheses give a mathematical theory of dispersion exactly like the classical theory is evident from the fact that the dispersion theory is not concerned with what becomes of the power $g\dot{x}^2$ after it is taken from the electron by the damping force.

To make them give a theory of heat radiation we need only an entropy condition for the oscillator. To get this we may use almost the same condition as in my heat radiation theory, referred to above, which was modelled to a great extent on Planck's. If $Q = h\nu$, the behavior of an oscillator in a given field of radiation is almost equivalent to what is assumed there, because the rate of absorption and mean rate of emission

are the same. To make them exactly equivalent, we have only to assume that an oscillator newly formed in an atomic readjustment will behave almost exactly like the one of the other theory, which absorbed energy continuously and stored it, radiating by quanta or multiples of quanta. The only difference is that in starting its career with its new frequency it must not start at the zero of the scale of absorbed energy, but at any point on it, the probability of starting at any one point being proportional to the density of oscillators on the scale of energy at that point; and when it changes frequency again it must not be considered to have a store of energy in it. In other words, the stored energy of the previous theory must have been merely a mathematical fiction and should not have been assumed to have any real physical existence.

If the energy Q is greater than $h\nu$ we have a case of fluorescence, the energy $h\nu$ appearing in a photoelectron and the rest as fluorescent rays. In this case we get into the question of the thermodynamics of fluorescent bodies, which needs a long discussion. In the absence of very definite evidence on the exact value of Q , as suggested above, the time does not seem ripe for such a theory.

SUMMARY.

We may summarize these conclusions as follows: The simple Bohr theory is good for explaining the phenomena of excitation of X-rays and light by impact, if we assume that in the normal atom all rings are full from the K ring to the $1.5S$ ring, inclusive, but none outside $1.5S$. But the theory is quite unsatisfactory for absorption phenomena. Even if the absorbing electron is not a member of a Bohr ring, but a "magneton" of the type assumed in my previous theory, situated in a stable position similar to a Bohr ring, and having the required resonance frequency, there are still some difficulties when the frequency is subject to change with time. Consequently it seems better to assume that in postulating the existence of stored energy in the oscillator we have carried the postulate of conservation of energy a step too far. We had better abandon it at this point and postulate a system of equations that give the conservation of energy as a statistical effect only and preserve the explanations of dispersion and heat radiation intact, at the same time explaining the phenomena of excitation of radiation by impact.

ROGERS LABORATORY OF PHYSICS,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, MASS.

IONIZATION POTENTIALS OF ARGON, NITROGEN, CARBON MONOXIDE, HELIUM, HYDROGEN AND MERCURY AND IODINE VAPORS.

BY CLIFTON G. FOUND.

SYNOPSIS.

Measurement of the Ionization potential of Gases was accomplished by means of a two electrode tube, by determining the point on the current-voltage curve at which the current increases at a rate faster than that given by Langmuir's equation

$$i = A(V + V_0)^{3/2}.$$

The effect of a voltage drop along the cathode was eliminated by a commutator arrangement which broke the filament heating circuit while the electron current was being measured.

The initial velocity of the electrons, V_0 , was determined directly from the current-voltage curve.

Ionization potentials were obtained by measurement as follows: Argon 15.6, nitrogen 15.8, carbon monoxide 15.0, hydrogen 15.1, helium 20.5, mercury vapor 10.1, iodine vapor 8.5. The ionization potential of *argon* was found to be constant for pressures between 1 and 200 bars.

INTRODUCTION.

IT has been shown by Langmuir¹ that for a pure electron discharge, when the electrons start from the cathode with zero velocity, the maximum current which will pass to the anode is given by

$$i = AV^{3/2}, \quad (1)$$

where V is the voltage on the anode and A is a constant, depending only on the geometry of the tube.

The limitation of current is caused by space charge or the negative electrostatic field due to the electrons in the space between the electrodes. If in any way the effect of this negative space charge is neutralized, the current to the anode will increase with the voltage faster than the rate given by equation 1. One way of neutralizing space charge is by the presence of positive ions. Hence, if an electron tube contains gas, as soon as positive ions are produced, the current increases more rapidly with increase of voltage, causing a point of discontinuity in the volt-ampere characteristic curve.

The voltage at which this kink occurs is a measure of the ionization potential of the contained gas.

¹ PHYSICAL REVIEW, Vol. 11, No. 6, Dec., 1913.

mutator is so great that there is no appreciable cooling of the filament during the intervals in which the heating current is off. The voltage of the anode and the anode current were measured on direct current instruments. Since these were on only a portion of the time, the readings obtained were average values and in order to obtain effective values, it is necessary to multiply the observed values by a constant—corresponding to the reciprocal of the fraction of the time during which current flowed. This constant was determined by taking the ratio of the voltage with commutator stopped, to the voltage reading with commutator running.

In all cases, the values given in the tables are effective values, obtained from the observed values by multiplying by the commutator constant. This varied from 2.18 to 2.22. The variation was due to resetting of the commutator brushes and in no case was a variation found during any series of measurements.

The electron tube which was used for these measurements consisted of two tungsten filaments, each wound in the form of a double helix, and molybdenum cylinder about 12 mm. diameter and 12 mm. long. The helices and cylinder were arranged coaxially. The inner helix, which was 5 turns of 0.125 mm. wire wound on a 2.25 mm. mandrel, was used as cathode. The outer helix, which had 3 turns of 0.125 mm. wire wound on 3.65 mm. mandrel, was connected to the molybdenum cylinder and the combination was made the anode. This arrangement has the advantage that the electrons travel from the outer helix to the cylinder with a uniform velocity equivalent to the anode voltage so that when ionization takes place a larger number of positive ions is formed than if the outer helix were not present.

Since most of the positive ions travel to the cathode, a larger number is present in the region between the helices than would be present if the electrons did not travel over a large portion of their path with the maximum velocity corresponding to the anode voltage.

CALCULATION OF IONIZATION POTENTIAL.

To determine the ionization potential of a gas, the volt-ampere characteristic of the tube was first taken with a good vacuum (about .001 bars), while it was still connected to a Langmuir condensation pump. The tube was then shut off from the pump by means of a mercury trap separated from the former by a liquid air trap and a known pressure of the gas let in. The characteristics were then taken again with the gas present.

The value of V_0 was calculated from equation (2). If i_1 and i_2 are the electron currents at two voltages V_1 and V_2 respectively, below the

The first vertical column gives the effective values of anode voltage raised to the three-halves power. Table II. gives the difference between the electron current in a good vacuum and that when gas is present.

Figure 2 gives the plot between the electron current and the effective

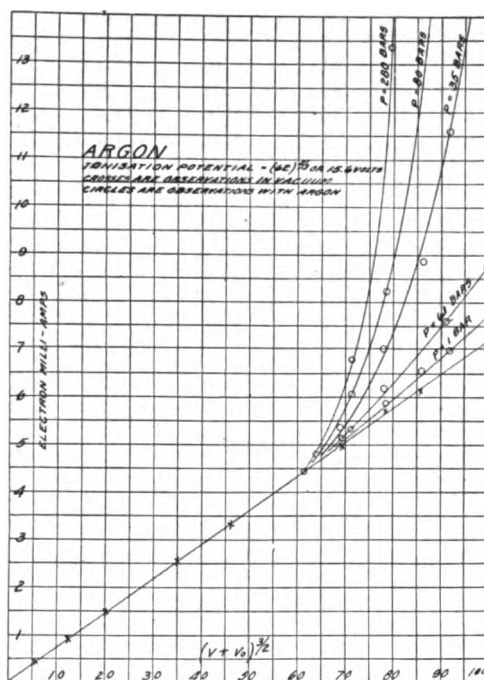


Fig. 2.

voltage raised to the three-halves power. It will be noted that the curve for a good vacuum is linear throughout the entire range while the curves when gas is present are linear only up to a certain point, above which the current increases faster than the linear relation. The curves for pressures from 1 to 280 bars apparently begin to depart from a straight line at the same point and the rate of departure increases with the pressure. At the lower pressures, it is difficult to tell the exact point at which the departure commences, but at the higher pressures the curve when gas is present meets that for a good vacuum at a sharp angle and there is no doubt where the effect of ionization sets in. For this reason, a high pressure of gas was used to determine the ionization potentials of the other gases.

Table III. contains the results for nitrogen. The first column gives the three-halves power of the effective voltage. The second and third

give the corresponding values of the electron current with gas present and in a good vacuum, while the last column gives the difference between columns two and three. Figure 3 shows plot of results with nitrogen.

TABLE III.

Nitrogen.

P (Bars)..... V_0 (Volts)..... K	300 1.6 1.14	.001 1.1 1.00	
$(V + V_0)^{3/2}$	Ki	i	Δi
4.45	.35	-.31	.04
10.9	.76	.74	.02
18.8	1.36	1.29	.07
28.0	1.96	1.92	.04
38.6	2.65	2.68	-.03
50.5	3.40	3.43	-.03
53	3.60	3.64	-.04
56	4.00	3.84	.16
59.5	4.20	4.08	.12
63	4.45	4.32	.13
66	5.10	4.52	.58
69	6.50	4.74	1.76
73	8.40	5.00	3.40
76	11.6	5.20	6.40

Ionization potential = $(63)^{2/3}$ or 15.8 volts.

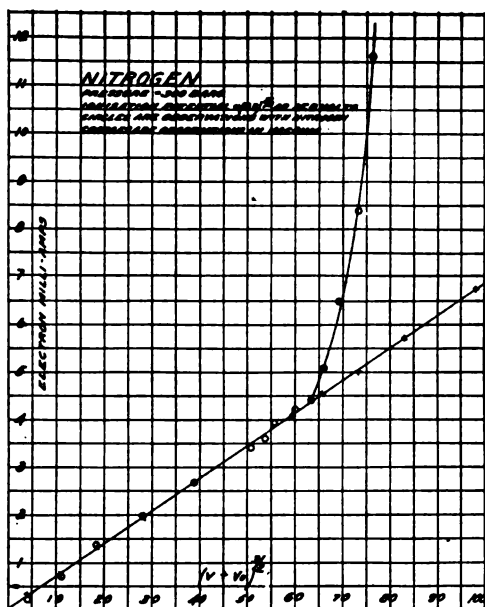


Fig. 3.

Tables IV., V., VI., VII., VIII. give the results for carbon monoxide, hydrogen, helium, mercury vapor and iodine vapor respectively, while Figs. 4, 5, 6, 7 and 8 give the corresponding curves.

TABLE IV.
Carbon Monoxide.

P (Bars)..... V_0 (Volts)..... K	200 2.2 1.03	.001 1.1 1.00	
$(V+V_0)^{1/2}$	Ki	i	Δi
6.0	.51	.57	-.06
12.9	1.14	1.19	-.05
21.5	1.90	1.96	-.06
31	2.80	2.80	.00
42	3.65	3.68	-.03
51	4.55	4.55	.00
54	4.85	4.85	.00
58	5.10	5.10	.00
61	6.00	5.40	.60
64	8.20	5.55	2.65
67	12.00	5.82	6.18
74	20.6	6.60	14.0
81	32.0	7.38	24.6

Ionization potential = $(58)^{2/3}$ or 15. volts.

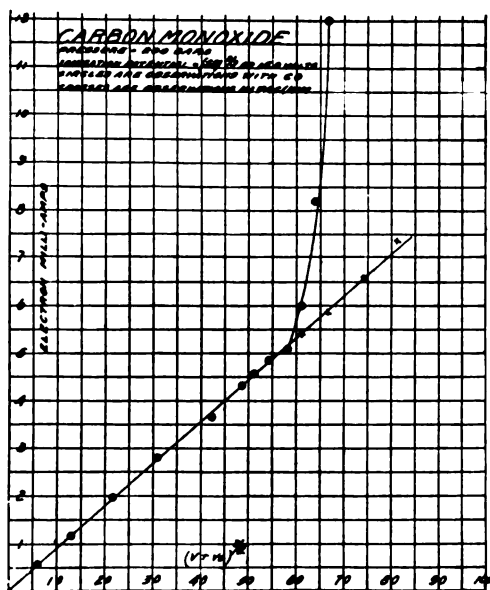


Fig. 4.

TABLE V.

Hydrogen.

P (Bars)	135	.001	
V_0 (Volts)	2.5	1.1	
K	1.2	1.00	
$(V+V_0)^{2/3}$	Ki	i	Δi
6.8	.38	.48	.10
13.7	1.00	.95	-.05
22.0	1.58	1.52	.06
31.5	2.23	2.18	.05
40	2.75	2.75	.00
48	3.35	3.32	.03
54	3.78	3.74	.04
57	4.01	3.96	.05
60	4.22	4.17	.05
63	4.45	4.38	.07
65	4.70	4.52	.18
68	5.10	4.72	.38
73	5.63	5.06	.57
86	8.00	5.92	2.08

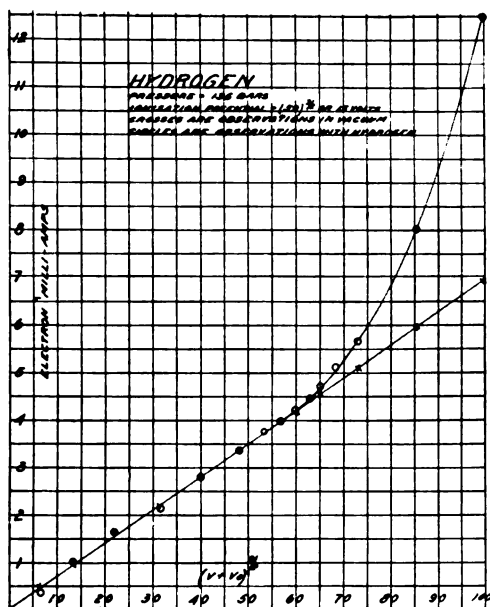
Ionization potential = $(59)^{2/3}$ or 15.1 volts.

Fig. 5.

TABLE VI.

Helium.

P (Bars)..... V_0 (Volts)..... K	650 2.2 1.60	Vacuum 2.2 1.00	
$(V + V_0)^{1/2}$	Ki	i	Δi
6	.123	.176	-.53
13	.220	.275	-.55
21.5	.358	.385	-.27
31.1	.520	.530	-.10
42	.700	.700	.00
54	.91	.900	.10
67	1.12	1.12	.00
74	1.28	1.23	.05
81	1.47	1.36	.11
88	1.68	1.50	.18
96	1.95	1.60	.35
103	2.47	1.74	.73
112	3.10	1.85	1.25
120	3.68	1.96	1.72

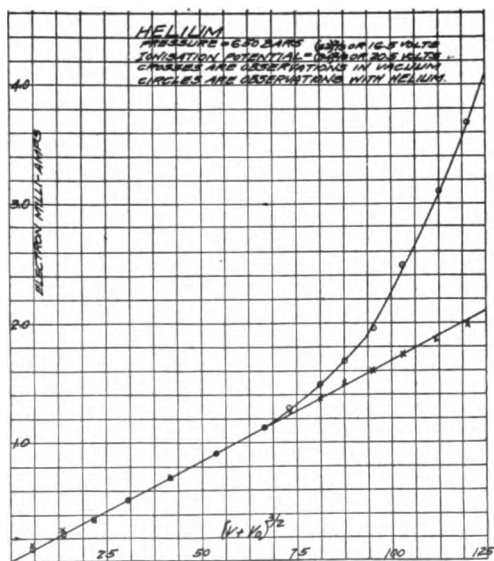
Ionization potential = $(94)^{2/3}$ or 20.5.

Fig. 6.

TABLE VII.

Mercury Vapor.

P (Bars)..... V_0 (Volts)..... A	2.0 2.75 2.0	.001 2.2 1.0	
$(V + V_0)^{1/2}$	Ki	i	Δi
4.6	.84	.84	.00
11.2	2.15	2.15	.00
15	2.75	2.85	-.10
19.4	3.63	3.74	-.11
24.0	4.45	4.55	-.10
26.5	5.20	5.20	.00
29	5.80	5.60	.20
32	6.20	6.10	.10
33	6.65	6.40	.25
34	7.20	6.50	.70
35	8.20	6.70	1.50
36	9.50	6.85	2.65
37	11.0	7.09	3.91

Ionization potential = $(32)^{1/2}$ or 10.1 volts.

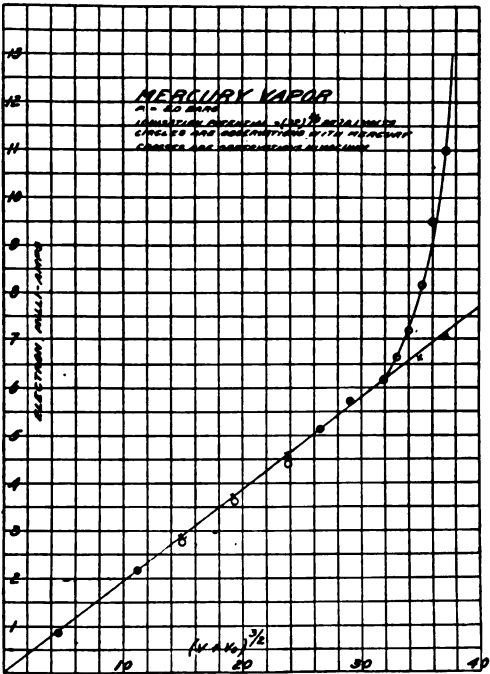


Fig. 7.

TABLE VIII.

Iodine Vapor.

P (Bars)	40	.001	
V_0 (Volts)	.35	1.6	
K	2.5	1.0	
$(V + V_0)^{1/2}$	Ki	i	Δi
4.3	.35	.32	.03
10.9	.78	.77	.01
19.4	1.44	1.45	-.01
24.2	1.80	1.80	.00
25.0	1.90	1.87	.03
26.1	2.00	1.95	.05
27.2	2.10	2.03	.07
28.2	2.30	2.10	.20
29.5	2.40	2.25	.15
30.5	2.54	2.32	.22
31.5	2.77	2.40	.37
32.6	3.25	2.48	.77
33.5	3.75	2.57	1.18
37.0	5.25	2.80	2.45

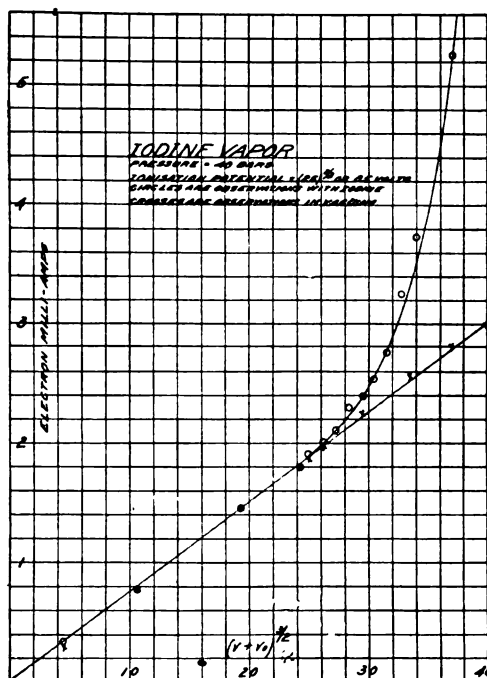
Ionization potential = $(25)^{2/3}$ or 8.5 volts.

Fig. 8.

The curve for helium shows two kinks, the first one at 16.5 volts and the second at about 20 volts. One explanation that might be given for these results is that helium has two types of ionization, a weak one occurring at 16.5 volts and a more intense one at 20 volts. Since no other experimenter has found ionization in helium as low as 16.5 volts, it seems more probable that this value is due to a slight impurity in the helium, although the latter was purified by passing through charcoal at the temperature of liquid air. The helium used was known to contain a slight admixture of neon, which would not be removed, as it passed through the charcoal at the temperature of liquid air. Since it would require only about 6 bars or 1 per cent. of neon to account for the magnitude of the kink at 16.5 volts, it seems very probable that this kink is due to ionization of the neon content, while the ionization potential of helium is given by the second kink at 20.5 volts.

SUMMARY.

1. It has been shown that the relation

$$i = A(V + V_0)^{3/2}$$

holds for the electron tube used in these experiments when no gas is present.

2. When gas is present, the above relation holds up to a certain voltage corresponding to the ionization potential of the gas, beyond which the current increases at a faster rate than given by the equation.

3. From the location of the point of departure from the above relation, the ionization potentials given in Table IX. were determined.

TABLE IX.

Gas.	Ionization Potential.
Argon	15.6
Nitrogen.....	15.8
Carbon Monoxide.....	15.0
Hydrogen.....	15.1
Helium.....	20.5
Mercury vapor	10.1
Iodine vapor	8.5

In conclusion, the writer wishes to express his appreciation of the kindly interest taken in these experiments by Dr. Langmuir, whose suggestions proved most valuable.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

NEW FACTS ABOUT SURFACE-FRICTION.

BY CH. TERZAGHI.

SYNOPSIS.

Friction between Smooth Surfaces: Physical Properties of Clays.—In the present paper the author discusses a number of preliminary experiments leading to an investigation of the physical properties of clays. Since plastic deformations of clay-bodies involve the sliding of the smooth clay particles upon each other, in order to determine the character and the laws governing the frictional forces, the author studies the behavior of smooth glass plates placed in contact under different conditions of loading. As a result of these experiments it is established that the frictional resistance between two surfaces is not due to the shearing strength of a cushion of air or water; but to the existence of microscopic particles that deposit themselves on the rubbing surfaces. Frictional resistance was found to be independent of the thickness of the air cushion.

Properties of Very Thin Layers of Water; Permeability and Water Contents of Clays.—A drop of water was placed between two glass plates and was allowed to evaporate. When the thickness of the layer of water was reduced to about $100\ \mu$, the evaporation stopped completely, suggesting that the molecules of a solid are able to exert forces over distances of the order of $50\ \mu$. Such thin layers of water have, beside viscosity, shearing and tensile strength. The author uses this result to explain the facts: (1) that layers of clay are impermeable unless the head exceeds a certain minimum; (2) that the water contents of clays do not drop below a certain limit; (3) that a drop of water does not spread over a surface wetted by water.

Theory of Surface Friction.—As a result of the observations mentioned and of a new set of experiments, the author concludes that the ordinary laws of friction of rest apply only where contact between the rubbing surfaces must be enforced by outside pressure. When two even surfaces are in contact the explanation of the causes of friction is not so simple on account of the presence of small particles on the surfaces.

THE following article may be considered as a preliminary report on a series of investigations carried out by the author in the Laboratories of Robert College, Constantinople (Branch institution of N. Y. State University).

I. INTRODUCTION.

Most of the properties of clays, as well as the physical causes of those few properties that have been investigated, are unknown. We know nothing about the elasticity of clays, or the conditions that determine their water capacity, or the relations between their water content and their viscosity, or the earth pressure that they exert and not even about the physical causes of the swelling of wetted clays. As a consequence

the civil engineer, dealing with this important material, is at the mercy of some unreliable empirical rules, and laboratory work carried out with clays leads only to a mass of incoherent facts. In order to establish a reliable theoretical basis for his practical investigations, the author has endeavored to solve the following questions: Is it possible to explain the physical properties of clays by means of the ordinary laws of physics (surface friction as a force, proportional to the surface pressure, acting merely in tangential direction, the laws of capillarity and the law of Darcy) or are we obliged to modify these laws in their application to the physics of colloidal matter?

II. SURFACE FRICTION BETWEEN BODIES WITH SMOOTH SURFACES.

Microscopical examination of clays shows that clay consists of a loose mass of small, transparent bodies with more or less smooth surfaces. Every plastic deformation of a clay-body involves the sliding of such surfaces on each other. No process can better be compared with this phenomenon than the sliding of two glass-plates on each other. But the question arises, as to whether the sliding of glass-plates on each other takes place under the same physical conditions as the relative motion of two grains of clay, and if not, what is the essential difference between the two cases of friction? The dealing with this fundamental question has forced the author to a new conception of the physical causes of friction of rest. This conception is evolved as a natural consequence of the following experiments:

1. A glass-plate with an even surface was laid on the table and covered by another one of the same size and quality. The surface of contact showed next to one of the corners a green spot surrounded by Newton's rings, the rest of the surface remained colorless. By pressing the colored spot with a glass-stick the color of the center became a yellow of the first order, indicating a thickness of $140\ \mu\mu$ ($0.00014\ \text{mm.}$) for the air cushion separating the two sheets, no further approach of the two glass plates being possible. The observation revealed the fact that the original green color of the center was of the fifth order corresponding to a distance of more than $1\ \mu$ ($0.001\ \text{mm.}$) and that this distance represented the minimum distance between the two bodies in unloaded condition. The experiment has been very often repeated, but the coloration and the location of the colored spot was always different, the only fact in common to the observed phenomena was the fact, that the minimum distance between the two plates never dropped below $1\ \mu$.

2. Two square glass-plates of $2 \times 2\ \text{cm.}$, $0.15\ \text{mm.}$ thick, were provided with a system of fine scratches. The average width of these scratches

was 0.2μ . By means of these scratches a map has been plotted of the air cushion separating the two sheets. The minimum distance between the sheets was found to be 3.7μ , the maximum distance 32μ . This experiment too was several times repeated, always with approximately the same result, but the spot of minimum distance changing its location.

3. Thirty four of the above-mentioned square glass-plates were placed one upon the other, so that they formed a prism of a total weight of 4.3 gr. This prism was placed under the microscope and gradually loaded up to a total load of 100 gr. The first application of the load resulted in a permanent settlement of 3μ per glass-sheet. At each new application the permanent settlement became smaller and finally the prism assumed the character of a perfectly elastic body whose modulus of elasticity increased rapidly with the load. By increasing the load from 2.85 gr. to 23 gr. per cm^2 , the total average compression was measured to be 4.17μ per sheet. The same experiment was repeated with the glass-sheets placed under water. The result of the loading test was practically the same, with the only difference, that an increase of the load from 2.85 to 23 gr. resulted in a compression of 4.85μ per sheet. In both cases the application of additional load was immediately followed by deformation.

The following experiments were devoted to an endeavor to find out the nature of the force that prevents the glass-sheets from touching. If the two glass-sheets were separated by an air or water cushion only, experiment No. 2 would have shown that:

- (a) The thickness of the cushion would decrease with increasing load.
- (b) The frictional resistance between the two surfaces would be due to the shearing strength of the cushion; and as experience shows that frictional resistance increases in approximately direct proportion with the load we would have to conclude:
- (c) That the shearing strength of a cushion of given area increases with decreasing thickness.

4. Two glass-prisms with particularly smooth surfaces were placed one above the other and the frictional resistance between the two bodies was directly measured by means of an accurate balance. After each experiment, the two surfaces were carefully rubbed with a cloth and freed from dust with a camel-hair brush. The distance between the two surfaces was indicated by the visible interference effect and varied between the very wide limits of 0.6 and more than 1.2μ . Frictional resistance was found to be *independent* of the thickness of the air cushion.

5. The same prisms were placed 2 cm. below the surface of distilled

water. Increase of water pressure up to 30 cm. did not effect their mutual distance. The third and fourth experiment show that the thick air and water cushions observed in the first and second experiment cannot be considered as the seat of the forces that keep the surfaces apart. The spacing was afterwards found to be produced by bodies of microscopical size, enclosed between the two surfaces. It must be simply considered as a statistical law, similar to the laws that determine the death-rate and the number of suicides in cities, that, as soon as a surface is exposed to air or water, a certain number of small fragments attach themselves to it and the dimensions of the three biggest particles determine distance as well as frictional resistance. That must be considered as the cause why there does not exist any definite coefficient of friction between glass and glass. The angle of friction varies between the limits of 1° and 10° and the case of friction at moderate pressure between the smooth surfaces of two bodies which are *not* separated by a third, one cannot be realized except in microscopical dimensions.

6. A glass sheet of 2×2 cm., 0.15 mm. thick was fixed to the upper surface of a glass plate of bigger dimensions by means of a drop of water enclosed between the two bodies. The temperature in the room being 28° C. the water evaporated rapidly until several wet spots remained, surrounded by Newton's rings. The thickness of the water-cushion could be estimated to be about $100 \mu\mu$. A great number of such couples were produced in a similar way and the result was always the same. Several water spots have been systematically examined under the microscope for enclosed microscopical bodies and typographical maps have been plotted indicating position and dimension of every obstacle. In every water spot a certain number of fine crystal-leaves could be located, but while in most of the spots the leaves were simply enclosed between the two surfaces, in one spot two particles have been discovered indicating by their coloration, that they stood under very high pressure. Simultaneous evaporation tests have been carried on. Water with its surface 2 cm. below the upper edge of a narrow glass-tube evaporated with a rate of about 1 mm. per day. A water surface 8 cm. below the top of a similar tube went down with a rate of about 0.3 mm. per day, while the water spots between the glass-sheets did not show any measurable sign of evaporation, even those which joined the outer edge of the glass-sheet, even although they have been purposely exposed for several days to the sun and to the wind.

The results of this experiment indicate clearly, that a water-cushion of less than $100 \mu\mu$ has properties which are different from the properties of larger bodies of water. The low rate of evaporation can be explained

in two different ways; Either the surface of the edge of the waterspot is covered by a layer of saturated vapor that is kept from diffusion by molecular attraction through the molecules of the solid body, or the water-molecules are kept in their position by the same forces. But if the molecules of the solid body are able to act over a distance of more than $50\ \mu\mu$ on the rapidly travelling molecules of saturated vapor, the more they will exercise their solidifying influence on the quieter molecules of water within the same range of distance. The distance of $50\ \mu\mu$ is identical with the distance (measured by Quincke)¹ up to which glass is able to influence the angle of contact between water and silver.

Combining the result of experiment No. 6 with the fact (observed by Seelheim and discussed by Forchheimer)² that layers of clay are impermeable unless the head exceeds a certain minimum, which is a function of the thickness of the layer, we are now obliged to attribute to the surface layer both the qualities of a solid and of a liquid, at least in the case where such surfaces meet each other between the surfaces of two solid bodies. This involves within the film of the liquid not only viscosity but shearing- and tensile-strength too.

The double thickness $0.1\ \mu$ of the contact-layer is the approximate width of voids in a thickly packed mass of grains whose size is smaller than 1 or $2\ \mu$. According to Atterberg³ no powder with uniform size of grains shows plasticity unless its particles are smaller than $2\ \mu$.

Experiment No. 6 explains the fact, observed by the author, that the water content of a clay exposed to the air at ordinary temperature does not drop below a certain limit ranging between 4 and 8 per cent. of the weight of the dry matter (the exact value depending on the size and particularly on the shape of the grains).

The fact that the influence of the molecules of the solid body is exerted up to a distance of more than $50\ \mu\mu$ from the surface seems to bear a relation with the unexplained fact, that a drop of water does not spread over a surface wetted by the water, but that its surface slopes down to the surface on which it rests, forming with it a certain angle of contact. If the influence of the molecules of the solid body were limited to a distance equal to the thickness of the surface layer of the liquid ($0.06\ \mu\mu$), the mentioned effect would not be possible as a simple statical consideration shows. The author explains the phenomenon as follows: The surface layer of the waterdrop is not anchored to the surface AB of the glass-sheet (Fig. 1) but to the semi-solid surface layer $ABCD$ and is therefore unable to spread because the area of this layer is limited. Another

¹ *Annales de Chimie et de Physique*.

² *Ztsch. d. Ver. d. Ing.*, 1901.

³ International reports on pedology, 1913.

simple experiment made with an eye-glass and a wet handkerchief indicated too the semi-solid character of thin films of water. Sweeping the handkerchief over the glass, the glass surface was covered with a thin film of liquid showing interference-colors. The surface of drops of water placed upon this film formed with it an angle of contact just as

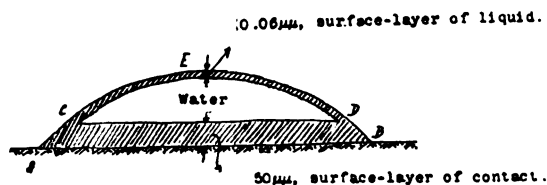


Fig. 1.

they would do it they were immediately placed on the glass surface, they showed no tendency to spread and their behavior was not at all influenced by the gradual evaporation of the film that surrounded them. While the film itself did not contract to drops as a thicker film would do but it retained its original relief until it evaporated.

III. ELASTIC THEORY OF SURFACE-FRICTION OF REST.

Under the influence of the results of his observations the author assumes that surface-friction stands in direct proportion to the area of those parts of their surfaces which are closer to each other than $0.1\ \mu$ (called area of contact) and distinguishes three different cases of friction:

Case (a). Very Smooth and Even Surface, Low Surface-Pressure.—The action is transmitted by intermediate bodies of microscopical size (Fig. 2, a). The area of surface of contact is, according to the formulas

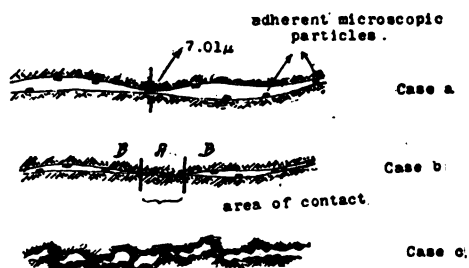


Fig. 2.

of Hertz, smaller the smaller the radius of curvature at the place of contact. Hence, as the shape of the intermediate bodies is a matter of chance, the coefficient of friction is extremely variable.

*Case (b) Very Smooth and Even Surfaces, High Surface Pressure (Fig. 2, b).—*The intermediate bodies disappear in depressions produced above and underneath them by high local pressure. The surfaces of the two main-bodies come in contact, the radius of curvature of this surface is by far bigger than the radius of curvature of the surfaces of the microscopical bodies, and the coefficient of friction goes rapidly up. With increasing pressure the peripheral parts *BB* of the surfaces whose radius of curvature is still greater than that of the central part *A* come in contact, the coefficient of friction increases therefore furthermore, although not as rapidly as at the limit between case (*a*) and case (*b*). These assumptions are verified by a series of facts published 1829 by Rennie.¹ This author found for the coefficient of friction *f* between steel and iron at different surface-pressures the following values:

Pressure $p = 8.79 \text{ kg. cm.}^{-2}$ $f = 0.166$ corresponding to a later stage of case (*a*).

$p = 23.62 \text{ kg. cm.}^{-2}$ $f = 0.333$, earlier stage of case (*b*).

With further increasing pressure p he found an increasing f until he obtained for $p = 47.25 \text{ kg. cm.}^{-2}$, $f = 0.403$.

Experiments carried out with other materials led him to similar results indicating that the coefficient of friction rises first rapidly and then slowly with increasing pressure.

*Case (c). Bodies with Rough Surfaces.—*The friction is a complicated result of the effect of molecular attraction due to contact and of the fact that the asperities of the two surfaces stand to each other in a similar relation as the teeth of two gears (Fig. 2, *c*). But as the surface of contact is already at the very beginning more considerable than in case (*a*), as it can never reach as big values as in case (*b*) and as furthermore the second element which determines the frictional resistance in case (*c*), the geometrical form of the two surfaces, is practically independent of the pressure, the coefficient of friction is a fairly definite figure and this explains the curious fact that the complicated causes of friction between rough bodies lead to a simpler effect than the seemingly simple causes of friction between two smooth surfaces.

The indicated theory removes the apparent contradiction between the fact of tangential frictional force and the molecular theory of matter. As long as the surfaces are in contact they adhere to each other not only in tangential direction but in every other direction too. But the adhesion perpendicular to the surface cannot be observed because as soon as the pressure is relieved the area of contact becomes again nearly zero, the surface resuming their original shape and this remark leads to a very

¹ Phil. Trans., 1829.

important conclusion essential for the understanding of the properties of clays:

The ordinary laws of friction of rest are only applicable to the friction between bodies whose original surfaces do not fit each other and whose contact must be enforced by external pressure. Two bodies joining each other with perfectly even surfaces would present not only a lateral resistance against displacement but resistance against separation too.

The latter fact has been demonstrated by the following striking experiment: A very dilute solution of a clay, distinguished by the flat shape of its particles and very strong Brownian movements was placed on a thin sheet of glass, inside a ring of a height of 2 mm. Five minutes after having filled the ring, the cylindrical space was covered with a glass cover, so that there was no trace of air between the bottom and the cover, and the whole was turned upside down. Twelve hours later the vessel was turned again, and after another twelve hours top and bottom were examined under the microscope. All particles smaller than μ adhered to the glass cover and all particles bigger than this size stuck to the bottom. The colloidal particles showed no trace of Brownian movement, apparently kept in their position by molecular forces. Turning the vessel upside down again the coarse particles remained in their places on the upper surface of the water-body. A drawing was made of some characteristic parts of the crystal groups and twelve hours later it could be stated that no grain had left the top-cover, a clear proof, that in the case of contact without external pressure the retaining forces act not only in a horizontal but also in vertical direction.

The author is deeply indebted to the College and to Mr. Tubini, A. M. I. E. E., acting dean of the engineering department of Robert College for very liberal assistance.

ROBERT COLLEGE,
ROUMELI HISSAR,
July 1, 1920.

SPEEDS IN SIGNALING BY THE USE OF LIGHT.

BY W. E. FORSYTHE.

SYNOPSIS.

Relative Length of Dot, Dash and Space for Light Signaling.—A preliminary investigation was carried out to determine the relative lengths of dot, space and dash necessary for the greatest speed. Three observers made readings for this test and all agreed very well with the following ratio for the maximum speed:

dot : dash : space :: 1 : 4 : 3.

Speed for Sharp Cut-off of Signals.—To test out the speed for a sharp cut-off of the signals, using this ratio for dot, dash and space, a sector was made with which seven different signals could be shown. This sector was mounted so that the observers saw the light source through a small opening. According to the criterion adopted, speeds were obtained that would correspond to 9 to 13 five-letter words per minute.

Speeds of Reading under Various Conditions.—It was shown that different observers were able to read the different signals at about the same speed with signals that were sharply cut off (as with the sector) and when the signals were made by turning on and off a special ribbon lamp filled with argon gas. The speeds of reading using other types of lamps were slower than for the ribbon lamp. With the observers at a distance and the lamp in a signalling unit, speeds somewhat slower were obtained. When an observer failed to read a given signal correctly the failure was generally due to his inability to see the dots.

IN connection with the design of some tungsten filament lamps to be used for signaling purposes it was found that little if any information was available as to the possible speed of signaling using flashes of light. This depends on two factors, the inertia of the eye and the characteristics of the source used. The purpose of this investigation was to ascertain if possible the maximum speed at which signals of this kind can be read and to compare this speed with those attainable when using certain types of tungsten filament signal lamps.

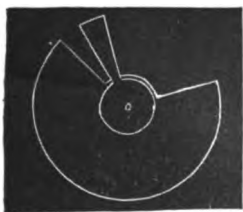


Fig. 1.

Disk with adjustable opaque sector.

A preliminary investigation was made to find the relative time intervals corresponding to a dot, space and dash which would permit of the maximum speed of reading. A disk was made up having a 120° opening and provided with a movable opaque sector which rotated with the disk (Fig. 1) but could be fixed at any position in the opening. The size of this sector determined the interval corresponding to a space while its

position defined the relative intervals for the dot and the dash. The speed of the disk was measured by a speedometer.

This disk was mounted between a lamp and a small opening (1.5 mm. in diameter) in front of which was placed a shutter controlled by a telegraphic key and relay, the latter connected with a commutator attached to the disk shaft. With the disk rotating, the shutter opened when the operator pressed a key, whereupon the observer, sitting about 4 meters in front of the opening, saw flashes of light of different duration with an intervening period of darkness and then the shutter closed. In this way a given signal was seen only once but by properly adjusting the position of the opaque sector it was possible to show any one of three signals, *i.e.*, dot space dash, dash space dot, or dash alone. Using a given size of sector and a given ratio of dot to dash interval, a number of these signals given in irregular order were presented for successively increasing speeds of the disk until a speed was reached beyond which they could no longer be differentiated. Then using the same sector, the ratio of the dot and dash intervals was changed and another series of readings was made until the maximum speed was reached. From the data thus obtained a curve was plotted with the ratio of dash to dot as abscissa and for ordinate the reciprocal of the time required to see the

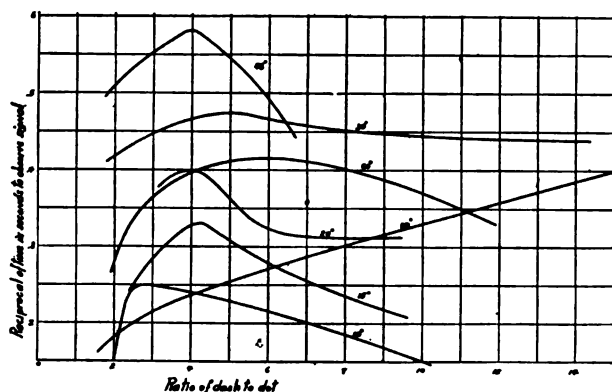


Fig. 2.

complete signal when the disk was rotated at a maximum speed. The procedure was repeated for a series of seven different sized opaque sectors, subtending angles of 10°, 15°, 20°, 30°, 45°, 60°, 80° and thus a family of curves was obtained as shown in Fig. 2. They represent the observations of a single observer, and show that for this observer and under the given conditions a ratio of dash to dot of approximately 4 to 1 corresponded to the maximum speed of reading for most of the sectors and particularly for the 45° sector. Moreover the speed using the latter sector was greater

than that obtained with the 30° on the one hand or the 60° on the other, and hence the correct size of sector for the greatest attainable speed would seem to be in the neighborhood of 45° . This corresponds to a proportionality of dot : dash : space :: 1 : 4 : 3. Later, a like test by another observer gave results slightly different from those shown in Fig. 2. The maximum speed when the movable sector was 45° occurred for a ratio of dot to dash of about 1 : 4 as in the first test. In this second test, observations were made with a $52\frac{1}{2}^\circ$ sector and the value for the maximum speed was about equal to that for the 45° sector, with the difference in this case, however, that the ratio of dot to dash for maximum speed was about 1 : 9, which gives 1 : 8 : 9 as the proportionality for dot, space and dash.

This second test seemed to show that about the same maximum speed would be obtained for the sectors above 45° up to 80° out of 120, with the difference, however, that there was a very great increase in the ratio of dot to dash for the very large sectors.

It is well known that in the case of any incandescent electric lamp a time interval elapses after the current is turned on before the filament glows with full brilliancy, and similarly time is required for the glow to disappear after the current has been cut off. If it were possible to construct a lamp in which full brilliancy and complete absence of glow were synchronous with the closing and opening of the circuit it would be possible to study the part played by the eye in determining the maximum speed of signaling independent of the characteristic of the lamp. Such a limiting case can be almost exactly reproduced by the use of a disk with proper openings rotating in front of a very small opening before a continuously luminous source. The relative time intervals for dot dash and space found in the preliminary investigation evidently apply rigorously only to this limiting case, and it should be noted that while the use of this proportionality for any signal lamp is an assumption, it is probably not far from right and applies to all of the data to be subsequently presented.

In the main experiment the first procedure was designed to ascertain the maximum speed of signaling for the limiting case, and to compare this with that obtained when the flashes were produced directly by a type of signal lamp which other experiments indicated would give the greatest speed. The previously mentioned disk might have been used for this purpose but in order to have a greater number and more complicated signals corresponding more nearly to those used in practice, a second disk was constructed using the ratios noted above for dot, dash and space. It is illustrated in Fig. 3 and with it the following seven

signals could be shown (a space of course occurs after each dot and each dash):

- 1 dot dash dot dash
- 2 dash dot dot dash
- 3 dash dot dot
- 4 dash dot dash dot
- 5 dash dash dot dot
- 6 dot dash dash dot
- 7 dot dash dash

This disk was then mounted with respect to the signal lamp in a manner quite similar to that described for the disk used in the preliminary experiment. The apparatus was so arranged that the different signals could be presented in any prearranged order while the disk was in rotation. The position of the observer and the use of the shutter were the same as previously described. A 6-volt, 2-ampere ribbon-filament signal-type tungsten lamp was used as a source, and held at a color temperature in the neighborhood of 2950° K. The intensity of the light was so great that a one per cent. transmission screen was interposed to make observation more comfortable. The intensity was still ample for accurate reading.



Fig. 3.

Disk with open and closed spaces to give seven different signals.

The following method was employed in making a test. A series of 12 signals arranged in a haphazard order as determined by the playing-card method, were presented at each of a series of successively increasing speeds of the disk, starting at a point such, that there was no question of ability to decide what the signal was. A record was made of all decisions and the experiment was continued until a speed was reached at which, from the number of errors, it was evident the observer had passed the limit of accurate judgment.

In order to make measurements with the lamp directly, a commutator was constructed by means of which the intervals during which current was turned on and off were in the same ratio for dot, space and dash as the disk-exposure times previously referred to. The relation of dot : space : dash is somewhat different in this case than when the sector was used. This is due to the fact that it takes a somewhat longer time for a lamp to heat up than to cool off. This shortens the dot relative to the dash and lengthens the space. From the results of the test for the relative length of dot to dash by the second observer, this would make but little difference, if any, in the final results. With the disk removed but

other arrangements the same, an exactly similar series of readings were made as before, using the 12 signals and successively increasing speeds. There was practically no resistance in series with the lamp. Each observer made runs with both the disk and the lamp, the two series of measurements following each other in rapid succession in order to maintain as nearly as possible the same eye conditions.

The results are shown in Table I. In column one are given the reciprocals of the time in seconds from the beginning to the end of the signal. In the succeeding columns are given for each observer the number of

TABLE I.

Results for Disk Compared with those for Ribbon-Filament Tungsten Lamp in Argon.

Reciprocal of Time in Seconds from Beginning to End	Number of Signals Read Correctly Out of the Twelve Signals Shown.					
	W. W.		J. R. C.		F. E. C.	
	Disk.	Lamp.	Disk.	Lamp.	Disk.	Lamp.
.20.....	12	12	12	12	12	12
.25.....	12	12	12	12	12	12
.30.....	12	12	12	12	12	12
.35.....	12	12	12	12	11	11
.40.....	11	12	12	12	11	11
.45.....	10	12	11	12	11	10
.50.....	10	12	12	12	9	10
.55.....	9	12	12	12	9	9
.60.....	9	11	12	12	7	7
.65.....	8	8	12	12	4	7
.70.....	8	11	12	11	5	9
.75.....	7	9	11	10	6	3
.80.....	5	5	12	10	3	7
.85.....	7	5	7	9	6	3
.90.....	3	3	9	5	1	5
.95.....	4	1	5	0	0	3
1.00.....	.	.	4	.	.	4

The results for W.W. indicate a breakdown at about the point .50 which corresponds to a speed of about 9 five-letter words per minute.

For J.R.C. the breakdown point would be at about .80 which corresponds to a speed of 13 five-letter words per minute.

The results are arrived at by allowing two mistakes out of the twelve signals shown.

signals read correctly out of the 12 presented at each speed, both when the disk was used and when the lamp was used directly with the commutator. The data show very little if any difference on the average between the results using the disk and those using the lamp and indicate that the eye and not the lamp was the limiting factor in the speed of reading the signals.

Having found that the special gas-filled ribbon-filament tungsten lamp

used gave results as good as those obtained by the use of the disk, it seemed desirable to compare this lamp with other equivalent types such as a lamp with a ribbon filament, in a vacuum, and also a gas-filled wire-filament lamp. The previous tests were all carried out in the laboratory where the observer was insured plenty of light and sharp contrast and had no difficulty in locating the flash. In order to carry on the tests under conditions more nearly like those which occur in practice, the lamp and commutator apparatus were moved up to the roof of the laboratory. The effect of distance between the observer and the apparatus was simulated by setting up the lamp without a reflector so that the observer saw the flash, against the sky as a background, with an intensity such as to give a flux equivalent to that which would be obtained by the use of the regular reflecting apparatus with the observer 20 or 30 times as far away. The procedure as to number and order of signals and speeds was the same as before. In the first test comparison was made with a lamp having a ribbon filament of the same shape but in a vacuum. The lamps were oriented so that the intensity in the direction of the observer was the same for each. For this run the observer was stationed at a distance of 100 yards and the results, given in the second and third columns of Table II. indicate that the gas-filled lamp is best for this work. A comparison was then made of the gas-filled tungsten-

TABLE II.

Comparison between a Gas-filled Ribbon Lamp and a Vacuum Ribbon Lamp; also between a Ribbon Lamp and a Wire Lamp, both Gas-filled.

Reciprocal of Time in Seconds from Beginning to End. of Signal.	Number of Signals Read Correctly Out of the Twelve Signals Shown.							
	W.W.		J.R.C.		W. W.		K.M.	
	Gas-filled Ribbon ¹ Lamp.	Vacuum Lamp.	Gas-filled Ribbon Lamp. ¹	Gas-filled Wire Lamp.	Gas-filled Ribbon Lamp. ²	Gas-filled Wire Lamp.	Gas-filled Ribbon Lamp. ²	Gas-filled Wire Lamp.
.20.....	12	12	12	12	12	12	12	10
.25.....	12	12	12	11	12	12	12	11
.30.....	12	11	12	12	12	11	12	5
.35.....	12	8	11	6	12	11	10	
.40.....	11	8	9	8	9	9	8	
.45.....	12	7	6		8	6		
.50.....	9	7	7		8	7		
.55.....	8	6			7			
.60.....	5	2			5			
.65.....	6							
.70.....	3							

¹ Distance of observer 100 yards.

² Distance of observer 170 yards.

ribbon lamp and a gas-filled tungsten-wire lamp using three observers at a distance of 170 yards. The former lamp was run at a color temperature of about 3000° K. and the latter at about 3100° K. As before the lamps were oriented so as to have approximately the same intensity in the direction of observation, and the results indicate superiority for the ribbon type. As operated for the most part in this test there was some

TABLE III.

Determination of Speed of Signalling under Various Atmospherical Conditions and at a Distance of 2,700 Yards with a Gas-filled ribbon-filament tungsten signal lamp mounted in a Standard Housing with a Reflector.

Reciprocal of Time in Seconds From Beginning to End of Signal.	Number of Signals Read Correctly Out of the Twelve Signals Shown.					
	1.	2.	3.	4.	5.	6.
<i>Results of W.W.</i>						
.15.....	11			12	10	
.20.....	12			12	12	12
.25.....	12	12	12	11	11	12
.30.....	11	11	11	12	8	12
.35.....	12	9	12	12	9	12
.40.....	12	8	11	11	7	12
.45.....	9	6	9		7	12
.50.....	11	6	7		9	11
.55.....	7	7	6		8	12
.60.....	8	4	6			9
.65.....	5	5	4			
.70.....	5		4			
.75.....	7	3	4			
<i>Results of K.M.</i>						
.15.....	11					
.20.....	12			12	9	
.25.....	12	12	12	12	12	12
.30.....	11	11	11	12	9	11
.35.....	12	12	7	12	9	11
.40.....	9	11	10	11	10	12
.45.....	7	10	5	12	7	10
.50.....	9	7	8		6	5
.55.....	8	6	8		6	0
.60.....	7	3	8		5	0
.65.....	0	0	7			
.70.....						
.75.....						

1. Cloudy, conditions changing.
2. Cloudy, some fog.
3. Cloudy, some fog.
4. Clear day.
5. Clear day.
6. Night.

resistance in the lamp circuit. The effect of this should be to lower the speed of the lamps. A test to see whether the effect would be appreciable and different in the two lamps showed a slowing up in both but a difference so small as to be almost within the limits of experimental error.

To still further reproduce actual practice two observers were placed at a distance of 2,700 yards. The gas-filled ribbon lamp was then placed in a regular signaling unit which gave about a 5-degree spread. The data are recorded in Table III. For observer *WW*, the limit of speed of receiving signals would appear to be lower on foggy days than on clear days and highest at night. On the other hand the results of *K.M.* showed little difference on clear or foggy days, but were better at night.

Assuming a five-letter word, and that the same speed was maintained for continuous sending of signals the 0.50 point in column one corresponds to about 9 words per minute, and the 0.80 point to about 13 words per minute.

A word might be said regarding the appearance of the signals when the speed approached the limit beyond which the signals could not be identified. Each observer was instructed to record what he saw, not by code letter, but by actually writing down the dots and dashes. A study of these records shows that on clear days inability to see initial dots caused uncertainty and eventually breakdown, dashes only being seen ultimately. At night, owing to the greater apparent brightness, the spaces seemed to vanish and a mere flicker appeared. In this connection it should be noted that in night work no red glasses were used such as are frequently employed in practice.

NELA RESEARCH LABORATORY,
CLEVELAND, OHIO,
March, 1920.

PHENOMENA IN OXIDE-COATED FILAMENT ELECTRON TUBES.

BY H. D. ARNOLD.

SYNOPSIS.

Thermionic Phenomena in Wehnelt Oxide Electron Tubes.—The author discusses some of the problems which have arisen in connection with the development of three-electrode vacuum tubes with special reference to the production of a highly active and uniform thermionic source of electrons. The materials and the methods for the preparation of both core and coating are given. Since tubes with these filaments are not self-evacuating by the clean-up effect to the extent found with tungsten filaments, greater care is necessary in the pumping process.

Pure Electron Discharge; the Influence of Pressure and of Bombardment by Positive Ions.—The evidence shows the electron emission from Wehnelt oxides to be purely thermionic. Preliminary measurements on the number of electrons emitted from these filaments under the bombardment of positive ions prove that no appreciable part of their activity can be attributed to this cause. Operation has been studied with pressures of the order of 10^{-10} mm. Hg., but no decrease in activity is found at these low pressures. A plot is given summarizing the constants of Richardson's equation for 4,000 filaments.

Rate of Evaporation of Oxide Coating.—Grams per cm.² per sec. evaporated from a filament coated by the above process are approximately given by

$$m = 4.6 \times 10^{-4} T^{-1/2} e^{-\frac{4.6 \times 10^4}{T}}.$$

Thinly Coated Filaments; Coating by Evaporation.—Preliminary results of work now being carried on by Dr. C. J. Davisson show that maximum thermionic activity may be reached with coatings considerably less than one molecule deep. This suggests that of the two important constants determining electron emission, the density of free electrons in the substance and the work function at the surface, the latter is modified by the presence of the oxides while the former may remain essentially that of the core.

THE most generally interesting phenomena in electron tubes are those relating to the flow of electrons through the space, the control of this flow by plate and grid voltages, and the operation of the tubes in the various circuits in which they are used. These phenomena are essentially the same whether the filament is pure metal or is oxide coated, and a considerable literature has already grown up in this field. The phenomena distinctive of oxide-coated filament tubes are those relating to the economic and scientific factors involved in the process of electron emission. I shall therefore confine myself as closely as possible to these factors as they have influenced the design of oxide filament tubes,

and as we have been led to investigate them in connection with our development of these tubes during the past seven years. For reference to the scientific literature in this field attention is called to the appended bibliography.

Wehnelt oxide filaments offer the most economical source of electrons at present available for use in three-electrode vacuum tubes, and thus from a commercial point of view they are at present of the greatest interest and importance. On purely scientific grounds they have long been of peculiar interest to physicists and chemists, offering as they have one of the most promising fields for speculations on the possible effect of chemical action in aiding the emission of electrons from hot bodies.

The early work of Richardson, H. A. Wilson, and others, established beyond reasonable doubt that for pure metals the emission of electrons is due to thermal action and not to chemical action in the ordinary sense of the word. Of the early evidence the most convincing was the proof that the emission did not decrease as more and more of the residual gas was removed, and the proof that the emission did obey the law of variation with temperature which would be expected if the effect were purely thermal. Further work¹ showed that the distribution of velocities among the emitted electrons was that which would be computed on the basis of the equipartition of energy between molecules and free electrons in the metal, the faster of the free electrons escaping in spite of the restraining influence of an electric field at the surface of the metal. Still other experiments² established that the value of this potential difference, as determined from the cooling of the metal due to the electron emission, corresponded with the value deduced from the observed relation between emission current and temperature. In view of these facts physicists have believed for the past ten years or more that there was no place for a chemical theory of emission as far as pure metals are concerned.

With respect to Wehnelt oxides, however, there was somewhat less certainty as to whether chemical action might not be necessary for electron emission. While it is true that early experiments established that with oxides as well as with pure metals the emission did not depend on the presence of gas in the evacuated chamber, the presence of the oxide itself on the filament seemed to offer sufficient chance for chemical actions, irrespective of any gas. Moreover, the fact that coating metals with these chemical compounds increased their electron emission many fold seemed to invite the hypothesis that this increase resulted from

¹ Richardson and Brown, *Phil. Mag.*, Vol. XVI., p. 353 (1908).

² Richardson and Cooke, *Phil. Mag.*, Vol. XXV., p. 624 (1913); Vol. XXVI., p. 472 (1913)

chemical action, although there has never been any experimental evidence clearly requiring such an hypothesis.

Data similar to those which established the purely thermal nature of thermionic emission from metals, were accumulated more slowly in the case of Wehnelt oxides, largely because of experimental difficulties which were encountered. The determination of the correspondence of the work function as obtained from the cooling effect, and as found from the exponent in the emission-temperature equation, followed several years behind the similar determination for pure metals, this in spite of the fact that the earliest experiments with the cooling effect were attempted by Wehnelt and Jentzsch¹ using Wehnelt oxides. As for the velocity distribution among the emitted electrons, this, as far as I am aware, has still to be satisfactorily determined for the oxides. Nevertheless, the evidence as it has been verified step by step has in every case supported the opinion of the physicists who believed that emission from oxides as well as metals was in all probability a purely thermal affair and did not require any assumption of chemical action for its explanation.

With the inception of the use of oxide filaments for commercial vacuum tubes, we had to face the same difficulties of technique in producing the filament that had retarded the scientific experimenters in their work, and it was in addition necessary to produce filament in large quantities and with a very high degree of uniformity. To do this a survey was made of the metals available for the core of the filament, and of the materials and methods of coating best adapted to the manufacture of a highly active and uniform filament. Considerations of mechanical strength, electrical resistance, non-oxidizability, availability and reproducibility of commercial supply, etc., led us to choose for the core a wire of platinum-iridium (about six per cent. iridium, with the other impurities usually found in commercial platinum-iridium). This wire was rolled to a ribbon to increase the surface, and the ribbon was twisted to secure a better mechanical structure. This core could be produced in quantity with electrical and mechanical properties sufficiently uniform for our purpose.

In the choice of coating materials we had available the oxides of barium, strontium, and calcium, with thermionic activities in the order given. Experiments with coatings of BaO, however, showed a mechanical disintegration during life which outweighed in importance its superior thermionic activity. Efforts to secure longer life resulted in the use of a mixture of BaO and SrO applied in a number of consecutive coatings.

¹ Wehnelt and Jentzsch, *Verh. der Deutsch. Physik. Ges.*, 10 Jahrg., p. 610 (1908); *Ann. der Physik*, Vol. XXVIII., p. 537 (1909).

In the process which we most commonly used, barium in the form of carbonate, and strontium in the form of hydroxide or carbonate, was mixed with some carrier such as resin or paraffin, which would burn away when heated in the air. In the coating process four applications of the strontium mixture were followed by four of the barium mixture, and this process was then repeated, making a total of sixteen separate applications. After each application the wire was raised momentarily to a temperature of about a thousand degrees, which burned away most of the organic carrier. When the coating was complete the wire was heated to about 1200° for two hours. At the end of this time there remains a fairly heavy coat of BaO and SrO (from 2 to 3 milligrams per sq. cm. surface), while next to the core is a firmly adhering layer which is built up due to chemical reactions between the coating and the core. Analysis shows this coating to consist of barium and strontium combined with platinum, rhodium and iridium, the compound present in largest amount being barium platinate (BaPtO_6). The compound with rhodium seems to be more readily formed, but due to the small percentage of rhodium present this compound makes up only a small fraction of the total.

The filament thus formed can be handled without undue precaution so long as it is not exposed to moisture or carbon dioxide. When stored in vacuum containers it shows no signs of deterioration even after a period of several years.

The time required for the proper evacuation of an oxide filament tube is determined almost entirely by the requirement that a large part of the occluded gases must be removed from the metal and glass parts inside the bulb. In case the metal parts can be heated before assembly, or can be heated electrically, either through leads from the outside or by Tesla currents, the evacuating process may be very much shortened. In any event it is desirable to carry the evacuating process considerably further in the case of oxide filament tubes than with tungsten filament tubes, since the clean up effect of the filament itself is not nearly so marked as is the case with tungsten.

During the pumping process the filaments are glowed for several minutes to liberate any gases they may have occluded. At the same time the compounds at the surface of the core decompose to a certain extent, as is evidenced by the fact that filaments taken from tubes after a considerable period of glowing show a marked diminution in the amount of BaPtO_6 and similar compounds.

With the exercise of proper care as to the purity of the materials used and with adherence to a definite schedule of coating and heat treatment,

filament can be produced with every expectation of uniformity and long life. During the war some half million vacuum tubes were made employing this filament, and the filament was prepared as a part of the regular manufacturing process by practically unskilled labor. Expert supervision was of course necessary to guard against the intrusion of chemical impurities or variations in the coating process.

We have naturally made a very thorough study of this so-called "standard" filament in order to determine its properties and particularly the factors which influence it during life. Having been developed in the first instance, about seven years ago, to meet the needs of telephone repeater service, severe requirements were imposed upon it from the start. Tubes made with it are required to be completely interchangeable. No adjustment of filament or plate voltages is made when tubes are replaced, and no appreciable change in amplification, or in electron current, may result from an interchange. The operation of the tube as an amplifier must remain unchanged when the filament voltage is varied between the limits customarily met with in storage battery service. The operation of the tube must remain sensibly unchanged through a life of several thousand hours, and when it does fail for any reason, it must give sufficient warning to ensure its being removed from service without causing the interruption of a telephone conversation. These and numerous other requirements are rigorously enforced, making it far more difficult to manufacture tubes for telephone repeater service than for the other uses for which they have since been adopted.

What terminates the useful life of an oxide filament is usually the development of local faults or "bright spots" due to the evaporation of the coating. They are practically free from the most common ageing effect of pure metal filaments, namely the gradual increase of electrical resistance caused by the evaporation of the filament. In the case of tungsten filaments used on constant voltage supply evaporation with the resulting increase in resistance causes a lowering in the filament temperature and therefore a decrease of the electron emission, while with a constant current supply the increase in resistance results in an increase of temperature and a progressive increase in rate of disintegration of the filament. With oxide coated filaments it is the coating alone that evaporates, at least until a bright spot is formed, and the temperature and operating characteristics remain unchanged throughout life on either constant current or constant voltage supply.

The cost of electrons in a vacuum tube device is determined by the characteristics of the filament, by the cost of the power used in heating the filament, and by the life and the replacement cost of the tube. The

physical factors necessary for computing the cost are the constants in Richardson's equation, the constants of the evaporation equation, and the radiation constant peculiar to the filament. In order to relate the life of the tube to the vaporization constants it is necessary in addition to know to what extent vaporization may proceed, on the average, before the useful life is terminated, and this can, of course, only be arrived at by exhaustive life tests.

For tungsten and molybdenum the physical constants are already available, but for Wehnelt oxide filaments these constants have not as yet been satisfactorily established. Preliminary results have, however, been obtained in our laboratory and Dr. C. J. Davisson is continuing the investigation. We have found that the total thermal emissivity lies between 0.45 and 0.70, for a considerable number of samples of our standard filament. The method followed in determining these limits was to get the resistance power relation for the filament under operating conditions, and later get the resistance-temperature relation by placing the tube in an electric furnace. The latter relation was checked by an observation at the melting point of potassium sulphate. In connection with measurements of filament temperatures it was found by the use of the optical wedge method that these oxides act practically like black bodies in the red region of the spectrum and hence the temperature can be read with a fair degree of accuracy by means of an optical pyrometer using the black body calibration.

The preliminary values of the evaporation constants, obtained by evaporating barium oxide from a tungsten boat, catching it on a platinum shield, and weighing the deposit show that with a fair approximation

$$m = 4.6 \times 10^8 T^{-1/2} e^{-[(4.6 \times 10^4)/T]},$$

where m is the rate of evaporation in grams per sq. cm. per second. A more definite determination of these constants for BaO, SrO and CaO is under way.

As for the determination of the constants in Richardson's equation for oxide filament, this work has covered about 4,000 filaments, and the results are of sufficient interest to warrant more detailed consideration. To simplify the investigation of this matter Dr. Davisson has devised a form of coördinate paper in which the coördinates are power supplied to the filament (abscissæ) and thermionic emission (ordinates). The coördinate lines are so disposed and numbered that if the emission from a filament satisfies Richardson's relation, and the thermal radiation satisfies the Stefan-Boltzmann relation, then points on the chart co-ordinating power and emission for such a filament will fall on a straight

line. For practical purposes the advantage of replacing temperature by power as the independent variable is obvious. From the slope and position of a line on this chart together with the area of the filament, it is possible to calculate $bE^{1/4}$ and $aE^{-1/8}$ where b and a are Richardson's constants and E is the total thermal emissivity of the filament. The lines shown in Fig. 1 give some idea of the characteristics of our standard

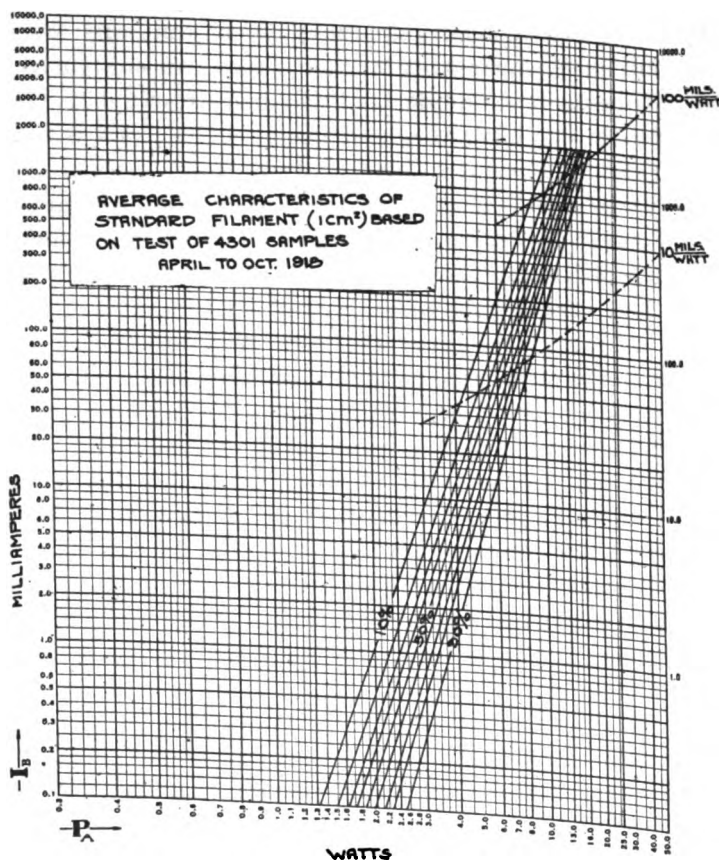


Fig. 1.

filament and the range of variability. Ten per cent. of the filaments have a greater activity than that given by the upper line while the activity for 90 per cent. of the filaments lies above the lowest line. In this connection it should not be overlooked that in the great majority of vacuum tube applications we are concerned only in maintaining the electron emission at a value greater than a certain fixed limit. It is in no way injurious for the emission to exceed this limiting value. In a limited number of cases, however, it is desirable to use tubes with as

nearly as possible the same voltage saturation value of emission. For these special problems pure metal filaments offer at present the best solution, since the uniformity of their saturation values can be more readily maintained.

The dotted curved lines on the figure are drawn through points of constant ratio of emission to power, that is, constant efficiency or economy. The customary operating power is from 8 to 9 watts per cm.², so that the range of efficiencies is seen to lie between 10 mils per watt and 100 mils per watt. Assuming the emissivity of the filament to be 0.6 we obtain from these lines the following values of thermionic characteristics of the filament.

Equivalent volts through surface $\phi = 1.55$ to 1.9,

Richardson's constant b , 19.4×10^3 to 23.8×10^3 ,

Richardson's constant a , $(0.5 \text{ to } 1.5) \times 10^{24}$ for electrons per sec. per cm.²,
(8 to 24) $\times 10^4$ for amps./cm.²

In our adaptation of Wehnelt oxide coated filaments to commercial vacuum devices we have found it desirable in a number of instances to carry on investigations which would appear to be more immediately of scientific than of commercial value. One of our early experiments which was reported by Dr. W. Wilson at the Christmas meeting of the American Physical Society in 1914 gave additional data as to the behavior of Wehnelt oxide filaments in very high vacuum. The experimental arrangement used consisted of a cylindrical anode, with suitable guard rings, surrounding a single strand of oxide coated filament placed along the axis of the cylinder. The vacuum was made as good as possible with a Gaede molecular pump and it was established that the electron current in the filament did not diminish even with these extreme vacua. It was expected that the current would vary with the $3/2$ power of the voltage according to Child's¹ space charge relation and that from the coefficient K , which involves among other factors the quantity $\sqrt{e/m}$ we might determine whether the carriers in the case of Wehnelt oxide filament were electrons or were in part carriers of greater mass. Wilson found that when proper consideration was taken of the geometrical form and of the voltage drop along the filament, the space charge equation took the form²

$$i = k \frac{2}{5} \frac{(V - V_0')^{5/2}}{V_0 - V_0'} \left[1 - \left(1 - \frac{V_0 - V_0'}{V - V_0'} \right)^{5/2} \right],$$

¹ Child, *PHYS. REV.*, Vol. 32, p. 492 (1911).

² See also E. R. Stoekle, *PHYS. REV.*, Vol. VIII., p. 545 (1916).

where V_0' is the potential of the negative end of the filament and V_0 the potential of the positive end. The value e/m obtained from the experimental data by the use of this equation was found under the best conditions to be about 5 per cent. lower than the value obtained for electrons by other methods. This deviation is explicable either by the presence of a very small number of negative carriers of molecular size or by the emission of secondary electrons from the anode under the bombardment of the primary electrons. The deviation is however, no greater than that obtained with tungsten filaments, and the concordance of the results indicates clearly that the discharge from Wehnelt oxides may be considered a pure electron discharge.

By means of the Knudsen absolute manometer, and later by the use of the Buckley ionization manometer,¹ we have measured vacua of the order of 10^{-9} and 10^{-10} millimeters of mercury during the operation of Wehnelt oxide filaments, and have never found any indication of the emission current falling off as the vacuum improved. With the introduction of various gases at pressures of the order of .001 millimeter, or so, the electron emission currents do suffer rather large changes, although not of the degree found with the emission from tungsten filaments under similar conditions. The presence of oxygen and carbon dioxide inhibit the electron emission, while a small amount of hydrogen in contact with a filament of abnormally low emission may result in restoring it to a normal condition. Because of their relatively smaller variations in emission under the action of gases we have found Wehnelt oxide filaments more adaptable for use in Buckley ionization manometers than pure metal filaments, as it is very rare that any blocking effects develop which cannot be overcome by a reasonable increase in the filament temperature.

In another paper² before the Physical Society in April, 1917, Wilson gave the results of a series of experiments on the relation between the work function ϕ and the exponent b of Richardson's equation. The method followed was a modification of that first attempted by Wehnelt and Jentzsch for oxide filaments and later used by Richardson and his collaborators in investigating the cooling effect due to the emission of electrons from filaments of osmium and tungsten. Previous experimenters who had attempted to determine the cooling effects with Wehnelt oxides had found it impossible to get consistent results due to the non-uniform behavior of the filament. During Wilson's experiments our standard methods of coating were employed, and the results obtained were remarkably consistent and reproducible. The values obtained were as follows:

¹ O. E. Buckley, Proc. Nat. Acad. Sci., 2, 683 (1916).

² Proc. Nat. Acad. Sci., 3, 426 (1917).

	bR/e Volts.	ϕ Volts.
BaO 50 per cent., SrO 50 per cent.....	{ 2.02 2.16	1.97 2.28
BaO 50 per cent., SrO 25 per cent., CaO 25 per cent..	{ 2.34 2.59	2.39 2.54
CaO.....	{ 3.28 3.49	3.22 3.51

We have recently been repeating some of these experiments with a slightly different bridge arrangement and find no difficulty in obtaining consistent results from a given filament over long periods of time.

The close correspondence between b and ϕ , is as pointed out above, one of the grounds for believing that chemical action does not play any part in thermionic emission. The correspondence is, of course, not a proof of the absence of chemical action, but it is certainly more easily explained on the assumption of thermal action than on the assumption of chemical action. The determination of ϕ is also of considerable importance in connection with the theory of photo-emission and of contact potentials.

In connection with our recent experiments Dr. Davisson is finding it of great convenience to use filaments which have been coated by active material evaporated from a standard filament. The standard filament and the wire to be coated are mounted close together in the same tube and the primary is run at a fairly high temperature for various lengths of time according to the purpose of the experiment. Observations are then made on the emission from the secondary filament. One advantage of this method of experimentation is that the core of the secondary filament may be any suitable material, for example, tungsten or iron, without meeting the difficulties of oxidation, which are often troublesome when these materials are coated in the open air.

These secondary filaments have many interesting properties which no doubt will prove of importance in establishing the process of electron emission from Wehnelt oxides. One of the most striking facts is that the secondary filament may show a high electron emission when only a very minute amount of active material has been transferred to it. In certain experiments where the secondary filament was tungsten, the standard filament was glowed for so short a time that only approximately one tenth of the surface of the tungsten filament was covered with active material. While we have not determined in just what form the deposit comes down on the secondary, it seems reasonable to suppose that the material is transferred essentially molecule by molecule, and that the surface of the secondary filament would be found to have on it a consider-

able number of single molecules of coating separated by distances of the order of one to ten molecular diameters, and in addition a certain number of groups of two molecules, a much smaller number of groups of three molecules, and so on. It is most interesting to note that the electron current obtainable at a given temperature from filament coated in this very tenuous fashion may be only a little smaller than that obtained when the entire filament is covered with a heavy deposit.

It is natural to ask whether this result is due to a difference in the a or in the b of Richardson's equation as applied to these partly coated filaments. Work along this line is not yet completed, but measurements of b taken at intervals while the deposit was forming have never shown an increase with thickness of deposit, and therefore it seems that the result must be due to a difference in a . If we assume that a completely coated filament emits uniformly over its entire surface, while a partly coated filament emits only in the vicinity of the molecule of active material, we compute an a for the partly coated filaments which is much greater than the values (0.5×10^{24} to 1.5×10^{24}) given above for standard filament. The fact that the b obtained for filaments coated merely with a few widely separated molecules is the same as that obtained with a complete coating shows that the reduction of the work function at the metal surface can be brought about by a very small group of molecules. Under these conditions it seems reasonable to suppose that the only important effect of the molecule of active material is to lower the restraining voltage in its own vicinity and thus facilitate the passage of electrons from the metal core. The number of electrons that can avail themselves of such a molecular opening is limited to those presenting themselves with a sufficient outward velocity, and this in turn is determined by the properties of the core material. Since the core materials used have values of a greater than those which we find for the standard filaments it does not seem so strange that the values of a for the partly coated filaments should be found to run higher than those for completely covered filament.

We hope that our present experiments will throw more light on the factors involved in the escape of electrons through these minute activated areas. Our information is at present too meager to warrant an opinion as to whether an electron on its way out remains for some time as a part of the molecule of active material or merely slips past it. The number of electrons passing out through one molecular opening in a second may be of the order of ten thousand, and that this rate may persist for some time is indicated by the rather slow rate of decay of activity. This proves at least that no irreversible chemical change in the active coating is involved in the emission of an electron. Perhaps a more

striking proof of this is found in certain of our filaments which through a life of twenty thousand hours have emitted fifteen times more mass of electrons than the mass of their coating.

Another factor of considerable importance in connection with the use of Wehnelt oxides is the facility with which they emit electrons when bombarded by electrons or by positive carriers. The factors governing the emission of secondary electrons are not well understood even in the case of pure metals. Experiments which we have performed with re-distilled mercury surfaces indicate that the secondary emission may become very small when an absolutely clean surface is obtained. With Wehnelt oxides the secondary emission may be comparatively large, several electrons being liberated by each one that strikes the surface. Under the bombardment of positive carriers, however, secondary electron emission does not appear to take place so readily, and our experiments have not as yet shown evidence of more than a few electrons emitted per impact. This negatives the idea, which a few people have held, that positive ion bombardment is a controlling factor in electron emission from oxide coated filaments.

In conclusion, it may be remarked that while oxide coated filaments, under the pressure of necessity, have been developed to the state where they can be manufactured and used on an immense commercial scale as evidenced by their employment in more than half a million vacuum tubes, and while they easily hold the field as the most economical, and longest lived, source of electrons available for electron tube devices, we have not as yet by any means reached the limit of their possible development. The physical constants delimiting the operation of pure metal filaments are well known for the metals now available, and there is no apparent avenue by which their efficiency can be materially increased. Marked advance can only be expected when we depart from pure metals by mixing or coating them with other materials. In the case of the oxides, which at present furnish the best material for coating, it is not at all unusual to find sporadic cases of enormously greater activity than the average which we obtain by commercial coating processes. The limit to which improvement may be expected to extend can only be determined when the factors governing electron emission from the oxides are as well understood as are at present those governing electron emission from the pure metals. It is more than probable that in the process of obtaining this knowledge about the oxides we will find new light thrown upon the process of emission from pure metals as well.

RESEARCH LABORATORIES OF AMERICAN TELEPHONE & TELEGRAPH
COMPANY AND WESTERN ELECTRIC COMPANY, INC.

BIBLIOGRAPHY FOR THE PHYSICS AND CHEMISTRY OF OXIDE CATHODES.

1. Cooke, H. L. and Richardson, O. W. Absorption of heat produced by the emission of ions from hot bodies. II. *Phil. Mag.* (6), 26, 472, 1913.
2. Deininger, F. Austritt negativer Ionen aus einigen glühenden Metallen und aus glühendem Calciumoxyd. *Ann. d. Phys.* (4), 25, 258, 1908.
3. Fredenhagen, K. Ueber die Elektronenemission des Platins und über die Wirksamkeit der Oxydelektroden. *Leipz. Ber., math.-phys. Kl.*, 65, 42, 1913.
4. Fredenhagen, K. Das Verhalten der Wehnelttelektrode in verschiedenen Gasen. *Phys. ZS.*, 15, 19, 1914.
5. Gehrts, A. Die Ursache der Elektronenemission von Oxydkathoden. *Verh. d.D.P.G.*, 15, 1047, 1913.
6. Germershausen, W. Die Elektronenemission der CaO-Elektrode im Vakuum. *Phys. ZS.* 16, 104, 1915. *Ann. d. Phys.*, 51.
7. Horton, F. Discharge of negative electricity from hot calcium and from lime. *Phil. Trans.*, A207, 149, 1907; also *Proc. Roy. Soc.*, A79, 96, 1917.
8. Horton, F. Ionization produced by certain substances when heated on a Nernst filament. *Camb. Phil. Soc. Proc.*, 17, 414, 1914.
9. Horton, F. On the action of a Wehnelt cathode. *Phil. Mag.* (6), 28, 24, 1914.
10. Horton, F. Origin of the electron emission from glowing solids. *Phil. Trans.* A214, 277, 1914.
11. Jentsch, F. Ueber die Elektronenemission von glühenden Metalloxyden. *Ann. d. Phys.*, 27, 129, 1908.
12. Richardson, O. W. Die Abgabe negativer Elektrizität von heissen Körpern. *Jahrb. d. Rad. and El.* 1, 300, 1904. (Synopsis).
13. Richardson, O. W. The kinetic energy of the ions emitted by hot bodies. II. *Phil. Mag.* (6), 18, 681, 1909.
14. Richardson, O. W. The ions from hot salts. *Phil. Mag.* (6), 26, 452, 1913.
15. Schneider, H. Die Energie der aus glühenden CaP entweichenden Elektronen. *Ann. d. Phys.*, 37, 569, 1912.
16. Schottky, W. Bericht über thermische Elektronenemission. *Jahrb. d. Rad. and El.*, 12, 149, 1915. (Synopsis.)
17. Wehnelt, A. Ueber den Austritt negativer Ionen aus glühenden Metallverbindungen und damit zusammenhängende Erscheinungen. *Ann. d. Phys.* (4), 14, 425, 1904.
18. Wehnelt, A. On the discharge of negative ions by glowing metallic oxides, and allied phenomena. *Phil. Mag.*, 10, 80, 1905.
19. Wehnelt, A. and Jentsch, F. Ueber die bei der Elektronenemission glühender Körper auftretenden Temperaturänderungen. *Verh. d. D. P. G.*, 10, 605, 1908.
20. Wehnelt, A. and Jentsch, F. Ueber die Energie der Elektronenemission. *Ann. d. Phys.*, 28, 537, 1919.
21. Wehnelt, A. and Liebreich, E. Energie der Elektronenemission glühender Körper. *Phys. ZS.*, 15, 548, 1914.
22. Willows, R. S. and Picton, T. Notes on the behaviour of incandescent lime cathodes. *Proc. Phys. Soc. London*, 23, 257, 1911.
3. Wilson, W. The loss in energy of Wehnelt cathodes by electron emission. *Proc. Nat. Acad. Sci.* 3, 426, 1917.

NOTE ON THE END CORRECTION IN THE DETERMINATION OF GAS VISCOSITY BY THE CAPILLARY TUBE METHOD.

BY WILLARD J. FISHER.

SYNOPSIS.

Assumptions Underlying Fisher's Formula for the End Correction.—Experiments by Benton having pointed to the conclusion that Brillouin's formula for the end correction represents the facts fairly well, whereas Fisher's does not hold even approximately; the writer points out that the experimental conditions assumed in the two formulas are essentially different. The use by Benton of comparatively short tubes with square cut ends gives a turbulent condition at the ends similar to that assumed by Brillouin, whereas Fisher assumes a slow stream-line flow such as could be attained only by the use of long tubes with gently tapered ends. The conclusions reached by Benton are therefore to have been expected.

IN reply to the note by A. F. Benton (PHYS. REV., 14, p. 463, 1919) "On the End Correction in the Determination of Gas Viscosity by the Capillary Tube Method," it may be said that the approximate formula for kinetic pressure drop proposed by me (PHYS. REV., 32, p. 216, 1911) is based on the following suppositions:

1. Such a low speed in the tube that the stream lines within are all parallel to the axis and expansion of the gas does not result in change in temperature, the necessary heat being supplied by conduction through the tube walls;

2. Both ends of the bore so gently tapered that the gas stream flows without anywhere leaving the glass; so that from stationary gas in the high pressure reservoir to stationary gas in the low pressure reservoir there is continuous stream line flow without turbulence.

Unfortunately neither of these conditions was explicitly stated in my note, though both clearly underlie the use made of Boyle's law and Bernoulli's theorem. I regret that my obscurity in expression should have led Mr. Benton to apply a correction formula in an experimental case to which its theory is entirely inapplicable; for the emergent jet from a square ended tube of diameter not small in proportion to its length is surely an illustration of stream line flow, and expansion within such a tube is known from published experimental data to be anything but isothermal; there is a decided temperature fall along such a tube, very pronounced as the exit is approached.

The correction formula of Brillouin is deduced for a tube cut off square at each end, and so for a turbulent squirt at the exit; also, the gas is assumed to flow as a liquid, without expansion within the tube, so that the kinetic energy wasted in turbulence and without pressure drop at the exit is entirely derived from pressure drop at the inlet. No account is made of the vena contracta.

It is interesting to note that Brillouin's correction, as applied by Mr. Benton, gives results which represent his experimental data much better than the correction based on stream line exit; and also that it gives for mass delivered amounts uniformly higher than experiment, and higher by amounts not to be attributed to errors of observation. This is to be expected as a consequence of reactions during expansion within the tube, and not only of vena contracta effects.

DEPARTMENT OF PHYSICS,
THE UNIVERSITY OF THE PHILIPPINES,
MANILA, P. I.,
February 13, 1920.

THE PHYSICAL REVIEW.

THE EXISTENCE OF HOMOGENEOUS GROUPS OF LARGE IONS.

BY OSWALD BLACKWOOD. .

SYNOPSIS.

Non-existence of groups.—Certain observers have found evidence indicating that the large ions produced by spraying water constitute several distinct groups each having a definite mobility. Using an apparatus similar to theirs, the author fails to confirm this result but finds that the ions distribute themselves continuously over a wide range with all intermediate mobilities present. In other words, he finds a *continuous* spectrum of mobilities and not a *band* spectrum. Conclusive proof of this has been obtained by a series of mobility determinations made with a Zeleney tube having a "resolving power" twenty times as large as that of the apparatus mentioned above.

Age and mean mobility.—The mean mobility of the ions decreases with time after formation at a rate indicating that the rate of condensation of water vapor on any ion is constant and independent of its size.

Growth of ions dependent upon the humidity of the air.—Having replaced the water sprayer by a red-hot platinum wire as a source of ionization, it was possible to control the water vapor density. Drying the air causes a decrease in the rate of growth of the ions.

Growth of ions probably due to condensation of water vapor.—The experimental facts are explained by assuming that large ions grow by condensation of water vapor on a nucleus. This assumption is consistent with the views as to the nature of the large ion held by Barus, Aitken, Thompson, and others.

NOLAN and McLelland¹ in 1916 and Nolan² in 1917 published papers presenting evidence which indicates that the spraying of distilled water or the bubbling of air through mercury produces large ions having a wide range of mobilities. The most mobile ion has a mobility much greater than that of the small ion produced in air by X-rays, while the most sluggish one approximates that of the stable Langevin ion found in the atmosphere. Moreover, the mobilities do not distribute themselves continuously over this range but constitute several distinct *groups*

¹ Proc. Roy. Irish Acad., Vol. 33, p. 9, p. 24, 1916.

² Proc. Roy. Soc., A, Vol. 94, 1917.

with intervening gaps. In other words, there seem to be band spectra of mobilities and not continuous spectra.

Recently the investigation has been extended, and similar indications have been found with regard to the ions produced by bubbling air through alcohol¹ or by passing it over phosphorus.²

TABLE I.

Water (Spray), <i>A.</i>	Mercury.				Alcohol, <i>F.</i>	Phosphorus, <i>G.</i>
	Ions Newly Produced.		Long Time Interval.			
	<i>B</i> , Undried Air.	<i>C</i> , Dried.	<i>D</i> , Undried Air.	<i>E</i> .		
6.5 ¹						
3.27						
1.56		•				
1.09					1.10 ?	
.53					.50	
.24					.22	.22
.12					.12	
						.092
.046			.043	.043	.049	.053
		.024				.028
.013	.014		.02		.017	.018
		.0068		.0064	.0077	.0074
.0043	.0040		.0045		.0040	.0041
		.0021		.0022	.0023	.0024
.0010	.0013	.00	.0013		.0014	.0012
		.00056			.00063	.00064
.00038	.00034				.00034	.00031
					.00015	.00015
						.000085
						.000053

Table I. gives a list of the mobilities of the groups produced by each method. In column *A* it will be noticed that the first two groups have mobilities greater than those of the small ions produced in air by X-rays. Column *G* gives a list of mobilities of the groups produced by passing air over phosphorus. This list is highly significant because the range of mobilities is large, and because the fourteen mobility values constitute an approximate geometric progression.

In view of the fact that no plausible hypothesis has been suggested to account for the production of several classes of ions of which the mobilities form a geometric progression, the groups have seemed especially worthy of investigation. In order to test out the apparatus before attempting to discover whether the groups exist in the ionization pro-

¹ Proc. Roy. Irish Acad., A, Vol. 34, p. 57, 1918.

² Ibid., A, Vol. 35, p. 1, 1919.

duced by other methods; Nolan's work on the mobilities of the ions produced by spraying distilled water was repeated with slight modifications of apparatus to be described later. The author, to his surprise, failed to confirm the existence of the groups.

The apparatus first used was essentially that of McLelland and Nolan and is shown in Fig. 1. The parts requiring especial notice are:

1. The air pressure regulator, *A*.
2. The sprayer, *C*.
3. The capillary *C'*.
4. The ion testing tube *F*.

1. The air pressure regulator *A* is a vertical tube partly submerged in water. The bubbling of air escaping from the lower end of this tube keeps the pressure approximately equal to the water pressure at that end.

2. The sprayer *C* was made entirely of glass and was especially designed for this investigation. With an excess air pressure of one half atmosphere—the value used in all the measurements—a dense fog of spray is produced and the quantity of air required is only 30 c.c. per second.

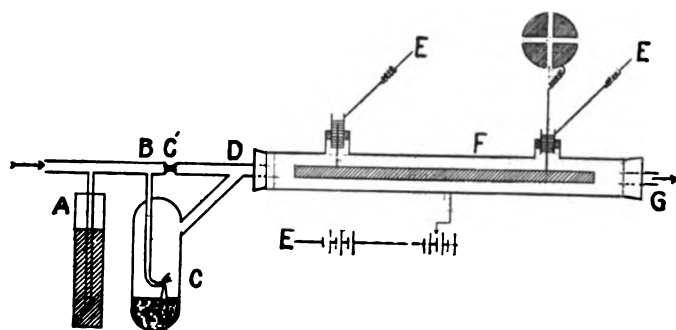


Fig. 1.

3. The capillary tube *C'* may be replaced by others of different bores, thus varying the velocity of the air current at will, while keeping the air pressure constant.

4. The ion testing tube *F* consists of a brass cylinder 160 cm. long and 5 cm. internal diameter, fitted with a co-axial cylindrical brass electrode 140 cm. long and 2 cm. in diameter. This electrode is supported by sulphur plugs mounted in grounded guard-rings. The electrode is connected to a pair of quadrants of a Dolezalek electrometer having a sensitiveness of 70 cm./volt with the scale at a distance of 200 cm. The outer cylinder may be connected to any portion of a battery of 100 fresh dry-cells.

On occasion, the rate of flow of air through the apparatus may be measured by attaching the tube of a gasometer at *G*.

METHOD OF EXPERIMENT.

Air from the laboratory system is filtered by passing it through a tube packed with glass wool. At *B*, Fig. 1, the stream is divided, part going through the sprayer *C* and part through the capillary *C'*. Reunited at *D*, the air enters the ion testing tube *F*.

The ion testing tube of the type described was designed by McLelland and has been used extensively in the determination of mobilities of large ions. These ions are carried forward in the tube by the air current and are simultaneously deflected toward the central electrode by the electric field. If the intensity of the electric field be gradually increased, the ion current to the electrode increases until the field becomes intense enough to drive all the ions to the electrode, beyond which point the current is saturated. It can be shown that—neglecting the effects of diffusion, recombination and distortion of the fields by electrode supports and by the presence of the ions—if ions of only one mobility are present the current-voltage curve is a straight line through the origin, breaking sharply at the critical voltage and becoming parallel to the voltage axis. If, however, ions of several mobilities are present, the resultant curve shows several breaks in slope, each corresponding to a certain mobility. (See Fig. 5.)

It can be shown that the critical voltage *V* for ions of mobility *K* is given by the equation¹

$$k = \frac{Q}{2\pi L V} \log_e b/a$$

a and *b* being the radii of the electrode and the cylinder, respectively, *Q* the quantity of air traversing a cross-section of the tube each second, and *L* the length of the electrode.

SOURCES OF ERROR.

In the early observations much error was caused by the falling voltage of the accumulators. The charge induced on the central electrode by a decrease of only 1/5 volt, in a total of 160 on the outer cylinder, caused an electrometer deflection of several centimeters. This variation became negligible when the accumulators were replaced by dry cells.

A persistent error was due to the variations of the sprayer. Consecutive readings usually agreed within two per cent., but the positions of

¹ This formula becomes exact for a cylinder of finite length if the part $\frac{\log_e b/a}{2\pi L}$ be replaced by the *measured* capacity of the portion of the insulated system which is exposed to the air current. See W. F. G. Swann, "The Theory of Electrical Dispersion into the Free Atmosphere," *Terrestrial Magnetism and Electricity*, Vol. 19, pp. 81-88, 1914.

the breaks in slope of the current voltage curves shifted to such an extent in successive curves that it was usually impossible to determine mean curves showing breaks. (See Fig. 2.) In the effort to eliminate

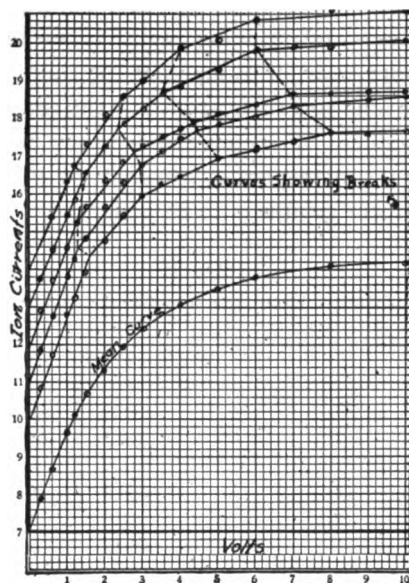


Fig. 2.

this shifting, three different types of medicinal sprayers were tried before the glass one already described was constructed.

RESULTS.

In certain cases the current-voltage curves are quite smooth. (See Fig. 3.) In others breaks in slope are present which are quite as definite as those exhibited by Nolan. (See Fig. 5.) His value for the ratio of successive mobilities, however, is not found in a single instance. This

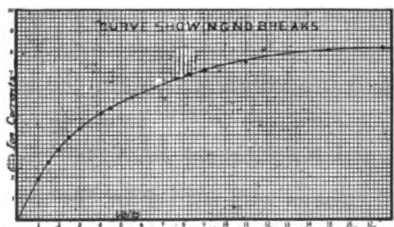


Fig. 3.

ratio, in the present investigation, is usually about 2.0 and is never as high as 3.4, the mean value determined by him.

Being convinced that the mobility values did not fit into the group system of Nolan, an attempt was made to ascertain whether my values determined from day to day fitted into any group system. Fifty-nine breaks were worked over in detail using various values of the air velocity and, in certain instances, introducing metal tubes between the sprayer and the testing cylinder in order to increase the time intervening between the production of the ions and their arrival at the testing tube. Table II. shows the mobility values. For convenience they have been arranged in classes, but if they are represented graphically by points along a straight line no tendency to form groups is to be observed.

TABLE II.
Mobility Values Determined by the Author.

Time After Production.	Mobilities of Ions.						
60 sec.		.0016	.0027	.0036 .0037	.0080 .0070	.016 .017	.028 .037
100 "	.0006	.0013 .0017		.0042 .0035 .0043	.0082	.012 .013 .014	
600 "	.0003 .0006	.0010 .0011	.0020 .0022 .0027	.0043 .0045	.0080		
1200 "	.00055 .00046 .00077 .00080	.0011 .0012 .0010 .0010	.0016 .0014 .0017 .0021 .0020 .0026	.0045 .0039 .0030 .0037 .0036 .0042 .0053	.0070 .0060 .0065 .0060	.015	

If then we disregard breaks, the smooth curve may indicate either a continuous distribution of mobilities or it may indicate that the sharpness of the break in the theoretical curve for a single class of ions has been obscured owing to the failure to obtain the ideal conditions of the theoretical method.

CRITICISM OF METHOD.

The McLelland ion tube is well suited for the measurement of mobilities which are widely separated, but if the number of breaks is large the distance between breaks is small and, owing to experimental errors, a smooth curve results. In the investigations carried on by McLelland and Nolan, in most instances not more than six breaks were located for

any individual current-voltage curve. It will be of interest to see how little such a curve differs from a smooth one corresponding to a continuous distribution of mobilities. Fig. 4 is a theoretical curve showing breaks corresponding to six different classes of ions all present in equal concentrations. The ratio of each mobility to the one preceding it is 1.9 which is the approximate value found by McLelland and Nolan for the ions produced by passing air over phosphorus. It will be noticed

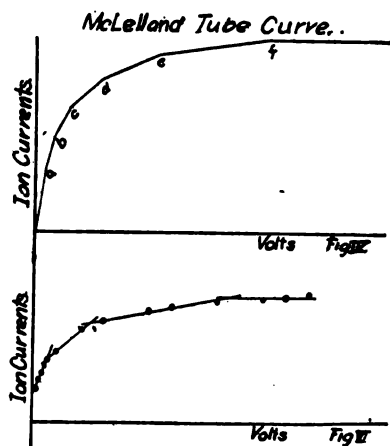


Fig. 4.

Theoretical Curve Indicating Existence of Groups.

Fig. 5.

Experimental Curve Determined by Nolan.

that the breaks, especially (a), (b), and (c) are so slight that they would easily be masked by experimental errors.

Nolan himself admits the possibility of a smooth curve being drawn through his points. Regarding the only curve showing several breaks which he exhibits (see Fig. 5), he writes:

"It might be said that a smooth curve might be drawn with almost equal exactness, showing that, instead of an abrupt step in mobilities, there was a gradual shading off from one to another, with ions of all intermediate mobilities present. With the object of eliminating this sort of uncertainty, and of obtaining as accurate values as possible for the mobilities of the different ions, the current-voltage curve was worked over in detail many times, each section being investigated under conditions specially chosen to bring out its features."¹

It is on the evidence of such breaks as these that Nolan in his earlier paper, based the claims for the existence of positive and negative ions

¹ Proc. Roy. Irish Acad., A, Vol. 33, P. 12, 1916.

whose mobilities are higher than any previously discovered for normal ions generated by X-rays, ultra-violet light, etc. To the writer's knowledge such abnormally high mobilities for positive ions have never been found by other observers though methods of much higher sensitiveness have been used, and though the experiments have been carried out not only in the presence of water vapor but also in carefully dried air in which the mobilities are distinctly higher. It is indeed true that abnormal mobilities have been found in the case of negative ions at low pressures for common gases, and even at atmospheric pressures for monatomic gases. These high mobilities, however, are believed to be due to the presence of free electrons.

In a more recent investigation¹ having modified his apparatus by replacing the cylindrical ion tube by a plate condenser, and by introducing the nozzle of the sprayer directly into the field between the plates, Nolan obtains discontinuities in his curves of an entirely different order of magnitude from those which occur in the earlier experiments. These discontinuities are certainly of a real nature and it is not in questioning their existence that one can attempt to correlate the present results with those of Nolan. It must, however, be pointed out that the breaks in his current-voltage curves may be produced as well through discontinuous changes in the electric fields driving the ions to the electrode as by a discontinuous distribution of mobilities of the ions themselves. The possibility of such an explanation of the curves of Nolan seems worthy of discussion. He himself admits regarding the observational values of his field strengths, that the applied voltage differs from that corresponding to the effective values of the fields by from zero to 4.5 volts. It is noticeable, also, that the discontinuities in the ion currents are comparable in magnitude to possible sudden changes in potential of the same order of magnitude as the values of the uncertainties in the potential difference which Nolan admits. Exactly how these discontinuities might occur, one cannot decide. It is possible that in increasing the negative value of the field strength, different sizes and different positively charged droplets produced by spraying, are successively removed from the space between the electrodes, thus discontinuously changing the values of the field. Nolan himself states that the uncertainty in his fields can be explained on the assumption of the disappearance of small positively charged droplets from between the plates. It is also to be noted that he obtains values of the positive and negative mobilities which are many times as great as the values obtained in dry air by the classical methods. It seems possible that the explanation of these mobilities

¹ Proc. Roy. Soc., A, Vol. 94, 1917.

lies in the fact that the absolute values of his fields are not definitely known. For example, for a negative carrier whose mobility he gives as 188 cm./volt $\times$ volt/cm. he finds that the break in his curve occurs at +3.5 volts. When the potential of the lower plate is further increased to +3.75 volts, no negative ions from the sprayer reach the upper electrode and the electrometer current becomes zero. Nolan assumes, therefore, that at +3.75 volts the field driving the negative ions toward the upper electrode is zero. What the experiment shows, however, is not that the field is zero, but that it is not strong enough to drive the ions to the upper electrode before they are carried past the electrode by the air current. The ions are probably driven *toward* the upper plate even though they do not actually reach it. For his experimental arrangement he finds $Vk = QA/LB = 47$ (in which A, L, B , are the dimensions of the plate condenser, Q is the quantity of air traversing it each second, k the ionic mobility, and V the potential difference between the plates). Having assumed that the critical voltage $(+3.75 - 3.50) = +0.25$ is necessary to drive the ions across the distance between the plates in the time during which they travel the length of the condenser, he interprets the mobility k as being equal to $k = 47/(3.75 - 3.50) = 188$. If one assumes under these conditions that he was actually measuring the mobility of the normal negative ion, which is 1.8, the same initial break in the curve could have been caused if the actual value of the field were $47/1.8 = 26$. This field strength could possibly have been caused by the electrification of the air produced by the sprayer which, as has been stated, is introduced directly between the plates of the condenser. As in his paper no mention is made of an attempt to verify the values of the fields by exploring electrodes or otherwise, there seems to be no *a priori* reason why one interpretation of this result is not just as adequate as the other. Too much confidence, then, cannot be placed on the interesting results of Nolan until there is more certainty as to the values of his fields.

THE ZELENKY METHOD.

It was obvious that, using the method of McLelland and Nolan no positive decision as to the real existence of the groups of ions could be obtained. Recourse was accordingly had to a modification of the Zelenky method which seems much better adapted for giving a decisive verdict.

The electrode of the ion tube already described was cut into two sections L' and L'' , 105 and 35 cm. long respectively, which were separated by a small air gap. The electrode L' was grounded and L'' was connected to the electrometer. (See Fig. 6.)

THEORY.

The ions, produced in the same manner as that described in the previous section, are carried by the stream in a direction parallel to the axis of the tube. Disregarding diffusion, if there is no potential difference between the outer and inner cylinders, none of the ions will reach the inner electrodes. If, now, the outer cylinder be raised to a potential

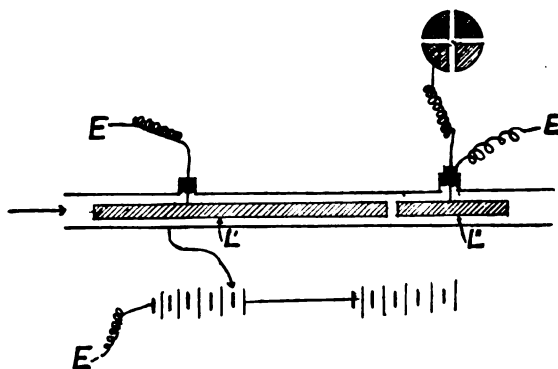


Fig. 6.

above the inner ones, all the positive ions entering the tube within a distance r of the axis are driven to the electrodes, while all others escape. As the potential difference increases, the value of r also increases and, consequently, with it the ion current to the electrodes. At a critical voltage V' , r becomes equal to the radius of the cylinder and all the ions are driven to the electrodes so that the current reaches its maximum. As the potential difference is still further increased, more and more ions are driven to the first, grounded, electrode, and consequently the electrometer current from L'' diminishes until, at a second critical voltage V'' , it becomes zero.

Neglecting the influence of recombination and diffusion of the ions, and assuming that the electric field is radial at all points in the tube, it has been shown that the Zeleny tube gives a current-voltage curve of the form shown in Fig. 7 (C).¹ The nature of this curve may be deduced from the curves A and B (Fig. 7) which are determined when the electrometer is connected (as in the McLelland arrangement already described) (a) to the first electrode L' and (b) to both electrodes. The curve (C) of which the ordinates are evaluated by subtracting the corresponding ordinates of A from those of B , therefore represents the ion current to L'' when L' is grounded.

¹ Bloch, *Ann. de Chemie et de Physique* (8), Vol. IV, p. 25, 1905.

From the curves *A* and *B* it is evident that V' is the voltage causing saturation when the electrode has the length $(L' + L'')$ while V'' causes saturation using an electrode of length L' .

For ions of mobility k , we have in the two cases:

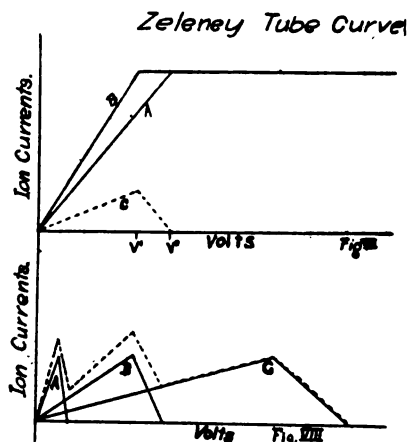


Fig. 7. Fig. 8.

$$k = \frac{Q \log_e b/a}{2\pi(L' + L'')V'} = \frac{Q \log_e b/a}{2\pi L'V''}; \quad (1)$$

$$\therefore \frac{V'}{V''} = \frac{L'}{(L' + L'')}. \quad (2)$$

The saturation voltage V' being given for ions of mobility k , it may be computed for ions of mobility K' from

$$k/K' = \frac{V_1'}{V'}. \quad (3)$$

If the maximum ion current be known, the current-voltage curve for the Zeleny apparatus may be constructed for ions of a given mobility with the aid of equations (1), (2), and (3).

Figure 8 (*A*), (*B*), (*C*) shows the forms and relative positions of the current voltage curves of the Zeleny apparatus for ions of mobilities .0043, .0010 and .0004. These are three mobility values for successive groups as found by Nolan for ions from spray. The dotted line shows the summation curve determined if the ions of the three groups are present in equal concentration.

If the number of groups should increase, and the interval between successive groups consequently become smaller, the broken line would be less deeply notched and, if the ions did not constitute distinct groups but distributed themselves continuously over the range from .0043 to

.0004, the curve would be smooth as indicated by the dotted line in Fig. 9 *A*. To decide whether the groups exist or not, it is therefore sufficient to decide whether the experimental current-voltage curve has one or more than one maxima.

Similarly, Fig. 9 *B* shows the theoretical curve given by the McLelland tube if the same three groups of ions are present in equal concentration. The dotted line shows a smooth curve such as might be given by a

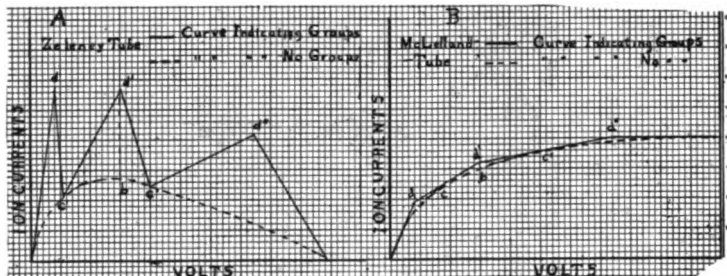


Fig. 9.

continuous distribution of mobilities. Comparing the curves *A* and *B* it is evident at a glance that the Zeleny apparatus affords a more decisive test as to the existence of the groups.

We shall arbitrarily define the "resolving power" of each instrument for any type of ions as the ratio bd'/cc' (Fig. 9 (*A*) and (*B*)). In the case of the three breaks in *each* curve, the "resolving powers" are approximately.

Group.	d .	d' .	d'' .	Mean.
"Resolving Powers" { Zeleny Tube	2.	1.	$\frac{1}{2}$	1.2
McLelland Tube	0.1	0.07	0.025	0.065

The resolving power of the Zeleny tube for the curves shown is accordingly about 18 times as great as that of the McLelland arrangement.

MEASUREMENTS WITH ZELENY TUBE.

Owing to the fact that the available time was limited, it was possible to determine only seven curves for spray ions using the Zeleny apparatus. In certain instances metal tubes were introduced between the sprayer and the ion tube in order to "age" the ions or increase the time interval between their production and arrival at the testing tube. Two specimen curves for different ages are shown in Fig. 10. Several interesting conclusions may be drawn from their forms and relative positions.

NON-EXISTENCE OF GROUPS.

The fact that this apparatus, having a "resolving power" many times greater than that of the McLelland arrangement, gives curves showing no discontinuities, seems conclusive evidence that a series of well-defined groups does not exist. Each curve shows one maximum and inversion point which indicates a preponderance of ions whose

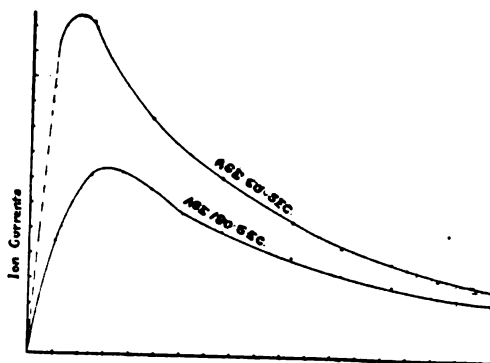


Fig. 10.

mobilities lie on a narrow range. The curves also show, by the fact that they approach the voltage axis asymptotically, that other ions of very low mobility are also present.

EFFECT OF AGEING.

When tubes are interposed between the sprayer and testing tube to "age" the ions, the center of gravity of the curve shifts to the right, indicating that the mean mobility has decreased. This agrees with the observations of Nolan. He explains this decrease as due to the disappearance of the smaller ions owing to their diffusion to the walls of the containing vessel; to their recombination to form larger neuclei; or to their growth by condensation of water vapor.

A more striking series of curves was obtained when the sprayer was replaced by a red-hot platinum filament as a source of ionization. This wire was found to afford a copious supply of positive ions, but a negligibly small number of negatives.

In this case, as before, tubes were interposed between the sprayer and the ion testing cylinder in order to age the ions. Several curves for different ages are shown in Fig. 11. In view of the fact that a preponderance of the ions for any age have mobilities lying on a narrow range on either side of the mobility value for the ion which is present in greatest numbers, an effort was made to interrelate the ages with

the respective mobility values for the maximum-concentration ion. The critical voltage corresponding to the maximum ion current was therefore determined for each curve, and the mobility values were computed with the aid of equation (1). The results are shown in Table III.

If one plots the logarithms of the values of these mobilities as ordinates

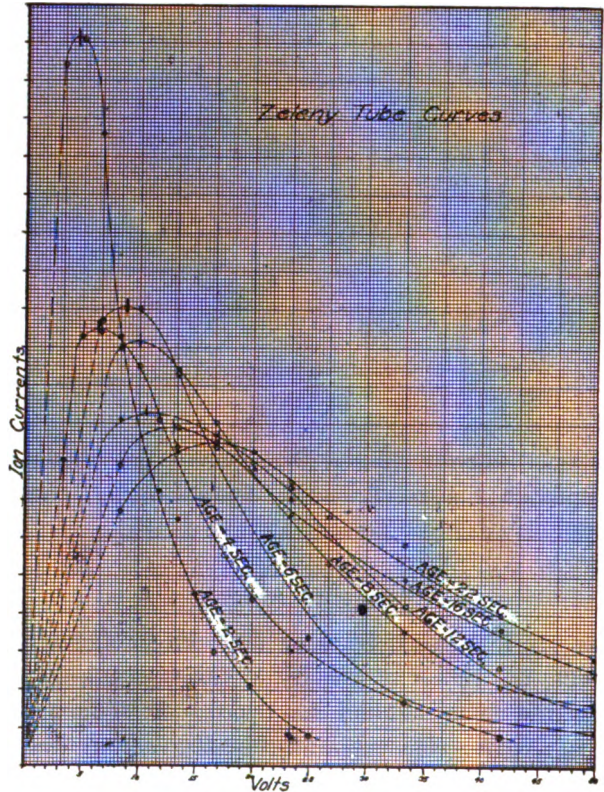


Fig. 11.

TABLE III.

Mobility Values for Ions Present in Greatest Concentration.

	Age.										
	2.	4.	6.	8.	12.	16.	22.	32.	45.	58.	70.
Mobility..	0.06	0.042	0.029	0.026	0.023	0.020	0.016	0.012	0.009	0.0075	0.0065
Log age ..	0.30	0.60	0.78	0.90	1.08	1.24	1.351	1.51	1.65	1.76	1.85
Log ($k \times 1,000$)..	1.78	1.62	1.46	1.41	1.36	1.30	1.20	1.08	0.96	0.87	0.81

against the logarithms of the corresponding ages as abscissas, the points are found to lie sensibly along a straight line of which the slope is 6/10 (see Fig. 12). This differs from 2/3 by 10 per cent. If we assume 2/3 to be the correct value, we may interrelate the age of the ions with the mobility of the maximum-concentration ion as follows:

$$\begin{aligned} 3/2 \log k + \log t &= \log C, \\ k^{3/2} t &= C, \end{aligned} \quad (4)$$

in which k represents the mobility of the maximum-concentration ion, t the age of the mixture of ions (defined on page 12), and C is an empirical constant. If in accordance with the "hard elastic sphere" theory of J. J. Thompson⁷ we assume that the mobility of a large ion varies inversely with its radius squared, equation (4) may be written:

$$\frac{C'}{4/3\pi r^3} \times t = \bar{C}$$

or

$$t = C'/C'v;$$

$$\therefore dv/dt = C'/C,$$

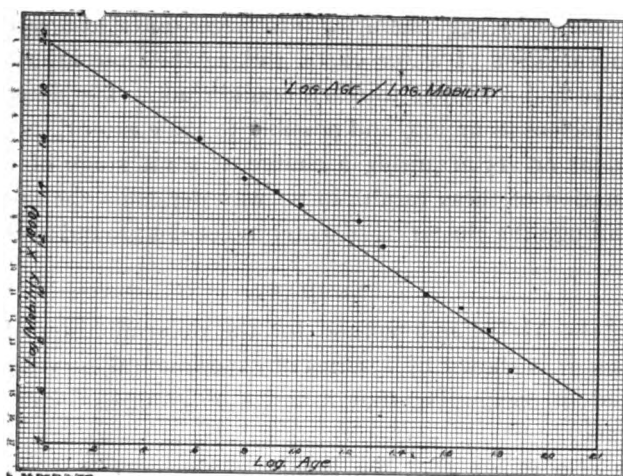


Fig. 12.

C' being a constant of proportionality, and v the ionic volume. This indicates that the ions are growing at a constant rate, independent of the radius.

⁷ Thompson, Conduction of Electricity through Gases.

HUMIDITY AND RATE OF GROWTH OF THE IONS.

Using the hot-wire method of ion production, it was possible to control the water vapor density in the air which was driven through the ion tube. The air was dried by passing it over CaCl_2 and P_2O_5 . The air was then charged with water vapor by bubbling it through a column of water, the temperature of which was raised in successive "runs," so as to increase the vapor density. It was found that for a given age increasing the vapor density decreased the mean mobility of the ions, which indicates that the presence of water vapor is favorable to the growth of the ions.

The conception which one gains from the above experiments as to the nature of the formation of the large ions is consistent with the views published in the papers of Barus,¹ Aitken,² Pollock,³ Lenard,⁴ and others. It is not at all inconsistent with the experiments of C. T. R. Wilson as explained by Thompson.⁷ According to our results, the original nucleus of the large ion may be an agglomeration of a few molecules of water or a few particles of dust which have gathered a charge either in the process of formation (spraying of water), or by picking up a charge while passing through the ionized gas surrounding the hot platinum filament. These nuclei, due in part to their charge, may continue to grow by condensation of water vapor or by agglomerating into larger units, at a rate depending on the concentration of water vapor and on the time interval in which the ions have had a chance to reach equilibrium. The detection of the particle or nucleus as an ion, and possibly to some extent, its dimensions, are dependent on its acquiring a charge at some stage of the process. The explanation of these results in no way demands the growth of the large ion through the clustering of water molecules around a single charge.

SUMMARY.

I. Using the McLelland method for the determination of mobilities of large ions, it is shown by a series of experimental curves that we are not justified in concluding that a series of groups of ions of definite mobility exist.

II. It is also shown that the breaks in the curves obtained by the author which indicate the existence of groups of definite mobility, are too uncertain to permit of their interpretation in this way in view of the possible magnitude of experimental errors.

III. It is also pointed out that the experimental curves obtained by

¹ *Amer. Jour. Science*, 33, p. 107, 1912.

² Aitken, *Roy. Soc. Edinburgh Proc.*, 37, p. 215, 1916-17.

³ Pollock, *Phil. Mag.*, Vol. 29, p. 514, 1915.

⁴ Lenard, *Ann. Physik*, Vol. 47, 44, July, 1915.

Nolan in a more recent paper in which the breaks are unquestionably present, may be interpreted in other ways not involving the assumption of the existence of groups of ions of abnormally high mobilities.

IV. The problem was also attacked using the Zeleny method of mobility measurement which gives a much higher "resolving power," than the McLelland method, with the result that no evidence was found for the existence of groups of ions of several different mobilities.

V. Further results are given showing a relation between the mean mobility of the ions present and the time which has elapsed between the formation of the ions and the measurement of their mobility, in the case of ions formed by spraying water and from hot wires. These results indicate that the mobility varies as the $2/3$ power of the reciprocal of the age, from which it is deduced that the rate of growth of the ions is constant.

In conclusion I wish to express my gratitude to Dr. Millikan under whose direction this investigation has been carried out, to Dr. A. J. Dempster for occasional assistance, and to Dr. Leonard Loeb for aid in the interpretation of results and the revision of my manuscript.

RYERSON PHYSICAL LABORATORY,
January 5, 1920.

ELECTRICAL DISCHARGES FROM POINTED CONDUCTORS.

BY JOHN ZELENY.

SYNOPSIS.

Surface Action in Electrical Discharges from Points.—The lag phenomena shown by the discharges from some points indicate a special surface action. Various possible surface actions are discussed and attention is centered upon an adhering layer of gas molecules. Further evidence is sought in discharges from points made of different materials.

Electrical Discharges from Water Points. Surface Electric Intensities and Stopping Voltages.—Discharges from water points begin impulsively. The surface intensity during positive discharges is independent of the current and a simple relation exists between values of this intensity, point radii and air pressures. Empirical relations are given connecting point radii and stopping voltages. The results are compared with those from similar metal points.

Electric Fields for Water Points in Air, Oxygen, Hydrogen and Carbonic Acid.—A table of these fields is given for two water points with different gas pressures ranging from 10 cm. to 87 cm. of mercury but no relation between these values and other constants was found.

Comparison of Electric Fields for Points of Water, Glycerine and Methyl Alcohol, and of Stopping Voltages for These Liquids and for a Brass Point.—The comparison was made in air at different pressures, and no differences between the different substances were observed with the exception that some of the results for methyl alcohol were smaller than those for the other substances.

Starting Voltages for a Brass Point Coated with Various Salts.—The surfaces were far from smooth but the positive discharge started at nearly the same potential from all of the different coatings, while the negative starting potentials varied greatly among themselves.

Dependence of Critical Fields upon the Curvature of Surfaces of Points.—An explanation is given of the fact that the critical field required for the production of a discharge is larger the smaller the radius of the point. The dependence of the field at the surface of a point during a discharge upon the divergence of the field is shown experimentally.

Discharge Currents for Points Made of Different Materials.—Testing for a possible ejection of ions from a surface by impact, similar points of platinum, brass, copper and water under the same conditions gave the same currents except that the negative current from water was slightly smaller than from the other substances.

POSSIBLE SURFACE FACTORS IN POINT DISCHARGES.

1. The theory of the flow of electrical currents from pointed conductors which is generally accepted assumes that the ions which carry the discharge current are produced solely by collision with molecules of the gas of the few ions normally created in the gas by radiations from radioactive substances. Many of the general characteristics of these dis-

charges may be explained qualitatively on this theory by the known properties of ions, without the need of assuming any action at the discharging surface such as ejection from it of ions by the impact of those colliding with it or such as would arise if the ions met with difficulty in discharging to the metal surface.

There are some features of these discharges, however, which indicate that at least under certain circumstances a special action does take place at the discharging surface.¹ Thus for example the discharges from some metallic points begin very impulsively as the voltage applied is gradually increased, as if some resistance had been suddenly overcome; and what is even more significant, on decreasing the voltage after a discharge has started in these cases the current breaks off abruptly. An explanation of this sensitivity of some points, as such behavior is often called, must be sought in some condition at the discharging surface because points to all appearances similar differ very markedly in this property; and a point not possessing the property may be made to acquire it by treatment which does not appreciably change the form of the point.

Edmunds² however ascribes the retardation in the commencement of discharges from points to the fact that the ions normally in the gas are few in number and that some time may elapse after a voltage is applied before ions may chance to assume a favorable distribution along that restricted region where the field is strongest and where accordingly ionization by collision can take place for the lowest permissible applied voltage.

It is difficult to see why this argument should apply to some points and not to others to all appearances of the same form which do not show the lag phenomenon. Moreover, the abrupt stopping of a current which accompanies the lag in starting cannot be explained in this way.

However, when a point is in a condition to show a lag in the commencement of a discharge, the discharge is often brought on by the sudden production within the field of force of an exceptionally large number of ions, a matter which will be considered more fully at a later time.

2. What possible actions may we suppose taking place at a discharging surface which surface conditions could aid or retard? We may suppose that electrons or ions are being pulled from the surface by the strong electric field; or that electrons or ions are being ejected from the surface by the impact of other ions against the surface; or that under some

¹ See J. Zeleny, *PHYS. REV.*, N.S., 3, p. 69, 1914.

² P. J. Edmunds, *Phil. Mag.* (6), 28, p. 234, 1914.

circumstances ions coming from the gas may find it difficult to discharge themselves to the surface. The state of the surface might influence greatly any of these actions.

That the pulling of ions from the surface by the field plays any appreciable part in point discharges is made improbable by the results of Almy¹ who working at distances of the order of a wave-length of light found that no discharge passed between the electrodes for fields as high as 1.7×10^7 volts per cm., although a discharge did pass with still higher fields. The field at the surface of a discharging point is ordinarily very much smaller than the field named and the only possibility of any such action actually taking place lies in the fact that in Almy's experiments the conditions were not favorable for the multiplication by collision of any ions that might have come from the metal.

The ejection of electrons from metal surfaces by the impact of ions is postulated in the explanation of some phenomena connected with discharges at low pressures, and a similar activity at the surface of a discharging point is not excluded. Ionization of gas molecules adhering to a metal surface by the impact of ions coming to it would produce effects similar to those resulting from ejection of electrons from the surface, and moreover might take place with either a positively or a negatively charged point. Neither of these effects however could change appreciably the voltage at which a current is observed to begin from a *positively* charged point, since a positive ion coming from the surface would be unable to ionize molecules of the gas by collision if the negative ion which produced it had not been able to do so, and without further increase these added ions could not give an observable current unless the highly improbable supposition were made that each original ion that strikes the surface gives rise to nearly a million new ions at the surface. The production of ions at the surface should result rather in an increased current for voltages above the critical one over the current due to the ionization produced by collision of those ions which originated in the body of the gas. See section 21.

What seems to be the most probable part that the surface plays in the discharge phenomenon is that under certain conditions, owing to the presence of a non-conducting coating, ions from the gas find more or less difficulty in discharging themselves to the surface, and hence a larger voltage than usual is necessary for the commencement of the discharge.

The lag in the commencement of the current from points can scarcely be laid to the accumulation of non-conducting dust on the surface, for

¹ J. E. Almy, *Phil. Mag.* (6), 16, p. 456, 1908.

Edmunds (*loc. cit.*) took great care to free the air from dust in the discharge vessel used; and points may be sensitive when no dust can be seen on the surface with a low power microscope. Small dust particles doubtless influence the magnitude of the current from a point to which they are adhering, but they would have to cover a large part of the surface to produce the large lag in starting, which is observed at times.

The sensitivity in question cannot be due to contamination of the surface by volatile substances since it occurs with metal points which have been heated to incandescence.¹

Gorton and Warburg (*loc. cit.*) have shown that with points made of copper or iron the sensitivity appears and disappears with the formation and reduction respectively of a layer of oxide. But sensitivity can scarcely be ascribed in general to the influence of non-conducting oxides, since it is known to occur with platinum points as well as with points made of other metals.

A possible cause of the lag phenomenon applicable to all materials, is to be found in the condensed layer, either of water molecules or of molecules of the surrounding gas, which is believed to cover the surfaces of solid and liquid substances, for if this layer is a poor electrical conductor it is probable that under certain conditions it exerts a marked effect upon these discharges by preventing or retarding the passage of electricity from the gas to the metal point.

That gaseous ions do not readily give up their charges to a metal surface is shown by the experiments of Gaede² who found that metal plates to which a discharge from a point had been allowed to flow exhibited a marked polarization when tested for the Volta effect. Gaede found that even 15 secs. after a measured quantity of electricity was allowed to flow from a point to such a plate, on immersing the plate in an electrolyte, over one half of this charge could be recovered from the plate. It is natural to suppose that it is the non-conducting layer of condensed gas that keeps the ions from discharging readily to the metal.

3. To account for the behavior of sensitive points by such a layer of gas or water molecules it is necessary to assume first, that this layer when solidly packed must be punctured and partially dissipated before a current of any considerable magnitude is able to flow to the surface, and second, that the layer is able to reform and thus interrupt the discharge when the current is below a certain value.

On these assumptions sensitive points should be those having smooth homogeneous surfaces on which closely packed layers of condensed gas

¹ F. R. Gorton and E. Warburg, *Ann. d. Phys.*, (4) 18, p. 128, 1905.

² W. Gaede, *Annalen der Physik* (4), 14, p. 669, 1904.

can form. Normal or insensitive points on the other hand should be those that have either surface roughnesses not large compared to a molecule, thus preventing a close packing of the adhering molecules, or those where the adhering molecules have such a loose packing as to leave places in the gas layer through which the oncoming ions can pass, and in so doing eventually perhaps clear the neighboring surface of the molecules adhering to it.

It may be cited that, in agreement with this view, it is found that points with freshly ground or filed surfaces are in general not sensitive, whereas among points showing a large lag are those made smooth by fusion and zinc points made smooth by amalgamation (Gorton and Warburg, *loc. cit.*) and liquid points, whose surfaces are inherently smooth. Professor Kovarik informs the writer that when the usual method of making a steel point sensitive by fusion fails, a microscopic examination of the surface reveals the presence of a slightly raised scale of oxide formed by a crack in the otherwise glossy surface.

There are some ways of making points sensitive, however, which cannot easily be reconciled with the idea that a smooth surface is always a necessary condition. Thus Gorton and Warburg (*loc. cit.*) found that a platinum wire could be made sensitive by heating to incandescence by the passage of an electric current, in which case the surface was not hot enough to smooth down slight inequalities by fusion. They also found that the sensitivity was produced when the wire point was thus heated in moist air or moist oxygen whereas when sensitive the point was reduced to its normal condition by heating in these gases after they had been dried by passage through sulphuric acid.

A circumstance of significance is that on certain days it seems almost impossible to prepare a metal point showing sensitivity (when used in the open air), whereas at other times nearly every point shows more or less sensitivity. This behavior rather indicates that something gathers on the point surface which is more abundantly present in the atmosphere on some days than on others. Water vapor naturally comes to mind, but Edmunds (*loc. cit.*) observed lag in gases which had been well dried.

Unless a wrong interpretation has been put upon the experimental evidence cited, none of the views presented gives a satisfactory explanation by itself of the lag phenomenon under all of the circumstances that are known to affect it.

Some of the experiments to be described below give additional evidence on this problem, although others are also included in this paper which are of interest mainly from other considerations.

In seeking for a possible effect of a surface layer, no attempt was made

to look for a very slight effect. The accuracy attainable in the measurements is usually not sufficient for distinguishing differences of less than one per cent. in the current or voltage which a change of conditions may produce. There are certain irregularities present in these discharges, especially noticeable with small negative currents, which often make it impossible to get exact repetitions of observations made with the same point. These very irregularities however point to some action at the surface. As the edge of the luminous area on a negative point is observed through a microscope, certain irregularities of outline are seen to be constantly changing as if something more is taking place at the surface than the mere delivery of charges by the ions coming from the gas. A similar effect is noticed in the glow areas on the electrodes of discharges at low pressures. A spark often shows a reluctance to move from one portion of a metal surface to another. These effects as well as the lag phenomena which appear with sparks and discharges in tubes at low pressures are probably of the same character as those observed with point discharges.

4. The amount of condensed gas upon a surface is generally supposed to depend upon the nature of the material of which the surface is composed, upon the pressure and the nature of the constituents of the gas itself, and upon the temperature; but Langmuir¹ has recently brought forward evidence favoring a mono-molecular layer which for different materials varies in closeness of packing.

Another factor which may possibly also enter into the problem is that the surface layer of gas on highly electrified points and even the density of the gas in the immediate neighborhood of such surfaces may be augmented by the attraction to which the gas molecules are subjected owing to their polarization by the strong field. A calculation shows that of itself this attraction is not sufficient to hold molecules against the surface; but when added to the forces already present which are capable of holding neutral molecules in position, it may for certain gases increase the number of molecules adhering to the surfaces of some materials, at least. This argument fails if, as postulated by Langmuir (*loc. cit.*) we are to look upon each adhering molecule as held by a definite bond which it completely satisfies in a classical sense.

Precht² found that coating a steel point with copper did not change either the voltage for which discharges commenced from the point or the magnitude of the currents at higher voltages, but Hovda³ concludes

¹ I. Langmuir, *Am. Chem. Soc. Jour.*, 38, p. 2221; 39, p. 1848.

² J. Precht *Wied. Annalen* 49, p. 150, 1893.

³ O. Hovda, *Göttingen Inaugural Dissertation*, 1913.

from a long series of careful measurements that the voltages for which discharges begin do depend to a slight extent upon the nature of the metal. But doubt is thrown upon this conclusion by the fact that there were noticeable differences in the sharpness of the ends of the conical points which were used, differences whose effect could not be wholly eliminated and which may easily account for the small differences in the results obtained.

LIQUID POINTS.

Some time ago the writer¹ did some work on electrical discharges from a new class of points, consisting of minute hemispherical drops of water protruding from the ends of fine tubes. A method of measuring the electric intensity at the surface of the drops was devised and a study was made of the value of this intensity in air at atmospheric pressure, for a number of points differing in size. It was noted that the surface of the water becomes agitated when the electric current starts to flow from the point. For positive discharges this agitation is confined to small values of the current, the surface being quiescent for larger currents. A careful study was made later² of these initial surface disturbances and it was shown that they arise from the surface becoming unstable when the electric intensity exceeds a certain limit. Under these conditions fine threads of liquid are rapidly pulled from the surface which break up into myriads of minute drops that act as carriers of the electric charge. When water, in air at atmospheric pressure, is used, the surface instability begins at a potential which is only a little below that at which the discharge would start from an undisturbed surface. For this reason the true cause of the surface disturbances was not discovered until some work was begun on discharges in other gases than air.

The values of the electric intensities given in the paper first mentioned and the relation found for the dependence of these intensities upon the curvature of the discharging surface apply to conditions for surface instability rather than to the initial stages of the electric discharge, although for the smaller points the intensities for the two phenomena are almost identical.

The electric forces at the surface of water points, when about to discharge positive electricity, which were indicated by those measurements, were considerably smaller than those found at the surface of platinum points by Chattock.³ It seemed desirable therefore to repeat some of the measurements on the electric forces acting with discharges from liquid

¹ J. Zeleny, *PHYSICAL REVIEW*, 3, p. 69, 1914.

² J. Zeleny, *Proc. Camb. Phil. Soc.*, 18, p. 71, 1915; *PHYSICAL REVIEW*, 10, p. 1, 1917.

³ A. P. Chattock, *Phil. Mag.* (6), 20, p. 270, 1910.

points under conditions free from surface instability to ascertain whether these forces are actually different from those that obtain at the surfaces of similar metal points. The difficulty arising from surface instability may be avoided by working with air at reduced pressures, since in this way the discharge voltage is lowered while the voltage at which instability sets in is not affected.

APPARATUS FOR DISCHARGES FROM LIQUID POINTS.

6. The apparatus used for work on discharges from liquid points was arranged as shown in Fig. 1. *T* is a brass cylindrical vessel 15 cm. high

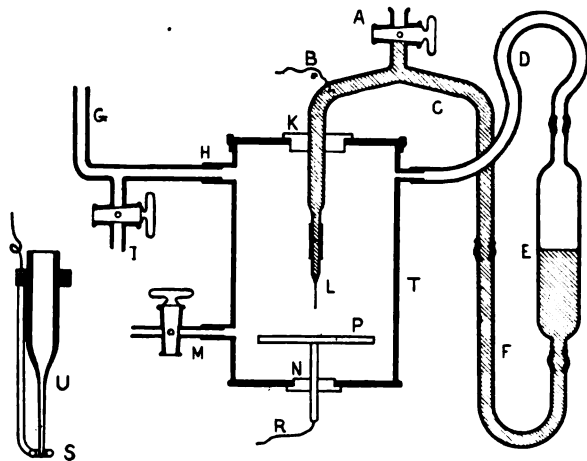


Fig. 1.

and 9 cm. in diameter. The outlets *G*, *I* and *M* at the left, lead to a pressure gauge, to the gas supply and to the pump respectively. A glass tube *C* passes through an insulating plug *K* in the upper end of the vessel and carries at its lower end the drawn out glass point *L* from which the hemispherical drop of liquid protrudes, the electrical discharge from which is under study. The brass disc *P* receives the discharge from the point and is connected to earth through a galvanometer. The glass tube *C* is connected, by the rubber tube *F*, to the bottom of the movable reservoir *E*, the top of which is in turn connected by a similar tube *D* to the vessel *T*. Liquid extends continuously from the reservoir *E* to the drop at the end of *L*, and the height that the liquid surface in *E* is above the end of the point *L* is a measure of the pressure in the drop. The platinum wire *B* makes connection with the liquid in the system and leads to a Braun voltmeter and a battery of Leyden jars charged by an electrostatic machine. Some calcium chloride was kept in the bottom

of the vessel T so that water overflowing to the sides of the point, as often happened, would be removed by evaporation. The drop at the end of the point L was observed with a microscope through glass windows which are not shown in the figure.

The distance from the tip end of a point to the plate opposite, which was 5 cm. in diameter, was 1.5 cm. in each case. This distance affects the voltage values but not the values of the surface electric intensities. The average temperature of the room was about 16° C.

The fifteen points which were used in the experiments had radii ranging from 0.117 mm. to 0.988 mm., and were made from quill glass tubing which was drawn down to the proper diameter and broken squarely across from a fine scratch. The distilled water used was very slightly acidulated with hydrochloric acid to increase its conductivity, so that during a discharge there was no appreciable potential drop between the end of the liquid meniscus and the voltmeter.

GENERAL BEHAVIOR WITH LIQUID POINTS.

7. The electric intensity f at the end of the drop is obtained from the distance p , that the liquid surface in E must be lowered to maintain the drop of the same form when charged as when uncharged, by means of the relation

$$f = \sqrt{8\pi p d g}, \quad (1)$$

d being the density of the liquid. When the surface is not discharging a current, the electric intensity is not the same over the whole surface, being greatest at the tip end. To maintain equilibrium the shape of the drop changes slightly from the hemispherical form. This does not apply to a surface discharging a *positive* current, for the current flows from the whole hemisphere (except for very small currents) and the intensity is found to be independent of the current density.

A noteworthy feature of the discharge from water points in air at reduced pressures is the retardation in the commencement of the current. As the voltage of the point is gradually increased the current does not begin gradually, but rises more or less suddenly to a value of the order of a microampere.

The retardation for any point is not constant in amount, but depends somewhat upon the age of the liquid surface and upon the time that has elapsed since a current last flowed from the point, and upon a chance element in the formation of ions in the gas as the applied voltage is raised.

As has been stated in section 1, a similar retardation in the current

often occurs from metal points, which is larger and more frequently present with negative discharges than with positive discharges. But with many metal points the retardation is very small or not present, whereas with water points a retardation is the general rule. This retardation would in many cases be still larger than is observed were it not for the fact that a voltage is reached first at which the surface becomes unstable and the discharge starts owing to a disruption of the surface.

When the current does start the water meniscus jerks back to a more flat position, because for the same voltage the electric pull is smaller with than without a current.

After the current has started, the meniscus is quiescent with a positive discharge (but not with a negative discharge), and large changes in the magnitude of the current produce very little or no effect upon the electric pull upon the surface. (See end of section 12.)

As is the case with sensitive metal points in a lesser degree, the discharge current may be diminished below the value it suddenly assumed at commencement, and as the voltage is lowered to within about 50 volts of the value at which the current would disappear if it kept on diminishing at its previous rate of diminution, the meniscus as a rule suddenly elongates and the current stops. The increase in the electric pull with diminution of voltage is so rapid during this final stage that it is extremely difficult to regulate both the voltage from the static machine and the liquid pressure necessary for maintaining the drop hemispherical, without having the water overflow to the sides of the glass tube. The most successful readings taken in this region showed that as the current fell to zero value, the total increase in the electric intensity as measured by the increase in the electric pull, may be as much as ten per cent. of the whole value.

8. The retardation which has been discussed, occurs both with positive and negative discharges, and accordingly it is not possible to get any very definite voltages nor surface electric intensities for which currents begin to flow from these points.

It is possible however to obtain definite values for the electric intensities at the surface of points from which positive currents above a certain minimum are flowing, and to obtain the voltages for which these currents stop; and this has been done. Such measurements cannot be made with negative discharges because with them the surface of the liquid is agitated, owing to an intermittent element in the current and to the fact that the negative discharge is confined to a minor portion only of the surface. Often with some of the larger negative currents, however, the surface becomes almost quiescent.

As explained above, the electric intensity at the surface of a point changes but little with the current except just as the current is about to stop, where its value rises rather rapidly about 10 per cent. The values which will be recorded were taken in the region of small currents where the intensity begins to be constant in magnitude, but they apply equally well to larger currents.

The range of pressures that could be employed in the experiments was limited, on the lower side, by the pressure (about 10 cms. of mercury) at which the water commenced to vaporize in the upper portion of the glass tube; and on the upper side, by the value for which the discharge voltage was smaller than the voltage for which the surface became unstable. For the smallest points, observations could be made up to atmospheric pressure but this pressure could not be reached with the larger points. Owing to experimental difficulties the limits mentioned were not always attained.

ELECTRIC INTENSITY AT SURFACE DURING POSITIVE DISCHARGE.

9. The electric intensity at the surface of a point discharging a small positive current was determined for each point for a number of air pressures by the method previously explained. The results for some of the points expressed in electrostatic units per cm. are plotted as broken line curves in Fig. 2 against pressures expressed in centimeters of mercury, the radius of the point in millimeters being indicated on each curve.

From the whole set of these curves, the full line curves in Fig. 2 were constructed, each giving the relation between the electric intensities and the radii of the points, for the air pressure affixed to the curve. It is seen that the intensity diminishes as the radius of the point increases, the rate of change being most rapid for the smallest points. An explanation of this fact is given in section 19.

EMPIRICAL RELATIONS.

10. The relations shown by the whole set of curves in Fig. 2 is expressed very well by the equation

$$f = 0.955 p + 5.60 \sqrt{\frac{p}{r}}, \quad (2)$$

f being given in electrostatic units per cm. when p is taken in centimeters of mercury and r in centimeters.

An approximate formula of this form was obtained by Edmunds¹ for the field at the surface of a point at the commencement of a discharge

¹ P. J. Edmunds, Phil. Mag. (6), 28, p. 234, 1914-

(positive and negative not distinguished) between a point and a plane, who adapted to this case Townsend's method for coaxial cylinders, in which use was made of the empirical relation found by Baille for the

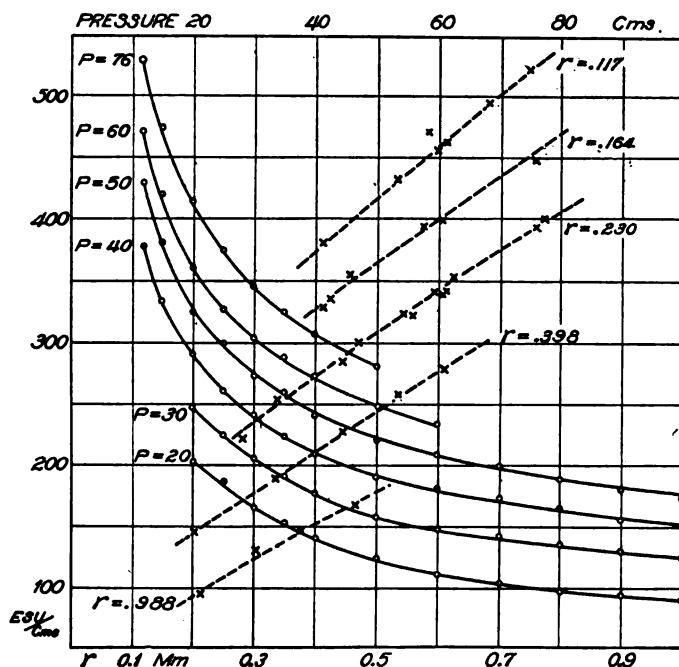


Fig. 2.

Electric intensities at surfaces of water points during positive discharge. Dotted line curves show the variation of the electric intensity with air pressure for individual points whose radii are given in millimeters. Full line curves show the variation of the electric intensity with size of points at constant air pressure expressed in centimeters of mercury.

dependence of the sparking potential between parallel plates upon the distance between them.

The constants in the formula as derived by Edmunds give values of f which are from 20 to 30 per cent. larger than those here determined with a current flowing.

II. Equation (2) may be written as

$$fr = 0.955 pr + 5.60 \sqrt{pr}, \quad (3)$$

showing that fr is a function of pr only. By taking different values of p and r whose products are constant, the products of the corresponding values of f and r are also constant. An illustration of this relationship is given in Table I., the data for which were taken from the curves in Fig. 2, a value of r being chosen from each pressure curve which gives the

constant product of pr placed at the top of each set of numbers, and the value of f corresponding to this value of r being read from the curve. The last column indicates the degree of constancy of fr for each case considered.

TABLE I.

$pr = 0.912.$				$pr = 2.8.$			
$r.$	$p.$	$f.$	$fr.$	$r.$	$p.$	$f.$	$fr.$
.012	76	520	6.24	.0368	76	318	11.7
.0152	60	413	6.28	.0466	60	255	11.9
.0182	50	342	6.22	.0560	50	210	11.8
.0228	40	270	6.16	.0700	40	171	12.0
.0304	30	202	6.14	.0933	30	128	11.9
.0456	20	134	6.11				

Taking the average values of fr thus obtained and plotting them against the corresponding values of pr we get the experimental relation-

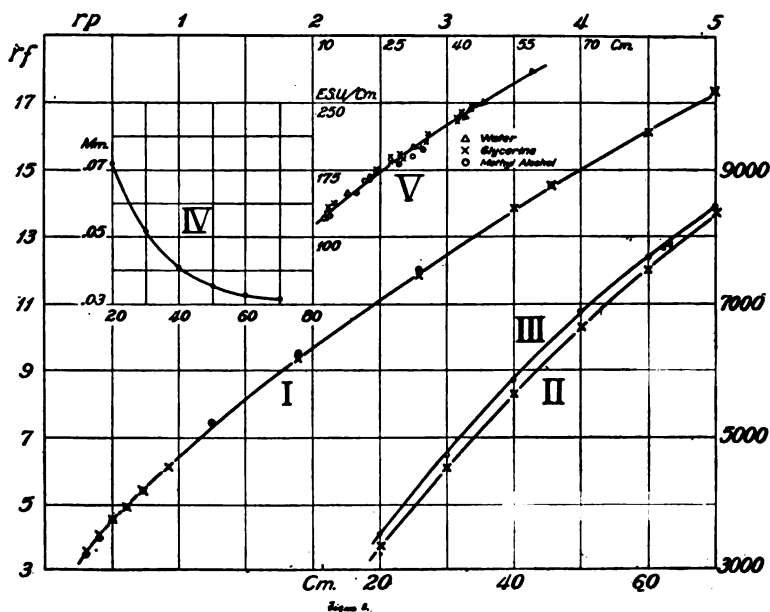


Fig. 3.

Curve I. represents the relation between rf and rp , r being in millimeters, p in centimeters of mercury and f in electrostatic units per cm. The crosses are experimental values and the circles values computed by equation (3).

Curves II. and III. show the variation with pressure expressed in centimeters of mercury of the constant a in equations (4) and (5) respectively, when v is in volts and r in millimeters

Curve IV. shows the variation with pressure of the constant b in equation (5), expressed in millimeters.

Curves V. gives the electric field in e.s.u. per cm., during positive discharge in air at different pressures expressed in centimeters of mercury, at the surfaces of the three liquids named for a point of 0.346 mm. radius.

ship shown by the crosses on curve I. in Fig. 3, which combines in one curve the data represented by all of the curves in Fig. 2. With the aid of curve I., it is possible to find the value of the intensity f which applies to a point of any radius in air at any pressure, within the limits used in the experiments. The points marked by circles on Curve I. represent values obtained from equation (3), and show the degree of exactness with which the equation represents the experimental values.

12. Now Townsend¹ has already shown that it follows from the theory of ionization by collision that if the same number of ions is produced by collision in two similar systems the product fr must be constant for all conditions where pr is constant, it being presupposed that the current flowing is sufficiently small for the volume charges present not to affect the field appreciably; and that this relation holds for the critical fields required to start discharges from metal points has been shown by Edmunds² and by Tyndall.³

The work just reported proves that the product fr remains constant for constant values of pr not only for the case where the current is the same and small as is contemplated in Townsend's theorem, but irrespective of the magnitude of this current. This follows from the constancy of f for all currents above a certain minimum value. This constancy of the surface field f for all values of the current indicates that any increase of this field owing to an increase of applied voltage is compensated by a diminution arising from larger volume charges present in the gas with the larger currents.

The nicety of this compensation is hardly fortuitous and it is probable that when this limiting field exists at the surface the electric forces in the space where ionization takes place have reached a value such that a slight change in field results in a large increase in the number of ions produced and hence as the voltage of the point is raised nearly the whole of the change goes to strengthen the field beyond the main region of ionization, this being necessary for the removal of the increased number of ions produced. This process does not continue indefinitely for the glow discharge eventually changes into a brush or spark discharge.

COMPARISON OF FIELDS WITH METAL AND LIQUID POINTS.

13. Only one result for the field strength at the surface of a metal point during positive discharge is available for direct comparison with values obtained at a water surface. This is found in a paper by Chattock⁴

¹ J. S. Townsend, *The Electrician*, 71, p. 348, 1913.

² P. J. Edmunds, *Phil. Mag.* (6), 28, p. 234, 1914.

³ A. M. Tyndall, *Phil. Mag.* (6), 30, p. 640, 1915.

⁴ A. P. Chattock, *Phil. Mag.* (6), 20, p. 273, 1910.

where a table is given from which the field at the surface of a platinum point of 0.18 mm. radius in air at atmospheric pressure (exact pressure not stated) with a positive discharge current of 0.79 microamperes is computed to be 413 e.s.u. per cm. as compared with 426 e.s.u. per cm. obtained for a water point of the same radius (as shown in Fig. 2).

A more extended indirect comparison may be made by making use of the critical fields determined at the surfaces of metal points when a current starts or stops. Such critical fields were obtained for a number of platinum points at atmospheric pressure by Chattock (*loc. cit.*) and the results embodied in an empirical equation, $f r^{0.46} = 85$ (f in e.s. units per cm. and r in cms.); and Tyndall's more recent paper (*loc. cit.*) shows the values of these fields at different pressures as well. The critical fields were computed from the square roots of the electric pulls on the ends of the points. A table in Chattock's paper (p. 273) shows that the square roots of these pulls is 7 per cent. greater as the current stops than it is with a current of the order of a microampere, and a curve in Tyndall's paper shows a difference of 11 per cent. for the same thing. The average of these values (9 per cent.) will be used for making the reduction.

There is an additional correction to be made. When a current flows from all parts of the hemispherical end of a cylindrical wire, which is the case for positive currents above a few microamperes, the electric intensity is the same over the whole surface and may be obtained directly from the electric pull on the surface. But when a current just ceases to flow, the electric intensity is greatest at the tip end of the wire, and to get the intensity at this place a correction of 8.5 per cent. must be added to the average value found from the electric pull, as was shown by Young.¹ This correction is included in the values of the critical fields represented by Chattock's empirical formula, and presumably also included in Tyndall's values. The critical fields for which a current stops may therefore be obtained approximately, by increasing by 17.5 per cent. the fields computed from the square roots of the electric pulls measured when a current above a certain minimum value is flowing.

Now the values of the critical fields computed by Chattock's formula are on the average about 15 per cent. larger than those shown by the curve for atmospheric pressure in Fig. 2, the difference being somewhat larger than 15 per cent. for the smallest points indicated on that curve and smaller than 15 per cent. for the largest points. On the other hand the values of the critical fields taken from Tyndall's curve are uniformly larger by about 13.5 per cent. than corresponding fields given on Curve I. of

¹ F. B. Young, *Phil. Mag.* (6), 13, p. 542, 1907.

Fig. 3 for liquid points with a current flowing. This indirect comparison shows therefore an average difference of over 3 per cent. between the results for metal points and those for liquid points, which is the same as that found above in the direct comparison made with the single result available for metal points. Considering all of the circumstances, great weight cannot be placed upon a difference of this magnitude, and it can only be said that if any difference exists between the fields during positive discharge, at the surface of a platinum point and a water point of the same radius, this difference is small.

STOPPING VOLTAGES FOR POSITIVE DISCHARGES.

14. The procedure followed in obtaining the stopping voltages was as follows. The current flowing from the point was measured for a number of decreasing voltages down to the one for which the water drop suddenly overflowed owing to the rapid increase of the electric pull. From these values the voltage was found graphically for which the current would have vanished if it had followed the previous rate of decrease. The voltage thus obtained was usually less than 100 volts below the last reading actually taken.

The experimental results on the voltages at which positive discharges ceased in air at different pressures are shown by the curves in Fig. 4. The two broken curves show the relation between the stopping

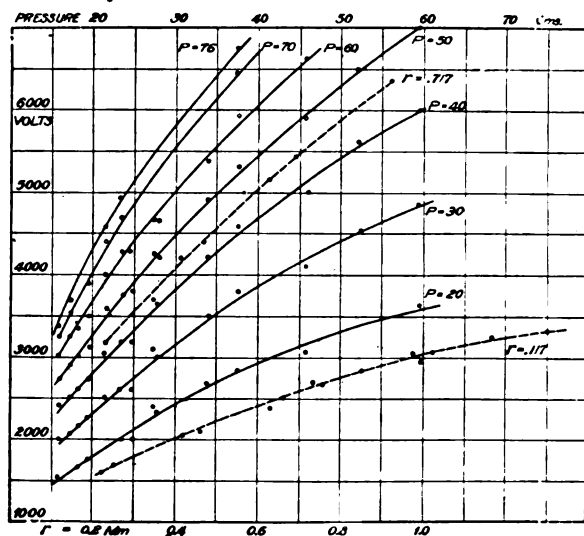


Fig. 4.

Stopping potentials in volts for positive discharges from water points of different radii expressed in millimeters, in air at different pressures given in centimeters of mercury. Distance to plate = 1.5 cm.

voltages and air pressures as obtained for two of the points whose radii are indicated on the curves.

It seemed preferable to plot all of the voltage-pressure results as radius-voltage curves and these are drawn in the figure as full line curves. Here each curve corresponds to a given pressure, whose value in centimeters of mercury is affixed, and represents the dependence of the stopping voltages upon the radii of the points used. The values for these curves were taken from the complete set of voltage-pressure curves similar to the two broken line curves shown in the figure.

The voltages at atmospheric pressure were obtained in the region bordering on the state of surface instability but they agree within experimental errors with results obtained previously¹ for positive discharges in air at the same pressure from the same sized points made of brass.

15. The radius-voltage curve for each pressure in Fig. 4 may be represented equally well by either of the two empirical relations,

$$v = a\sqrt{r} + b \quad (4)$$

or

$$v = a\sqrt{r + b}, \quad (5)$$

v being the stopping potential, r the radius of the point, and a and b constants.

The term b in equation (4) is small compared to the second term and hence its value is not obtained accurately. The numbers found for it from the different curves showed no regular tendency to vary with the pressure, and so the average value 340 volts,² was taken as common to all of the curves, and values of the constant " a " were found on this assumption. These values of a , for r expressed in millimeters, are plotted against the corresponding air pressures expressed in centimeters of mercury as curve II. in Fig. 3.

In equation (5), both of the constants a and b are dependent upon the pressure of the air. Values obtained for the constant a are shown by curve III. in Fig. 3 and are seen to follow in general the values of the corresponding constant in equation (4), represented by Curve II. The values of the constant³ b vary from .072 to .032 mm., decreasing as the pressure increases, and are shown by Curve IV. in Fig. 3. Obviously a

¹ J. Zeleny, *Phys. Rev.*, 25, p. 313, 1907.

² This constant may be interpreted to signify the lowest potential at which a discharge can occur; and its value is actually within a few volts of the minimum spark potential obtained for air by Strutt (341) and by Carr (350).

³ A physical meaning which may attach to this constant b , is that the effective radius of the point in the discharge is greater by this value than the real radius, which would be the case if for example the region of the luminous glow (whose thickness increases as the pressure is reduced) were highly conducting.

combination of equations (4) and (5) would represent the experimental results equally well or better, and would involve quantities having the significance of both of the constants b in the two equations. It is to be noted that at the higher pressures the stopping voltages for the larger points become approximately proportional to the square roots of the radii of the points.

FIELDS IN DIFFERENT GASES.

16. The electric intensity during positive discharge at the surface of a water point was also measured at different pressures in the gases, carbonic acid, hydrogen and oxygen, as well as in air. The general behavior of the discharge in these gases was similar to that found for air. The electric intensity at the discharging surface was found to be very nearly independent of the magnitude of the current, but in oxygen the discharge was less steady than in the other gases, especially with small currents, the liquid surface becoming quiescent only after the current approached about two microamperes. The numerical results given in Table II. were taken from curves drawn through the experimental values obtained. The different upper limits of pressures used are imposed by surface-instability. No simple relation is apparent between these values and other related quantities.

TABLE II.

Electric Intensities (in E.S.U. per cm.) at Water Surface during Positive Discharge in Different Gases.

Radius of Point = 0.346 Mm.					Radius of Point = 0.162 Mm.	
Pressure of Gas in Cm. of Mercury.	CO ₂ .	Air.	H.	O.	CO ₂ .	Air.
10	157	117	97		250	189
20	205	155	128		330	238
30	248	191	155	183	387	281
40	286	225	180	212	436	325
50		255	203	239		367
60		283	225	270		397
70			246			
87			277			

FIELDS FOR DIFFERENT LIQUIDS.

17. Some experiments were made with a point of radius 0.346 mm. using the liquids water, glycerine and methyl alcohol in succession in air at different pressures, in order to determine whether the nature of the liquid has any effect upon the electric field at the surface during a positive discharge. The behavior of the discharge with glycerine and methyl alcohol was in general the same as has been described for water,

but both liquids were even more difficult to work with than water. The experimental errors in the measurements with these liquids were therefore greater than was the case with water.

The results of the measurements made are shown by Curve V. in Fig. 3, and it is seen that the three sets of points fit one curve within the experimental error with the possible exception that the values for methyl alcohol are somewhat low at the highest pressures used with this substance. Owing to the great difficulty of making measurements at all in this region, which is near the instability voltage for methyl alcohol, and owing to the fact that at lower pressures the results for this liquid are in agreement with those for the other liquids it is believed that there is no significance in the slightly lower values obtained with methyl alcohol for pressures above 30 cms.

A comparison was also made, over the same range of pressures as used above, of the voltages at which the positive discharge stopped from the three liquids and from a metal point made of brass wire with a rounded end of the same radius as that of the liquids. Here again no difference above the experimental error was found between the different substances with the same exception of the values for methyl alcohol above 30 cms. pressure which were again below those for the other substances.

METAL POINT COATED WITH VARIOUS SUBSTANCES.

18. In seeking evidence of the possible effect of the nature of the material of which a point is made, upon the potential at which a discharge starts from it, experiments were done in which a brass point (diameter = 0.5 mm.) was used when coated in succession with different substances, including cadmium sulphate, thorium oxide, potassium iodide, caustic potash, fluorescein, methyl violet and the chlorides of sodium, tin, copper, mercury, iron and cobalt. It was not possible in general to get a thin, uniform coat of the material on the point, either by evaporation from solution or by application in the form of a fine powder or by fusion, but notwithstanding this unevenness of the surfaces, the potential at which positive discharges started and stopped from the coated points was approximately the same throughout as from the uncoated point. The total variation among the results was about two per cent. which is somewhat greater than twice the experimental error of the voltage determinations. Within the limits named, therefore, the positive starting potential is independent of the nature of the material of which the point is made and equally independent of a considerable roughness of surface.

With negative discharges, however, the starting potentials showed a total variation of about 25 per cent., the values ranging on both sides

of the value for the positive discharge.¹ Without more evidence, this behavior should be ascribed not to the influence of the material of the surface but rather to the effect of its inequalities, the greater localization of the negative discharges making such inequalities of moment for these discharges.

DISCUSSION OF FIELDS AT SURFACES HAVING DIFFERENT CURVATURES.

19. It is surprising at first sight that the surface field intensity required to start and maintain discharges should be the larger the smaller the radius of curvature of the point, but an explanation of the fact is found on the theory of ionization by collision by considering the nature of this process and the character of the field in the neighborhood of such surfaces.

The field intensity near the surface of points decreases very rapidly with distance from the surface, and the decrease is the more rapid the smaller the radius of curvature of the point. For illustration consider the nearly analogous case of two charged spheres, one of radius 0.01 cm. and the other of radius 0.1 cm. For the smaller sphere, the field at a distance of 0.01 cm. from the surface is only 25 per cent. of its value at the surface and at a distance of 0.04 cm. it is but 4 per cent. For the larger sphere, however, the fields at the same distances from the surface are 82 per cent. and 50 per cent. respectively of the value at the surface.

Now ionization by collision does not begin abruptly at a definite field strength, but owing to the chance elements entering, the process is a statistical one and hence if there is an appreciable rate of increase of ions by this process in the field at the surface of a point there must be an appreciable though smaller rate of increase at a distance from the surface where let us say the field is but half as strong.

Consider two points of different size so charged that the field intensities at their surfaces are the same, and imagine the same number of ions drawn toward each surface from the surrounding gas. Consider the multiplication of these ions in volume elements formed by equi-distant surfaces parallel to the surfaces of the points. In the two volume elements adjacent to the surfaces of the two points the rate of increase of the ions will be the same because the fields there are assumed to be equal but in all of the following volume elements the rate of increase will be larger for the point of larger radius of curvature because for this point the field strength falls off less rapidly with distance. Accordingly the total number of ions reaching the larger point per second will be greater than the number reaching the smaller point, and it follows that, in order to have

¹ For a point of this diameter the normal starting potential is the same for the two kinds of discharges. See J. Zeleny, *PHYSICAL REVIEW*, 25, p. 305, 1907.

ions reach the two point surfaces at the same rate, not so large a field intensity is necessary at the surface of the larger point as is required for the smaller point. It is thus made apparent why the field strength at the discharging surface is not the sole determining factor for the production of a discharge and that the manner in which the field changes with distance from the surface is of much importance; the field at the surface being least when the field is uniform and increasing more and more as the divergence of the field increases.

In discharges between a cylindrical wire and a concentric cylinder the field at the wire changes less rapidly than it does at the end of a wire of the same diameter, and accordingly the critical field required to start a discharge should be less in the former case than in the latter case. Watson¹ has computed the critical fields at different pressures at the surface of the inner of two concentric cylinders from the critical discharge voltages. The smallest wires he used correspond to the largest of those given in Fig. 2 and for these the values are almost identical with those of the figure named, showing when the correction discussed in section 12 is applied that the field at the cylindrical wires is about 17.5 per cent. smaller than for points of the same diameter.

EFFECT OF DIVERGENCE OF FIELD.

20. The effect of the divergence of the field at the surface upon the intensity of the field that obtains there during a discharge, is well illustrated by the following experiment in which the divergence of the field near a given point was artificially altered.

A small ring, *S* in Fig. 1, 4 mm. in external diameter, made of wire 1 mm. in diameter, was held by a vertical stem so that it surrounded the glass point *U* used (diameter = 0.53 mm.). The ring was adjustable vertically and was metallically connected to the liquid inside the point. The presence of this ring near the end of the point made the field at the point less divergent by an amount which depended upon its vertical position, the effect being greater the lower the ring. The electric intensities, *f*, at the surface of the point obtained when a positive discharge was passing in air at a pressure of half an atmosphere for different distances, *d*, that the tangent plane through the lower surface of the ring was above the end of the discharging point are given in Table III.

TABLE III.

	<i>f</i> .
2.0 mm.	249 e.s.u. per cm.
0.34	236
0.10	218
0.026	212
0.00	201

¹ E. A. Watson, *The Electrician*, Feb. 11, 1910.

The voltage required to maintain a current was increasingly larger as the ring was lowered, but the intensity at the surface became smaller and smaller as the field was made less and less divergent by changes in the position of the ring. The experiment was repeated at other air pressures with a like result and confirms the reasoning in the last section.

COMPARISON OF CURRENTS FROM SURFACES OF DIFFERENT MATERIALS.

21. Since the velocity with which the ions impinge against the surface of a discharging point is large, it is possible that the impacts may liberate other ions from the surface which may appreciably increase the current due to the ions produced in the volume of the gas. If such is the case, it is probable that the number of ions so ejected by a given number of impacts should depend upon the material of which the point is made, and accordingly if similar points made of different materials are used the current obtained with the same voltage should not be the same from the different points. As already stated (Sec. 3) Precht found that after plating a steel point with copper the current obtained with a given voltage remained unchanged. Additional experiments were made on this subject by obtaining the voltage-current curves for both positive and negative discharges in air from points with rounded ends made of platinum, copper, brass and water. The points were used at a distance of 1.5 cm. from a plate and all had a common diameter of 0.41 mm. After a slight correction for differences in atmospheric pressure was applied, the results with positive discharges from the different materials were identical within experimental error. For negative discharges also, the metals gave identical results. The negative discharge from the water point produced an agitation on the surface. This became less violent as the current was increased until when the voltage was between 7,000 and 8,000 volts (starting voltage being 4,700) the surface was almost quiet and showed a hazy edge only under the microscope, owing to a small amplitude oscillation. Under these conditions the current from the water point was a little smaller than that obtained with the metal points, so that for example 7,600 volts was required on a water point to produce the same current as was obtained with 7,500 volts when the metal points were used. This small difference may be explained by any one of three causes. Either electrons are ejected in smaller number from a water surface than from a metal surface during the discharge, or the smaller currents with water are due to the changes in the shape of the point caused by the small oscillations present, or the reduction in current from water points arises from a diminution in the mobility of the negative ions owing to the presence of water evaporating from the point. On the whole, the results

indicate that if any ions are ejected at all from the surface by the impact of approaching ions, the number of such ions must be small compared with those producing them; unless indeed owing to the influence of volume electrification the currents from points in general are controlled almost entirely by the applied voltage and but slightly by the number of ions produced in the neighborhood of the point.

DISCUSSION.

The ideal smoothness of liquid surfaces favors their use for the investigation of a possible surface action in point discharges, since with metal points the physical state of the surface and in most cases its chemical composition as well is a matter of constant concern, mainly because of the changes wrought by the discharges themselves.

Unfortunately the chemical composition of these liquid surfaces also was subject to change as evidenced by changes in surface tension. The action of a discharge appears to cleanse the surface, since the value obtained for the surface tension was always largest after a discharge and gradually diminished with time. It is possible that some of this surface ageing is to be attributed to the adsorption of gas molecules by the surface. While all of the liquids used showed a lag in starting for both positive and negative discharges, the amount of lag observed for any point may vary greatly on different trials, and it is probable that this fact is to be ascribed to the changes in surface composition, just noted.

The general presence of lag with liquid points lends support to the working hypothesis that the lag phenomenon is dependent upon a compact non-conducting layer of molecules upon the surface, since the smooth surfaces are favorable for the formation of such layers. The comparisons made of surface electric intensities during positive discharges show these to be virtually independent of the material of the surface, and hence it may be assumed that any non-conducting layers which may have been present at the start have under these conditions been removed.

Further discussion of the results given in this paper are reserved until some other experiments which have been made are reported in a second paper.

SUMMARY.

Considerations are given for attributing the lag phenomenon in point discharges to the presence of a non-conducting coating on the discharging surfaces which may consist of adhering gas molecules.

Both positive and negative point discharges from surfaces made of water, glycerine or methyl alcohol show a lag in starting.

The electric intensity f at the surface during positive discharges is

practically independent of the current and of the material of the point. For water points f (e.s.u.) is related to the point radius r (cm.) and the air pressure p (cm. of mercury), by the relation $fr = 0.955 pr + 5.60 \sqrt{pr}$.

The stopping voltages v for positive discharges from water points of radii r may be expressed by either $v = a\sqrt{r} + b$ or $v = a\sqrt{r + b}$ where a and b are constants for any pressure and have been obtained for a number of pressures.

The surface electric intensities during positive discharges in air, hydrogen, oxygen and carbonic acid at different pressures are given for two different points.

Coating a metal point with various substances was found to affect very little the potential at which positive discharges commenced whereas the effect upon the potential for the negative discharges was large, but this may be due to the unavoidable roughness of the surfaces used.

The influence upon the discharge of the divergence of the electric field at the discharging surface is shown to explain the fact that the field necessary for a small point is larger than for a large point.

Ejections of ions from a surface by the impact of other ions is made improbable in point discharges from a study of the voltage-current relation from surfaces of water, platinum, copper and brass.

SLOANE LABORATORY,
YALE UNIVERSITY,
March 29, 1920.

SOME PHYSICAL PROPERTIES OF NICKEL-IRON ALLOYS.

By L. R. INGERSOLL AND OTHERS.¹

SYNOPSIS.

The specific heats, thermal conductivities, thermoelectric powers, and specific resistances have been determined for a series of iron-nickel alloys of exceptional purity and definitely known composition.

The *specific heat* (25° to 100° C.) varies only slightly with change in composition, but gives, nevertheless, a well defined maximum at 35 per cent. nickel. The *thermal conductivity* (20° to 100° C.) on the other hand, shows a much larger variation, with a value for 35 per cent. nickel of only one fifth of that for either pure iron or pure nickel. The *thermoelectric power* (0° to 96° C.) against copper exhibits a marked minimum at 35 per cent. nickel with maxima at about 20 per cent. and 50 per cent. The *specific resistance* at 0° C. shows a marked maximum at 35 per cent. nickel, more than five times the value for either pure iron or pure nickel. The relative increase is not so great as the temperature is raised to 700° C. The temperature coefficient of resistance, considering the whole range (0° to 700° C.), is a minimum for the 35 per cent. nickel.

These facts are in general agreement with the anomalies in other properties shown by nickel-iron or nickel-steel of this composition and point to the formation of the definite compound Fe₃Ni.

WHILE nickel-iron and nickel-steel alloys have been very extensively investigated² as regards their mechanical, magnetic and similar characteristics of practical interest, other properties of equal importance to the physicist have been little studied. It is true that K. Honda³ has measured the thermal (as well as the electrical) conductivities of a series of nickel steels, and W. Brown⁴ the specific heats for a number of specimens containing up to 31 per cent. nickel, but no attempt has been made in the way of determining and correlating these and other physical properties for the same series of specimens.

In an extended research on the properties of alloys of iron with nickel and copper, carried out in the electrochemical laboratory of the University of Wisconsin some years ago, a considerable number of ferro-nickels were produced. These formed a graded series, of nickel content

¹ The measurements for this work were carried out at various times by Messrs. O. F. Mussehl, D. L. Swartz, H. F. Smith, C. G. Thompson, M. A. Mahre and Misses J. F. Fredrickson and D. R. Hubbard, working under the direction of Professor Mendenhall, Professor Terry or myself.

² Vide Bureau of Standards Circular No. 58 for a résumé of such work.

³ Tohoku Univ. Sci. Reports, 7, 59, 1918.

⁴ Roy. Soc. Dublin, Trans., 9, 6, 59, 1907.

varying from 1 to 90 per cent., and of such purity of material and definitely known composition as to afford a rather unusual opportunity for experimental studies of the sort described here. They were furnished through the kindness of Mr. James Aston, by whom, in connection with Professor C. F. Burgess, they had been made and their mechanical and electrical properties studied.¹ These showed unusual changes when the nickel content was between thirty and forty per cent.—in keeping with the remarkable properties of 35 per cent. nickel-steel, *e.g.*, invar. It seemed of importance, then, to determine if the same was true of certain other of their physical properties and accordingly their specific heats, thermal conductivities, thermoelectric powers and resistances as a function of temperature have been studied in this connection.

TABLE.

Alloy.	Per Cent. Nickel.	Specific Heat 25-100° C. (Cal./Gm.).	Thermal Conductivity 20-100° C. (C. G. S. Units).	Thermoelectric Power (Against Copper) 0-96° C. (Microvolts / Deg. °C.).	Specific Resistance 20° C. (Microhms / Cm. ²).	Temperature Coefficient of Res. 0-100° C.
Fe.....	0		.1428			
144E....	1.07	.1162	.1035			
144F....	1.93	.1170	.1009			
150J....	4.0			2.32	20.9	.0020
150L....	7.0			7.32	25.2	.0023
144J....	7.05	.1163	.0727			
157D....	10.20	.1168	.0687			
166A....	13.0			16.9	33.0	.0018
144M....	13.11	.1160	.0534			
166B....	14.0			17.2	33.9	.0016
166E....	18.0			21.0	35.9	.00084
144P....	19.21	.1163	.0502			
150S....	21.0			23.5	38.8	.0018
166G....	22.11	.1163	.0490	21.0	40.0	.0018
154S....	25.20	.1181	.0320			
166I....	26.40			16.7	35.9	.0016
166C....	28.42	.1191	.0278			
166L....	35.09	.1228	.0262	9.79	92.0	.0011
166M....	40.0			22.4	74.1	.0022
166N....	45.0			29.0		
166O....	47.08	.1196	.0367	31.9	47.5	.0036
166Q....	75.06	.1181	.0691			
173W....	90.0			17.9	15.5	.0034
Ni.....	100.0	.1168	.1402			

Alloys.—The alloys had been prepared with the aid of a resistor furnace by melting² weighed amounts of iron and nickel in a magnesia crucible

¹ Met. & Chem. Eng., 8, 23, 1910.

² For a more detailed description of this process see Univ. of Wis. Bull. No. 346, Eng. Series, Vol. 6, No. 2, p. 6.

supported by a graphite jacket. The iron was obtained by a process of double electro-deposition and was 99.97 per cent. pure. The nickel was also electrolytic material of high purity. The resulting ingots, which weighed about 500 g. each, had been forged into rods and then machined into bars about 1 cm. in diameter. In the case of most of the specimens tested, the exact composition had been determined by chemical analysis, and the close agreement of the analytical results with the percentage of nickel added to the charge may be taken as evidence of the perfect alloying of the iron and nickel. The carbon content was estimated at considerably less than 0.10 per cent.

As will be noted on inspection of the table, the series of specimens on which the specific heat and conductivity measurements were made was not identical with that on which the other experiments were performed. While this is perhaps to be regretted, the fact that all these alloys were made in exactly the same way and of materials of the same purity, practically does away with any objection arising from this circumstance. It may be remarked that the specimens for which the nickel content in the table is given to two decimal places (*e.g.*, 22.11 per cent.) are those for which an exact analysis had been carried out. The composition of the others (*e.g.*, 13.0 per cent.) was determined from the amounts of materials used in forming the alloy and hence it is not so accurately known.

Specific Heat.—The specific heats were determined by means of a Joly steam calorimeter, using a delicate Sartorius balance. Before testing, samples of the alloy which weighed about 38 grams each were annealed by packing them in iron filings in an electric furnace and heating to 900° C. for over an hour, then cooling very gradually. The specimens were then carefully polished with various grades of emery paper. Three different sets of measurements were made, all of which gave results in excellent agreement. The mean values only are included in the table and plotted in Fig. 1.

The results are in good agreement with the measurements of Brown, already mentioned, although he did not investigate specimens with more than 31.4 per cent. nickel. The maximum in the curve at 35 per cent. nickel is very marked although the actual change in specific heat is small.

Thermal Conductivity.—This was determined by the well-known method of Gray.¹ The specimens used were those for which the specific heats had already been measured. They were between 5.1 and 6.7 cm. in length and of an average diameter of about .98 cm. To minimize losses,

¹ Proc. Roy. Soc. London, 56, 199, 1894.

the copper ball, to which the heat was conducted, was surrounded by a jacket through which flowed water at room temperature. The rod itself was jacketed with cotton wrapping. Curves were plotted of the temperature of the copper ball as a function of time and a tangent to these curves drawn at the point of room temperature enabled one to

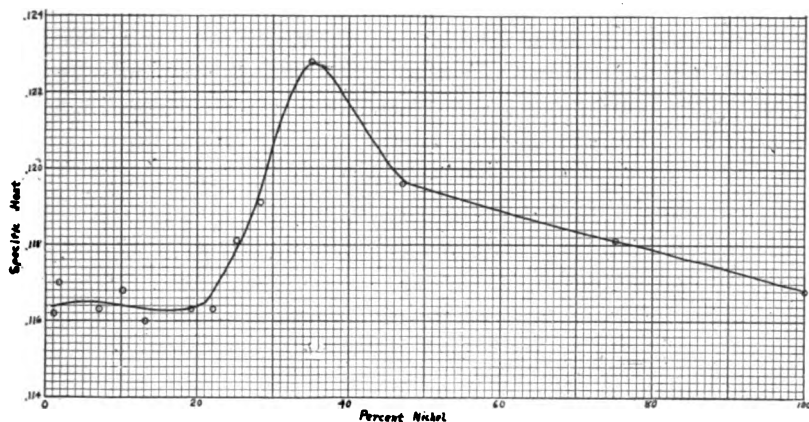


Fig. 1.

Specific heat of pure ferro-nickels.

determine the rate at which heat was being conducted along the rod to the copper ball. Three determinations, in general showing good agreement, were made for each specimen and the average results are given

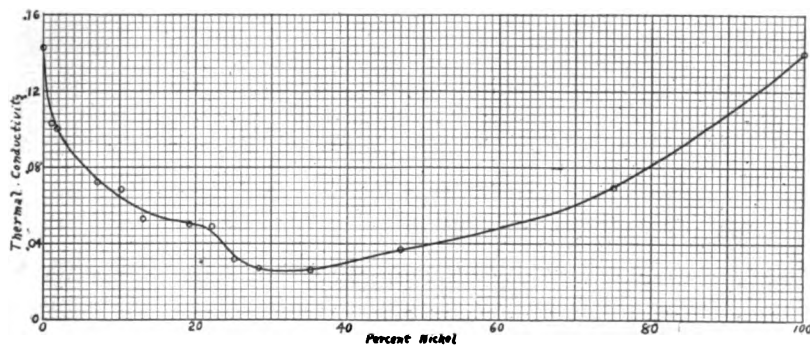


Fig. 2.

Thermal conductivity of pure ferro-nickels.

in the table and plotted in Fig. 2. The conductivities of pure iron and pure nickel are taken from measurements of Jäger and Diesselhorst.¹

The results are in substantial agreement with those of Honda (*loc. cit.*) for alloys annealed in the same way, save that this investigator finds the

¹ Abh. d. phys.-tech. Reichsanstalt, 3, 269, 1900.

minimum conductivity at 30 per cent. nickel instead of 35 per cent. as in the present case. It may be remarked, however, that Honda's alloys were prepared from low carbon steel and ordinary commercial nickel, while those used here were composed of materials electrolytically purified.

Thermoelectric Power.—The thermoelectric powers relative to pure copper were determined with a Leeds and Northrup potentiometer. The specimens of the alloy used in this case varied in length from 3.5 cm. to 5.8 cm. and were turned down until the diameter was only .12 cm. to .18 cm. The hot junction was placed in a glass tube through which steam passed, the cold junction being kept in a stirred bath of ice and water. The average difference between the temperatures at the ends of the rod under these conditions was found to be about 96° C.

It was found that a further decrease in the diameter of the rod increased the measured thermoelectric powers somewhat but the general form of the curve was not in any way affected. The results are given in the table and in Fig. 3. It may be remarked, however, that they are

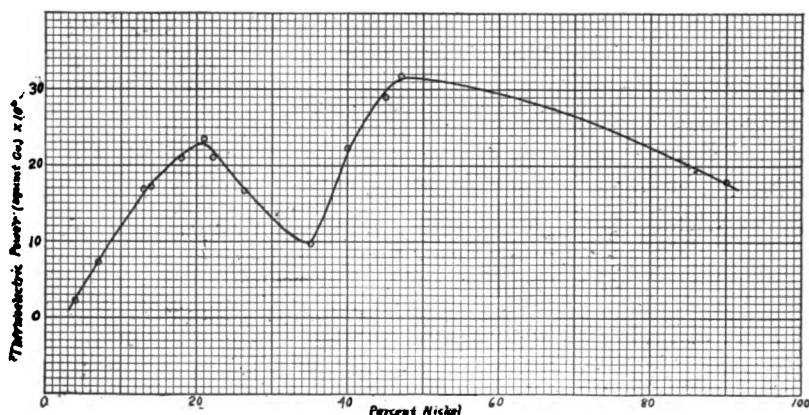


Fig. 3.

Thermoelectric power (against copper) of pure ferro-nickels.

perhaps not quite as trustworthy as the preceding series on specific heat and conductivity, inasmuch as the exact composition of the alloys was not as accurately known in all cases and there was some uncertainty as to how well the specimens had been annealed.

The curve shows a marked minimum at 35 per cent. Ni. This is in agreement with the work of Haken¹ who found, particularly for the binary alloys of bismuth, marked anomalies in the thermoelectric and electrical conductivity curves for compositions giving a maximum in the melting point curve. Eagan and Emmett² observed a similar minimum

¹ Ann. d. phys. 32, 291, 1910.

² Univ. of Wis. Thesis, 1913.

in the thermoelectric curve for bismuth-thallium alloys for a composition showing a maximum melting point.

Electrical Resistance as a Function of Temperature.—This was determined by the potentiometer method, a current of some 7.5 amperes being passed through the rod under investigation. A potentiometer of special type designed and built in this laboratory was used. The rods were heated in a nichrome wound furnace and the temperature measured with a copper-constantan thermocouple. The results are plotted in the curves of Fig. 4, while the specific resistance at 20° C. is included in the

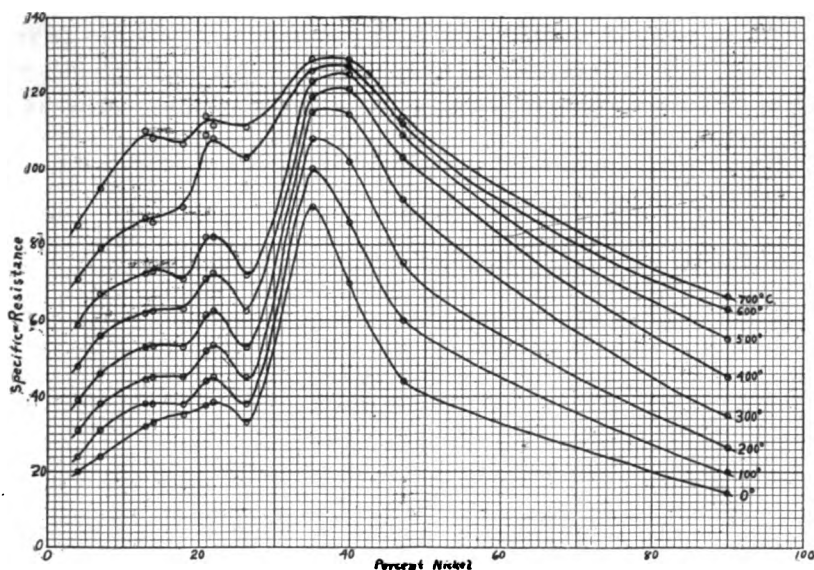


Fig. 4.

Specific resistance at different temperatures of pure ferro-nickels.

table, as well as the temperature coefficient for the range 0° to 100°. While it will be noted that the temperature coefficient for this range shows minima at 18 per cent. nickel as well as 35 per cent., it is evident from the curves of Fig. 4 that if a somewhat larger temperature range were taken the lowest point of the curve would be for the 35 per cent. alloy. The general form of these curves (aside from the minima at 26 per cent. Ni) is in agreement with the resistance curve given by Burgess and Aston (loc. cit.) for ordinary temperatures.

The thermal conductivity and electrical resistance measurements could be used to prove the law of Wiedemann and Franz that the thermal and electrical conductivities of a metal are proportional. A superficial examination of the table is enough to show that this is at least approxi-

mately true in this case, although a careful comparison is not possible since the two measurements were not carried out on exactly the same set of alloys. The law is very well proved for the nickel-steels, however, by Honda (loc. cit.).

Conclusions.—The results of the present work show that the 35 per cent. nickel alloy, corresponding to the composition Fe_2Ni , has physical properties more or less markedly different from the other ferro-nickels. When the measurements of these various physical characteristics are plotted as a function of the composition the following general facts are brought out: the melting point curve¹ shows a maximum, specific heat a maximum, thermal conductivity a minimum, thermoelectric power a minimum, specific resistance a maximum and temperature coefficient of resistance a minimum—all at, or very near, the composition of 35 per cent. nickel.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
Feb. 5, 1920.

¹ Vide Guertler and Tammann, *Zeitschrift für Anorg. Chem.*, **24**, 205.

THE SELECTIVE REFLECTION OF HEAT WAVES BY LINEAR RESONATORS.

BY E. C. WENTE.

SYNOPSIS.

Vacuum-thermocouple.—The construction of a vacuum-thermocouple of high sensitivity and quick action for measuring heat radiation is described.

Isolation of Long Heat Waves.—A method is given for isolating radiation of great purity and having a mean wave-length of $96\ \mu$. This radiation is obtained by combining a quartz lens focal isolation apparatus with reflecting surfaces of potassium iodide.

The Reflection of Radiation by Linear Electric Resonators of Microscopic Dimensions.—Resonators were ruled from films of silver chemically deposited on glass plates. The reflecting power of such resonators was measured for the $96\ \mu$ waves. Square resonators, the length of whose sides are small compared with the wave-length reflect little more than bare glass, provided the resonators are separated sufficiently to prevent conduction between adjacent edges. Resonators of equal width and separated by a distance of approximately one half of a wave-length were found to produce a maximum of reflection when their length was equal to 0.3 of the wave-length. The metal strips, although microscopic in size, thus show electrical resonance when stimulated by long heat waves. The results are in every way comparable with those that have been obtained with the longer electric waves.

SINCE the work of Hertz on electric waves a large number of experiments¹ have been carried out on the transmission and reflection of such waves by groups of linear resonators.² It has been found that a maximum amount of radiation is reflected from a plane surface over which such resonators are distributed in parallel rows and columns when the wave-length is from two to three times the resonator length, the exact value depending principally upon the position of the individual resonators relative to each other, and upon the dielectric constant of the material with which they are in contact. Only a few experiments of this nature have been undertaken with radiation of shorter wave-lengths such as is emitted from hot bodies, chiefly on account of the difficulty of making resonators of microscopic dimensions. The first experiments

¹ A. Garbasso, Atti. Acc. di Torino, 28, 470 and 816 (1893), Garbasso and Aschkinass, Ann. d. Phys., 53, 534 (1894). Aschkinass and Schaefer, Ann. d. Phys., 5, 489 (1901). Cl. Schaefer, Ann. d. Phys., 16, 106 (1905). Blake and Fountain, Phys. Rev., 23, 257 (1906). M. Paetzold, Ann. d. Phys., 19, 116 (1906). Woodman and Webb, Phys. Rev., 30, 561 (1910). Nelms and Severinghaus, Phys. Rev., 1, 429 (1913).

² The term linear resonator as here used may be defined as any metallic rod or rectangular piece of metal foil whose length is at least twice its greatest width.

on reflection by groups of resonators in the field of infra-red spectrum were carried out by Rubens and Nichols¹ in 1897 with plane polarized residual rays from fluorite, which had a mean wave-length of $23.7\ \mu$. Silver films, chemically deposited on glass, were ruled with a diamond on a dividing engine in such a way that the whole surface was left covered with regularly spaced, rectangular pieces of silver $5\ \mu$ wide and with separations of $5\ \mu$. Four such surfaces were prepared, on each of which the resonators were of different lengths. When the residual rays were allowed to fall on these surfaces, it was found that a greater proportion of the incident radiation was reflected when the electric component was parallel than when it was perpendicular to the axis of the strips, and that in the former case the amount reflected was greatest when the resonators had a length approximately equal to an even multiple of a quarter of a wave-length. The results were hardly more than qualitative, however. Only four of a larger number of resonator plates that were ruled were deemed suitable for the experiments. On all the rejected plates 10 per cent. or more of the resonators were torn away during the difficult process of ruling. The spacing between the resonators was insufficient to produce a very sharp maximum in the reflecting power regarded as a function of the resonator length. Although the radiation as obtained by reflection from fluorite plates had a maximum at $23.7\ \mu$, it was not as homogeneous nor as completely polarized as was desirable for these experiments.

In 1912² Wood performed experiments of a similar nature with the very long waves obtained from a Welsbach burner by the method of focal isolation. This investigator ruled the metal film of a "half-silvered" quartz plate into small squares. The film cut in this way was found to be entirely opaque to the long heat waves, although the linear dimensions of the squares were less than one tenth of a wave-length. Wood also performed some experiments with plates on which were deposited minute spherical metal particles, but found no indication of resonance.³ Thus Wood failed to verify the results obtained by Rubens and Nichols.

The experiments described below were undertaken for the purpose of making a more complete study of the reflection of heat waves by linear resonators, particular attention being given to the highly important consideration of having the resonators well spaced in order to make the resonance as sharp as possible. The radiation with which Rubens and Nichols carried out their experiments had a mean wave-length of only $23.7\ \mu$. Methods are now available for isolating radiation of much

¹ *PHYS. REV.*, 5, 164 (1897), and also *Ann. d. Phys.*, 60, 418 (1897).

² *Phil. Mag.*, 25, 440 (1913).

³ *Phil. Mag.*, 25, 440-443 (1913).

longer wave-length,¹ so that the difficulty of making resonators of proper dimensions and spacings is considerably less.

CONSTRUCTION OF VACUUM THERMO-COUPLE.

In order to be able to rule metallic films into rectangles of the proper size it is advantageous to work with as long heat waves as possible. On the other hand, the amount of energy available decreases rapidly with the wave-length. For instance, in the case of the radiation obtained by Rubens and Wood by their quartz lens focal isolation method, the deflection of the radio-micrometer, even when the radiation was unpolarized, was only about 1 cm. A measuring instrument of the highest sensitivity is therefore necessary.

Of the various types of apparatus that have been used for radiometric measurements in the infra-red region, a vacuum thermo-couple with galvanometer seemed to be best adapted for use in this investigation. Recent experiments² have shown that the sensitivity of a thermo-couple is increased from four to seven times when used in a high vacuum; at the same time convection currents, which may cause fluctuations in the readings, are eliminated. In a theoretical paper on the design of vacuum thermo-couples Johansen³ has discussed the values that must obtain for the dimensions of the lead wires and the area of the receiving surfaces at the junctions of a vacuum thermo-couple for maximum sensitivity. The thermo-couple here described was designed so as to conform to these values as nearly as possible.

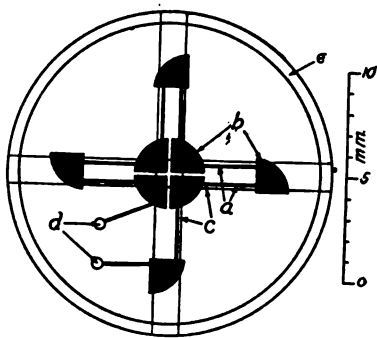


Fig. 1.

Front view of thermo-couple.

The general arrangement of the thermo-couple finally constructed is shown in Fig. 1. The leads are of bismuth and bismuth-tin alloy (Bi 95 per cent.; Sn 5 per cent.). Bismuth wire, made by Heraeus, was kindly supplied to the writer by Prof. H. M. Randall, of the University

¹ Rubens and Wood, *Phil. Mag.*, 21, 249 (1911). Rubens and Hollnagel, *Sitzber. der Preuss. Akad.*, Jan. 20, 1910.

² P. Lebedew, *Ann. d. Phys.*, 9, 209 (1902).

W. H. J. Moll, *Arch. Neerland des Sc. Ex. et Nat.* (II), 100.

A. H. Pfund, *Publ. Allegh. Obs.*, 3, p. 43 (1913); *Phys. Zeit.*, 15, 876 (1913).

E. S. Johansen, *Ann. d. Phys.*, 33, 517 (1910).

Reinkober, *Ann. d. Phys.*, 34, 348.

W. Coblentz, *Bull. Bur. of St.*, 11, 621 (1915).

³ Loc. cit.

of Michigan. The alloy was made from commercial bismuth and tin. A junction formed from these materials gave a thermo-electric power of 135 microvolts per degree centigrade. The leads, *a*, are 2.5 mm. long and were all made of such a size that their resistance was 2.5 ohms each. In order to obtain leads of this size, small pieces of the material were pressed between glass plates and then cut into narrow strips with a razor blade. The resistance of each strip was measured separately, and those pieces not having the proper resistance were rejected. The ends of the leads were attached by a very small amount of solder of low melting point to the receivers, *b*, which consist of sectors of silver foil of 0.0005 cm. thickness and of 2 sq. mm. area.

Although a junction formed from bismuth and bismuth-tin alloy gives a high thermo-electric power, these materials have not been used extensively because the thermo-couples become easily broken, the alloy particularly being quite brittle. To avoid this difficulty the receivers were attached with a trace of shellac to the stretched quartz fibers, *c*, having a diameter of about 0.002 cm. The fibers are fastened to one end of a short piece of glass tubing, *e*. The heat conductivity of these fibers is quite negligible. The copper terminal leads, *d*, were attached by a glass seal to the other end of the glass tubing as shown in Fig. 2. A thermo-couple put together in this way will withstand a great deal of jarring without becoming broken.

The receiving surfaces, instead of being blackened on the front side in the usual way, were coated with a thin layer of water glass to which had been added a small amount of India ink. While unblackened surfaces are not good absorbers of ordinary light, Rubens and Wood¹ found water glass to be an especially good absorber of radiation of long wavelengths, whereas lampblack is almost perfectly transparent to these waves.

The thermo-couple was mounted in a glass container as shown in Fig. 2. The glass tube, *e*, on which the thermo-couple was mounted was placed inside of another glass tube, *c*, and attached to it with sealing wax in order to keep the receiving surfaces about a millimeter away from the quartz window, *a*. This quartz window, had a thickness of 1 mm. The vacuum was maintained by liquid air and charcoal. By means of a small discharge tube, attached as shown, the general state of the vacuum could be determined.

One of the chief disadvantages of a thermo-couple for use in radiation measurements has generally been considered to be its sluggishness. In order that a thermo-couple may respond quickly to variations in the

¹ Phi. Mag., 21, 249 (1911).

intensity of the radiation, the leads should be short and the junctions should have a small heat capacity. In the case of the couple just described, because of the high conductivity of silver, it was possible to make the receivers of silver foil of very small thickness. Supporting all the parts of the couple on quartz fibers eliminates the usual long lead wires going to the supporting frame.

Although the receiving surfaces of the couple were not entirely "black"

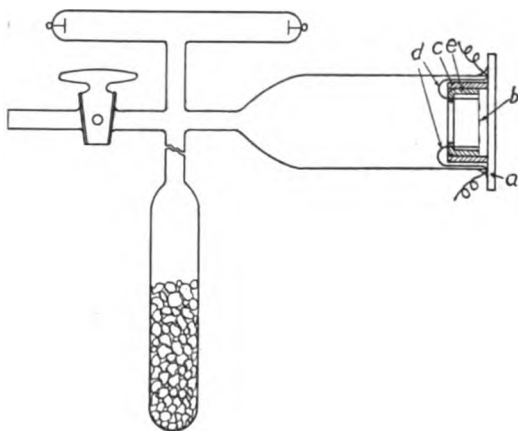


Fig. 2.

Mounting of thermocouple in evacuated tube.

for ordinary light, nevertheless, with the couple mounted as shown in Fig. 2, when connected to a galvanometer having a resistance of 25 ohms and a sensitivity of 1×10^{-10} amperes per mm., a deflection of 2,500 mm. per candle meter was obtained. This sensitivity compares favorably with that of any instrument heretofore described, having the same receiving area.

THE GALVANOMETER.

The galvanometer used with the thermo-couple was identical in design with that described by Nichols and Williams¹ except for a slight modification of the shields. Instead of a cylinder of silicon steel, the intermediate shield consisted of 35 turns of 0.04 cm. transformer iron, wound spirally, the layers being separated from each other by paper 0.04 cm. thick. The efficacy of this type of shielding has been investigated both theoretically and experimentally by Esmarch.² The three shields combined were found to have a shielding ratio of about 40,000.

The zero position and sensitivity of moving magnet galvanometers

¹ *Phys. Rev.*, 27, 250 (1908).

² *Ann. d. Phys.*, 39, 1550 (1912).

have generally been controlled by means of a special magnet provided for that purpose. This has usually been necessary in order to overcome a certain amount of residual magnetism either in the shields or in some of the other parts in the immediate neighborhood of the moving magnets. With this arrangement slight variations in the strength of the residual field will cause a drift of the zero position, which may impair the accuracy of the readings. In the instrument here described this residual magnetism was reduced practically to zero by very carefully demagnetizing the inner shield and by winding the galvanometer field coils with non-magnetic copper wire. No control magnet was necessary, as the moving system followed almost exactly any angle through which the upper end of the quartz fiber suspension was turned. The quartz fiber was selected of a thickness to give the desired sensitivity, which in most of the work was 5×10^{-11} amps. per mm., with the scale at a distance of one meter. With this sensitivity the galvanometer had a period of about 12 sec. for a complete oscillation.

The galvanometer was supported by a Julius suspension, and to prevent disturbances from air currents and temperature changes the frame of the Julius suspension together with the galvanometer was surrounded by a box. Heavy copper leads were used to connect the thermocouple to the galvanometer. Soldered connections were used throughout, and all junctions were protected as much as possible from circulating air currents. In spite of all these precautions, during certain hours of the day the galvanometer zero reading varied several millimeters; these fluctuations were apparently caused by earth currents as during other periods the zero reading was constant to a small fraction of a millimeter.

METHOD OF ISOLATING RADIATION OF LONG WAVE-LENGTH.

In order to make satisfactory resonators on glass by ruling silvered surfaces, it is desirable to work with the longest wave-lengths possible. The method of focal isolation with a Welsbach burner as a source, according to the experiments of Rubens and Wood,¹ gives radiation which is distributed over a fairly wide spectral region, but has quite a distinct maximum of energy at about 100μ wave-length. A modification of this method was adopted for this investigation.

The general arrangement of the apparatus is shown in Fig. 3. a and a' are quartz lenses having a thickness of 0.7 cm. at the center and 0.35 cm. at the edge, a diameter of 4.4 cm. and a focal length for ordinary light of 13.5 cm. c shows the position of the surface whose reflecting power is to be studied. The opening, O and the reflecting surface at C

¹ Loc. cit.

have diameters of 0.5 and 1.0 cm. respectively. The lens a' is placed in a position such that the radiation of long wave-length is brought to a focus at c . Coming from c , it is reflected from plane surfaces at e and f and then again brought to a focus on the central junctions, h , of the thermo-couple by the lens, a . The short waves, which would otherwise pass through the central parts of the lenses, are stopped by the metal

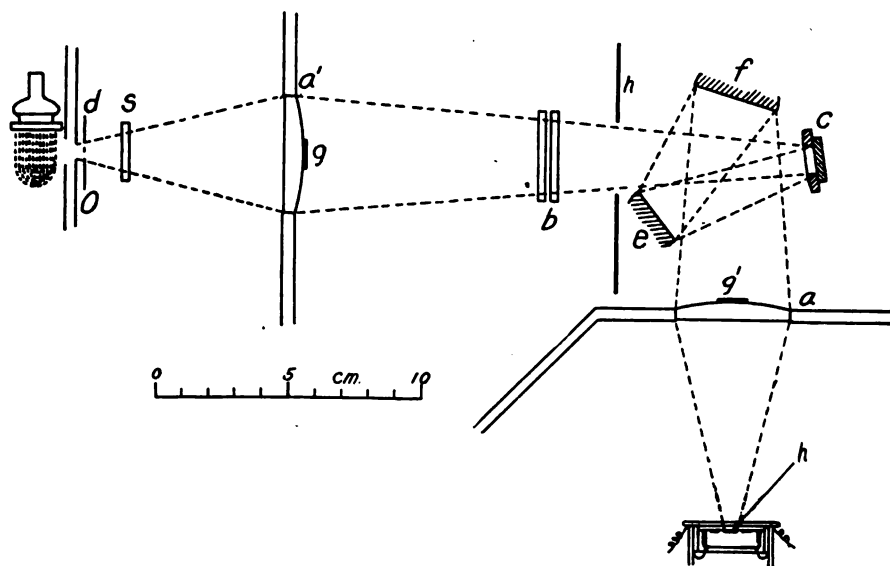


Fig. 3.

discs g and g' . The shutter, s , is made of a plate of rock salt 3 mm. thick. Rock salt is almost perfectly transparent up to the point where quartz is a total absorber, and is opaque to radiation beyond 80μ , where the quartz begins to transmit freely. At b is placed a Fabry and Perot interferometer such as was first used by Rubens and Hollnagel¹ for measurements of wave-lengths in the infra-red region. The plates are made of quartz 3 mm. thick the inner surfaces of which are plane to within a few wave-lengths of sodium light.

At d is a polarizer which consists of a plane grating made of platinum wires 0.025 mm. in diameter separated by a distance of 0.025 mm. A grating of this type has been investigated by DuBois and Rubens,² who found that for long heat waves the transmitted radiation was almost completely polarized. The grating, which had a diameter of only 1 cm., was easily made by winding the wires on a metal frame and spacing them

¹ Phil. Mag., 19, 761 (1911).

² Ber. der D. Phys. Gesell., 9, 431 (1911).

by means of a guide fastened to the carriage of a dividing engine. The wires were then soldered to the frame and those on one side of the frame, removed. In this manner little difficulty was experienced in making sufficiently good gratings of the small size required.

The apparatus as described above is very similar to that used by Rubens and Wood,¹ who found that the radiation reaching the thermocouple, when a Welsbach burner was used as a source, had a mean wave-length of approximately $100\ \mu$, but the energy was distributed over a rather wide spectral region. For this investigation it is highly desirable that the radiation should be as nearly monochromatic as possible. The radiation was therefore purified further by introducing reflecting surfaces of potassium iodide at e and f . The residual rays from potassium iodide have a wave-length somewhat less than $100\ \mu^2$. This value coincides very nearly with the mean wave-length of the radiation obtained by the focal isolation method with a Welsbach burner.

These reflecting surfaces of potassium iodide were prepared by grinding the salt to a powder, and forcing it against a piece of heavy plate glass with a pressure of about 450 kilograms per square cm. When the pressure was relieved, the salt was left in a solid mass, which could easily be removed from the glass plate, the surface adjacent to the glass being left in a perfectly smooth condition, so that no grinding or polishing was required.³

Fig. 4 shows an interferometer curve which was taken with the apparatus arranged as described but with silver instead of potassium iodide surfaces at e and f . The ordinates represent deflections of the galvanometer resulting from the opening of the shutter, s . The abscissæ give the separation of the interferometer plates, except for an additive constant, as no special pains were taken to determine the exact distance between the plates for the smallest separation.

If the energy distribution with respect to wave-length had but one maximum, the curve obtained in this way should have a form similar to that of a damped sine-wave, that is, the amplitudes of the periodic variations in the galvanometer deflections should decrease gradually with the separation of the interferometer plates. In the curve shown, these amplitudes first decrease and then increase and finally decrease gradually. This fact shows that the spectral distribution curve of the radiation has more than one maximum.⁴ The radiation so obtained

¹ Loc. cit.

² Rubens and Hollnagel, loc. cit.

³ This method of preparing reflecting plates appears to have been used first by Miss G. Langford, *PHYS. REV.*, 33, 137 (1911).

⁴ Rubens and Hollnagel, loc. cit.

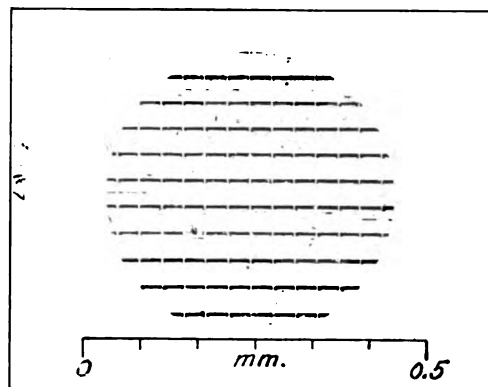


Fig. 7.

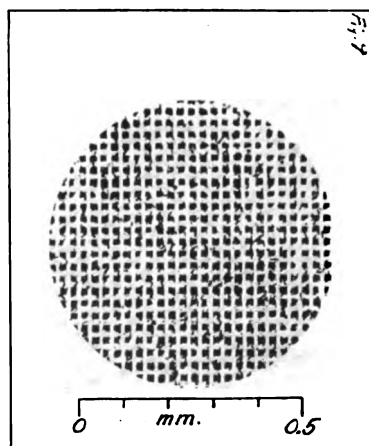


Fig. 9.

E. C. WENTE.

is therefore quite unsuited for investigating the reflecting power of resonators. Rubens and Wood concluded from the curve obtained by them with apparatus of the same type that the spectral distribution curve had only one maximum; but apparently they did not continue

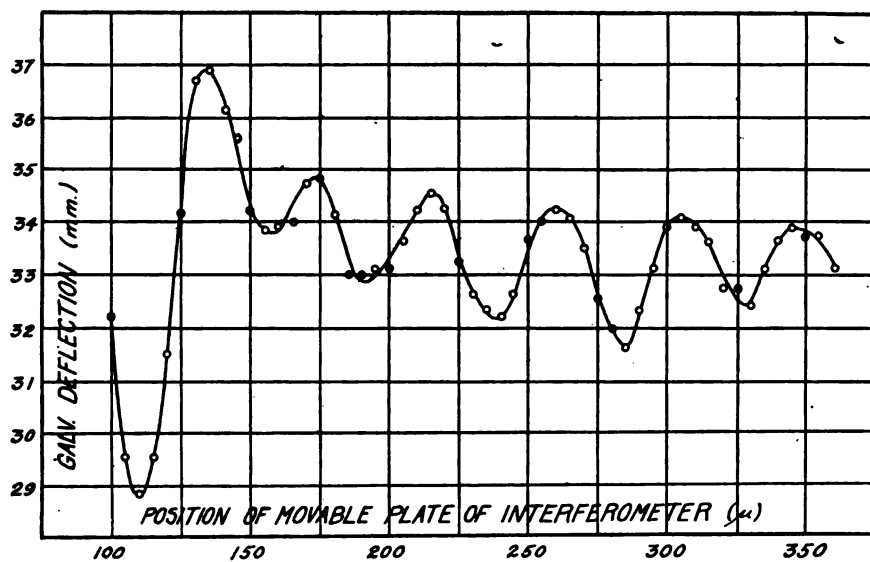


Fig 4

their observations to sufficiently large separations of the interferometer plates to determine the real character of the energy distribution of the radiation. The fact that there are several maxima is probably due to absorption bands of water vapor.¹

The curve of Fig. 5 corresponds to that shown in Fig. 4 except that the

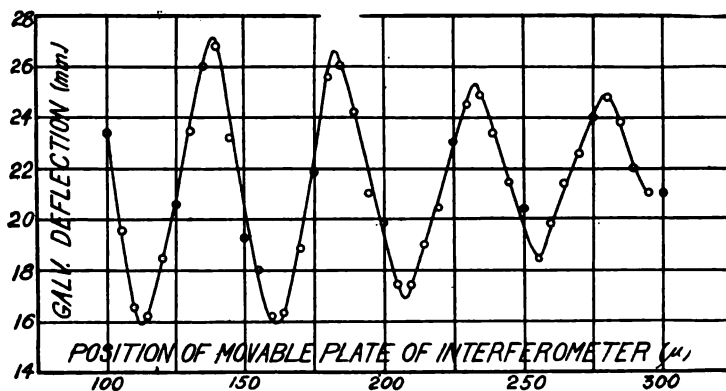


Fig. 5.

¹ H. Rubens, Sitzber. der Preuss. Akad. der Wiss., 28, p. 513 (1913).

silver plates at e and f were replaced by the potassium iodide surfaces. This curve, being very similar to a damped sine-wave, indicates that the energy of the radiation reaching the thermo-couple is grouped principally about a single wave-length. This wave-length may be calculated from the distances the interferometer plates were moved between successive maxima or minima. The mean of all these distances is 47.9μ from which we determine the principal wave-length as 2×47.9 or 95.8μ . An approximate idea of the energy distribution curve in this case may be obtained, if, following the method adopted by Rubens and Hollnagel, we assume that this has the form of a resonance curve when plotted as a function of the wave-length, *i.e.*,

$$\Phi_{\lambda} = \Phi_0 \frac{\delta^2}{\delta^2 + 4\pi^2(\lambda - \lambda_0/\lambda)^2},$$

where λ_0 is the wave-length for which the energy of the radiation has a maximum value, Φ_0 , which in this case we assume to be 95.8μ . δ is the logarithmic decrement, as obtained from the interferometer curve. δ ,

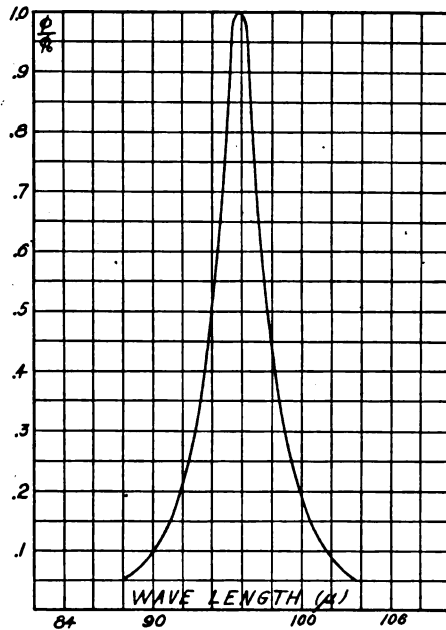


Fig. 6.

Distribution of the energy of the radiation as a function of the wave-length.

as determined from the curve of Fig. 5, assumed as a damped sine-wave, has an approximate value of 0.25. With these values of λ_0 and δ Φ_{λ} , plotted as a function of λ , gives the curve shown in Fig. 6. This curve

shows that the greater part of the radiation lies within a narrow band of wave-lengths.

All the preceding measurements were made with a Welsbach burner as source. A quartz mercury vapor lamp was tried but apparently gave less radiation of the desired wave-lengths even when operated at very high voltages.

CONSTRUCTION OF RESONATORS.

In order to make resonators of the small size required for these experiments silver was chemically deposited on pieces of plate glass 0.4 cm. thick and the fresh deposit ruled into rectangles of the proper size with a diamond on a dividing engine. For this method to work satisfactorily the silver deposit should be uniform in thickness and adhere firmly to the glass. These conditions seemed to be best satisfied by the use of the formaldehyde silvering process.¹ Best results were obtained only when the glass plates were first immersed in a hot solution of chromic acid. The silvering process was so regulated as to give coatings of practically the same thickness for all of the plates. This thickness was such that the silvered surfaces reflected the 96μ radiation as completely as a plate of silver several millimeters thick, although they were not entirely opaque to light in the blue region of the visible spectrum.

In order to obtain resonators that are widely separated it is necessary to remove the silver deposit in wide strips. If an attempt is made to make wide cuts with a diamond, a large proportion of the resonators will be carried away, whereas clean cuts result, if the diamond is set so as to rule a fine line by having one of its natural edges parallel to the direction of the cut. To remove the silver in wide strips it was found most

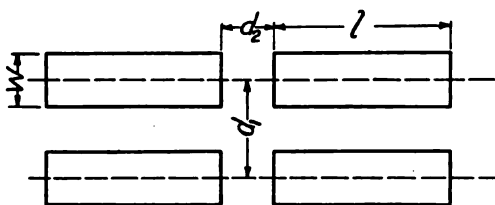


Fig. 8.

satisfactory therefore to rule fine lines but so close together that no metal remained between adjacent rulings. In this way it was possible to leave very narrow strips of silver and separated by as wide a space as desired. Light watch oil, flowed over the surface during the ruling, carried away the chips of silver removed by the diamond and so prevented

¹ Wood, Physical Optics, p. 281, 1914 edition.

their accumulation under the cutting edge. Fig. 7 is a photomicrograph of a part of a set of resonators ruled in this way. This particular photomicrograph was taken of the plate listed as No. 7 in the table below. It shows the character of the resonators on all the plates numbered from 1 to 8. The following table gives the dimensions and spacings of the resonators of all the plates that were used in this study. Fig. 8 will make clear the meanings of the terms used in the table.

In the experiments of Wood¹ on the reflection of long heat waves by

No. of Plate.	l (μ).	w (μ).	d_1 (μ).	d_2 (μ).
1.....	13.0	8	44.5	4
2.....	17.2	"	"	"
3.....	21.35	"	"	"
4.....	25.5	"	"	"
5.....	29.7	"	"	"
6.....	33.85	"	"	"
7.....	38.0	"	"	"
8.....	42.2	"	"	"
9.....	15.0	15.0	17.0	2
10.....	15.0	15.0	27.0	12

ruled silvered surfaces it was found that single narrow cuts made by the diamond did not alter the resistance of the film appreciably. This result was verified. A single fine cut of the diamond increased the resistance of the film only slightly, although as far as could be determined from the image given by the microscope the silver had been removed completely. When several adjacent cuts were made so that the gap was 2 or 3 μ wide the resistance was increased, but only after the total width of the space was roughly 5 μ did the resistance become practically infinite. Apparently when the diamond makes a narrow single cut, the silver is not completely removed from the gap, although it does appear so when observed with a microscope. However, if the surface was ruled so as to leave widely spaced strips of metal 5 or 10 μ wide, a single cross cut made the resistance along the strips practically infinite. In this case the diamond undoubtedly removes the silver in its path completely.

EXPERIMENTAL RESULTS.

Wood² found that when a silvered surface was ruled into regularly spaced squares, the reflecting power of the surface was the same as before the rulings were made, even if the width of the squares was only a small fraction of the wave-length of the incident ray. As one would expect

¹ Loc. cit.

² Loc. cit.

such a surface to reflect scarcely more than the bare glass it was thought worth while to make a further test of this point. Two silvered surfaces were therefore ruled into squares $15\ \mu$ wide. In the first of these the cuts were about $2\ \mu$ wide and the resistance across each cut was approximately 100 ohms per cm.; in the second set the cuts were made wide enough to make the resistance between the squares practically infinite. A photomicrograph of this plate is shown in Fig. 9. When these plates were used in turn as reflectors at c (Fig. 5), the former was found to reflect 70 per cent. of the incident radiation, whereas the latter reflected only 30 per cent., the reflecting power of a plate of silver being assumed as 100 per cent. A glass surface without any metallic deposit reflected 20 per cent. The surface with the wide cuts, before being cross-ruled, reflected only 34 per cent. of the radiation when the electric component of the incident waves was perpendicular to the strips, but 86 per cent. was reflected when it was parallel to the strips.¹ From these data it appears that the negative result obtained by Wood was very probably due to the fact that the squares were not separated completely from each other.

The ideal method of studying the selective reflection of waves by a group of linear resonators is to determine the reflecting power of such a group for monochromatic radiation of varied wave-length. However, this method is impracticable because of the difficulty of obtaining pure radiation of the different wave-lengths required.

A method virtually equivalent to this is to keep the wave-length of the radiation constant and to measure the reflecting power of a number of groups of resonators of different dimensions, but in each of which the quantities, w , l , d_1 , d_2 (Fig. 8), bear the same relation to each other. It is rather difficult to rule sets of resonators of different sizes with a fixed relation between all the dimensions on account of the many different settings that have to be made on the dividing engine. How-

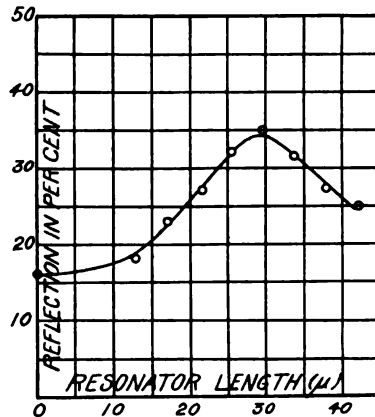


Fig. 10.

ever, experiments on electric waves have shown that the quantities, d_2 and w (Fig. 8), affect the sharpness of resonance but little, provided l is

¹ These particular measurements were made with the radiation obtained without the use of the KI reflecting surfaces. The radiation was therefore not as pure as is indicated by the curve in Fig. 6.

at least four times as great as w .¹ The plates numbered from 1 to 8 in the table were therefore ruled so that all the dimensions except l were the same in each case.

The percentage reflection of each of these plates was measured for the radiation obtained by the combination of the quartz lenses and potassium iodide reflecting surfaces (Fig. 3). These values, plotted as a function of the resonator length, l , are shown in Fig. 10.

CONCLUDING REMARKS.

The curve shown in Fig. 10 has a distinct maximum for a resonator length of about $29\ \mu$. Although this maximum is not very sharp, it is sufficiently pronounced to show that the microscopic metal strips on the plates function as electrical resonators when stimulated by the heat waves $95.8\ \mu$ in length emitted by a Welsbach burner.

It may be of interest to compare the results here given with those obtained by other experimenters using electric waves and linear resonators made from tinfoil. Most of the work with electric waves was done with resonators in air. For this case the theoretical value² for the ratio of wave-length to the resonator length for resonance is given by some investigators as 2 and by others as 2.5. The ratio found by different experimenters varies between these values. Perhaps the most recent experimental investigation of this point is that of Nelms and Severinghaus,³ who found that this ratio was a function of the axial separation of the resonators, but when the separation was greater than twice the resonator length, it approached the value 2.5. The curve shown in Fig. 10 indicates that for resonance the wave-length ($95.8\ \mu$) is 3.3 times the resonator length. Because of the high dielectric constant of glass the electrostatic capacity of the individual resonators is greater than is the case when they are completely surrounded by air, and for this reason it is to be expected that the ratio of wave-length to resonator length for resonance should be somewhat greater than 2.5. A similar result was found by Blake and Fountain⁴ for electric waves having a wave-length of 10 cm. and resonators made from strips of tinfoil. These investigators found that when resonators attached to a glass plate were separated axially a distance equal to 0.5 of a wave-length the ratio of resonator

¹ Nelms and Severinghaus, loc. cit.

² MacDonald Electric Waves, p. 111.

M. Abraham, Ann. der Physik, 66, 435 (1898).

Rayleigh, Phil. Mag., 8, 105 (1914).

Oseen, Mat. Astron. Och Fysik (Stockholm), 9, 30, p. 1 (1914).

³ Loc. cit.

⁴ Loc. cit.

length to wave-length for resonance was 3.5. It is not possible to make any accurate comparison between the values found for resonators on glass for electric waves and for the long heat waves used in the investigation described in this paper; the dielectric constant, even if the same kind of glass were used, might be very different in the two cases because of the difference in frequency. The composition of the glass is undoubtedly also an important factor. In spite of these different conditions the agreement between the value as found in this investigation and that given by Blake and Fountain for the ratio of wave-length to resonator length for resonance is very close.

It would be very desirable to separate the amount of energy reflected by the resonators from the amount reflected by the glass. Rubens and Nichols¹ assumed that the resonators and the glass reflected independently. It does not appear that this assumption is altogether legitimate, as the experiments of Blake and Fountain show. They found that for certain resonator lengths less energy was reflected by a glass plate covered with resonators than from a plate of bare glass. There appeared to be no simple way of separating these two quantities so that the results are here given only for the composite structure.

In the experiments of Rubens and Nichols it was found that a much larger percentage of the energy was reflected by linear resonators when the wave-length was equal to twice the resonator length. Not a sufficient number of resonators of different lengths were tried, however, to show the exact position of the maximum. The distance between the resonators which they used was a small fraction of a wave-length. Now, experiments² on electric waves have shown that as the separation between the resonators is decreased the length of the resonators for resonance is increased. For this reason it is probable that a resonator length equal to one half the wave-length was very nearly the proper length. It is therefore more than probable that the phenomenon observed by them was due to electrical resonance. The maximum reflection observed by Rubens and Nichols was 66 per cent. as compared with 35 per cent., the value found in the experiments here described; this difference is probably due to the fact that the number of resonators per unit area was much greater. The latter value agrees more closely with the value obtained by Blake and Fountain with electric waves.

Although the results obtained in these experiments do not indicate that metallic resonators of the type considered reflect highly selectively, yet the reflection is so much greater within a certain region that the use

¹ Loc. cit.

² Blake and Fountain, loc. cit.

of such plates of resonators for isolating heat waves of a certain wave-length by multiple reflection does not seem altogether impracticable. Severinghans and Nelms¹ used a similar method for obtaining electrical waves of much greater purity than was otherwise possible. Although the measurements of Rubens and Von Baeyer² on the radiation obtained from a quartz mercury vapor lamp indicate the presence of wave-lengths as great as $600\ \mu$, by far the larger proportion of the radiation was of much shorter wave-lengths. If this radiation were reflected a number of times from plates of resonators of proper dimensions the longer wave-lengths would be isolated more definitely. The outstanding difficulty in the use of such reflecting plates is that the energy finally obtained would be very small, perhaps too small, for accurate measurement with apparatus at present available

In conclusion the writer wishes to acknowledge his indebtedness to Professor E. F. Nichols, who suggested the foregoing investigation, for his kindly encouragement and very helpful suggestions.

SLOANE LABORATORY,
YALE UNIVERSITY.

¹ Loc. cit.

² Sitzber. der Berl. Akad., 1, 666 (1911).

THE CRYSTAL STRUCTURE OF SODIUM NITRATE.

BY RALPH W. G. WYCKOFF.

SYNOPSIS.

The crystal structure of sodium nitrate has been determined from a study of the Laue photographs obtained by passing X-rays through crystal sections. The *method of interpretation* is similar to that used by Nishikawa in studying spinel.

The unit rhombohedron of sodium nitrate is a body-centered structure containing two molecules of NaNO_3 . The marshalling of the atoms in the crystal as a whole resembles that within a crystal of sodium chloride with NO_3 groups replacing the chlorine atoms of NaCl .

MEASUREMENTS of the reflection spectra from the (100) and (111) faces of sodium nitrate have been made by W. L. Bragg.¹ Because of the close crystallographic relationship between calcite and sodium nitrate and because of their similar spectra it was concluded that the atoms in sodium nitrate are arranged in the same fashion as are the atoms in the more thoroughly studied calcium carbonate. If this arrangement for the atoms in sodium nitrate is the correct one, then univalent sodium replaces divalent calcium, and tri- or quinquivalent nitrogen replaces quadrivalent carbon without changes in the crystal structure. The inevitable conclusion is that in these two compounds there is no *peculiar* connection between the valencies of the atoms concerned and their arrangement in space.

This conclusion is of such great importance that it seemed desirable to assure its truth by a more detailed study of the structure of sodium nitrate. Consequently a determination of the crystal structure was made from a study of the Laue patterns obtained by passing rays through sections of sodium nitrate cut parallel to the (100) and the (111) faces. The method of interpretation is a modification of that used by Nishikawa² in studying spinel.

THE SPECIMENS.

The crystals used in these experiments were obtained by the slow evaporation over sulphuric acid of a solution of sodium nitrate. The specimens used were perfect rhombohedra about one centimeter on a side.

¹ Bragg, W. L., Proc. Roy. Soc. (A), 80, 468 (1914).

² Nishikawa, S., Tokyo Sugaku-Buturigakkwai Kizi (2), VIII., 199 (1915).

Sodium nitrate crystallizes in the ditrigonal scalenohedral class of the hexagonal system. The crystallographic data are given as follows:¹

$$\alpha = 102^\circ 42.5'; \quad a : c = 1 : 0.8297.$$

The best determination places the density as 2.271.²

THE LATTICE AND THE NUMBER OF ASSOCIATED MOLECULES.

A comparison X-ray spectrum of tungsten using NaCl (100) face, (a crystal of known spacing) and NaNO_3 , (100) face, was prepared. The equation for reflection is:³

$$n\lambda = 2d \sin \theta. \quad (1)$$

λ (the wave-length) for the various lines of tungsten is known;⁴ $\sin \theta$ (where θ is the angle of diffraction) can be determined by measurements upon the plate and from a knowledge of the distance from the crystal to the plate. The result of a number of closely agreeing values for d/n (the ratio of the spacing to the order of the reflection) is 3.035×10^{-8} cm.

The volume of a rhombohedral unit of structure of angle $102^\circ 42.5'$ in terms of the spacing for the face (100) is

$$V \text{ (the volume)} = d^3_{(100)} \times 1.077,$$

$$V = \frac{mM}{\rho} = d^3 \times 1.077,$$

where m = the number of molecules associated with the unit of structure, M = the weight of one molecule of sodium nitrate ($85.01 \times 1.64 \times 10^{-24}$ gm.), and ρ = the density.

$$d^3/n^3 \times 1.077 = m/n^3 \times M/\rho,$$

$$n^3/m = M/(d^3/n^3 \times \rho \times 1.077) \quad (2)$$

$$= 2.038 \text{ if the unit is a rhombohedron.}$$

It will be seen that the reflection closely corresponds with the second order spectrum from a rhombohedron containing four molecules of sodium nitrate. This is in agreement with the measurements of W. L. Bragg (op. cit.) and suggests a face centered arrangement similar to that deduced for calcite. Comparison of experiment with the other possible unit, the hexagonal prism, indicated that a structure having the hexagonal lattice is highly improbable.

¹ Groth, P., *Chemische Krystallographie*, II., 72.

² Groth, P., op. cit.

³ Bragg, W. H. and Bragg, W. L., *X-rays and Crystal Structure*, Chap. 2.

⁴ Siegbahn, M., *Jahrb. d. Radioakt. u. Elektronik*, XIII., Heft 3, 296 (1916).

THE LOCATION OF THE ATOMS WITHIN THE UNIT OF STRUCTURE.

Several Laue photographs were made both with the rays at right angles to the (111) face and also inclined at small angles to this normal position. One photograph was made with the rays perpendicular to the (100) face. The two symmetrical cases are shown in Figs. 1 and 2.

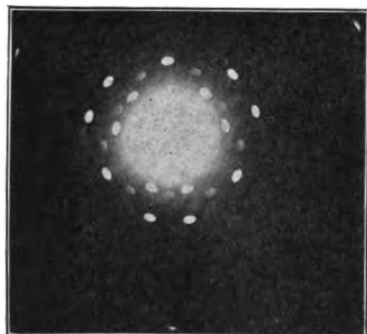


Fig. 1

Laue photograph obtained by passing X-rays through a section of sodium nitrate in a direction normal to the (111) plane.

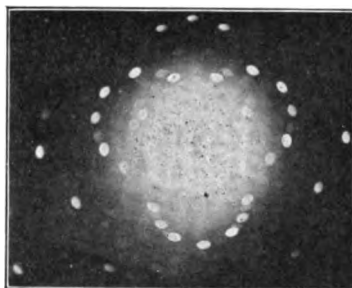


Fig. 2.

Laue photograph obtained by passing X-rays through a section of sodium nitrate in a direction normal to the (100) plane [original axes].

From a knowledge of the position of the crystal, the distance from the crystal to the photographic plate and the crystallographic data, the stereographic projection can in each case be prepared and the planes producing the various spots identified. When this is done using the commonly chosen angle between the axes ($102^{\circ} 42.5'$) there is no simple correspondence between spots and planes. If, however, the face diagonals of the rhombohedron formed by taking the old axes as edges are chosen as axes, the observed spots prove to be due to simple planes.

CALCULATIONS FROM LAUE PHOTOGRAPHS.

Equation (1) connecting the spacing, angle of diffraction and wavelength of the X-rays holds true. The spacing d between like planes for a rhombohedral lattice is¹

$$d = \frac{a \sqrt{1 + 2 \cos^2 \alpha - 3 \cos^4 \alpha}}{\sqrt{(h^2 + k^2 + l^2) \sin^2 \alpha + 2(hk + hl + kl)(\cos^2 \alpha - \cos \alpha)}},$$

where h , k , and l are the Miller indices of the plane, α is the angle between the rhombohedral axes and a is the length of the side of the unit rhombohedron.

From equation (1) then

$$n\lambda = \frac{2a \sin \theta \sqrt{1 + 2 \cos^2 \alpha - 3 \cos^2 \alpha}}{\sqrt{(h^2 + k^2 + l^2) \sin^2 \alpha + 2(hk + hl + kl)(\cos^2 \alpha - \cos \alpha)}} \quad (3)$$

$\sin \theta$ is readily obtained from measurements upon the photographic plate. The value of $n\lambda$ calculated from equation (3) is plotted for the various spots estimated intensity of the spot.

X-rays of a wave-length $0.44 - 0.47 \times 10^{-8}$ cm. have a maximum effect upon the photographic plate. Consequently clustered about this first value of the wave-length, planes are found which are reflecting in the order, about twice this value of $n\lambda$ planes reflecting in the second order, and so forth. In the tilted photographs corresponding planes will appear at different distances from the central spot and will be reflecting rays of different wave-lengths. If, in the prepared graph, these corresponding planes are connected, a set of curves of similar shape will be obtained, one above another and each representing the relative effect of different wave-lengths upon the photographic plate. If the atoms in the

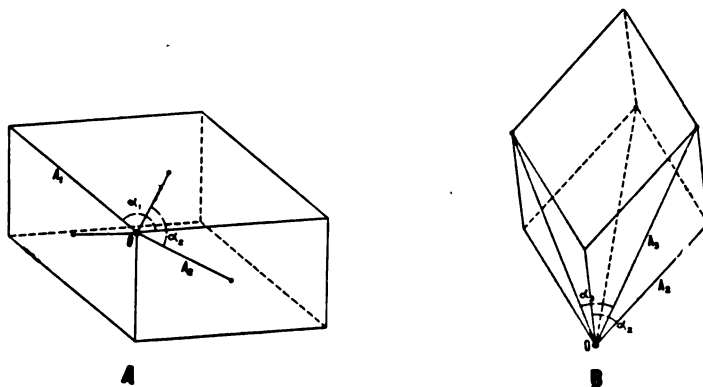


Fig. 3.

A_1 is axis of crystallographic unit. $\alpha_1 = 102^\circ 42.5'$.

A_2 is axis of the unit used in preparing the Stereographic Projections. $\alpha_2 = 77^\circ 24'$.

A_3 is axis of true unit. $\alpha_3 = 47^\circ 14'$.

crystal are in the simple cubic array of a Bravais point lattice, the curves for planes with smaller spacing appear above those having larger spacing. Departures from these ideal conditions will be shown by planes of small spacing, giving more intense reflections (for a particular wave-length of X-rays) than the simpler planes. The non-appearance of a reflection from a plane placed in a favorable position for reflection is as important as the observation of a reflection. Many times the connecting of similar points by a curve is inconvenient. This is quite unnecessary as long as

comparison is limited to only those points which are produced by waves of the same length.

When such a plot is constructed using the indices of planes obtained from the second set of axes (angle = $77^{\circ} 24'$), it is found that a number of points, all having odd indices, *appear* to reflect strongly in the neighborhood of 0.24×10^{-8} cm. This abnormality, however, could be removed by the choice of a third set of axes which are the diagonals of the faces of the unit rhombohedron made by the second set and are the true axes of the unit of structure of sodium nitrate. The relation between the

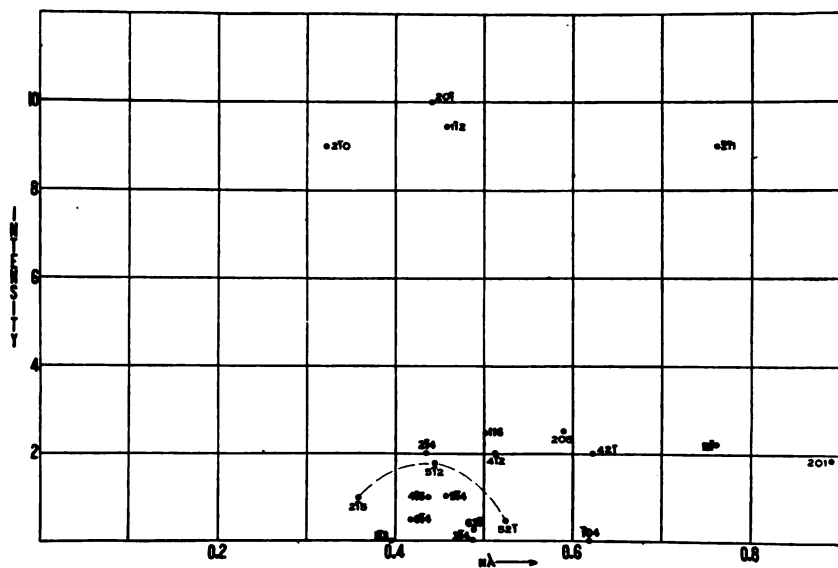


Fig. 4.

The points which appear in this figure were chosen not because of their special importance but for the purposes of illustration only.

three sets of axes is shown in Fig. 3. The graph for certain points of the first unsymmetrical (tilted) photograph (indices those of the third set of axes) is given in Fig. 4.

THE ARRANGEMENT OF THE ATOMS.

The symmetry of sodium nitrate is that of the point group D_3^d . The spectrum measurements and the inspection of Laue photographs indicate that the fundamental lattice is rhombohedral. There are two space groups¹ of D_3^d which have Γ_{rh} as the fundamental lattice: D_{3d}^5 and D_{3d}^6 . The group in this case was found to be D_{3d}^6 . For D_{3d}^6 the positions of

¹ Schönflies, A., *Krystallsysteme u. Krystallstruktur*.

equivalent points are:

$$(1) \quad xyz, yzx, zxy; \quad \bar{y}\bar{x}\bar{z}, \bar{z}\bar{y}\bar{x}, \bar{x}\bar{z}\bar{y};$$

$$(2) \quad \tau_x - x, \tau_y - y, \tau_z - z; \tau_x - y, \tau_y - z, \tau_z - x; \tau_x - z, \tau_y - x, \tau_z - y; \\ \tau_x + y, \tau_y + x, \tau_z + z; \tau_x + z, \tau_y + y, \tau_z + x; \tau_x + x, \tau_y + z, \tau_z + y;$$

where the axes XYZ are taken along the newest crystallographic axes and $\tau_x = \tau_y = \tau_z$ (in length) is a translation of half the length of an edge of the unit cell in the direction of the subscripted axis. Two molecules of sodium nitrate must be placed within this unit cell of D_{3d}^8 by the following choice of coördinates.

$N = 000$ and $a/2, a/2, a/2$ where a = the side of the rhomb.

$Na = a/4, a/4, a/4; 3a/4, 3a/4, 3a/4.$

$O = \beta, a - \beta, 0; 0, \beta, a - \beta; a - \beta, 0, \beta; a/2 - \beta, \beta - a/2, a/2; \\ \beta - a/2, a/2, a/2 - \beta; a/2, a/2 - \beta, \beta - a/2,$

where β/a is fractional part of a .

For this arrangement of atoms the formula giving a quantity proportional to the intensity of the n th order reflection from a plane (hkl) will be

$$A^2 + B^2 \propto \text{Intensity} \quad (4)$$

$$A = (1 + \cos n\pi s) \left\{ \bar{N} + \bar{N}a \left(\cos n \left(\frac{\pi}{2} \right) \right) s + \bar{O} \left[\cos 2\pi n\beta(h - k) \right. \right. \\ \left. \left. + \cos^2 \pi n\beta(l - h) + \cos 2\pi n\beta(k - l) \right] \right\}.$$

$$B = (1 - \cos n\pi s) \bar{O} [\sin 2\pi n\beta(h - k) + \sin 2\pi n\beta(l - h) + \sin 2\pi n\beta(k - l)],$$

where $\bar{N}$, $\bar{N}a$ and $\bar{O}$ represent the diffracting (scattering) powers of nitrogen, sodium and oxygen respectively and $s = h + k + l$.

Because of the uncertainty of the effect of spacing upon the intensity of reflection¹ only spots from planes of nearly equal spacing which were undoubtedly reflecting in the first order were used in comparisons of intensity.

The results of the study of the Laue photographs were used

I. To test the truth of the general arrangement as outlined above.

II. To obtain the probable value of β .

III. To compare the calculated data with the observed when this probable value of β is used and to search for discrepancies which could be applied to the more accurate placing of oxygen.

When $n = 1$ two cases of expression (4) arise according to the value of s ($h + k + l$).

¹ Bragg, W. H., and Bragg, W. L., op. cit., Chap. XI.

Case 1.—When s is odd (the indices are either all odd, or two even and one odd).

$$A = 0,$$

$$B = 2O[\sin 2\pi\beta(h - k) + \sin 2\pi\beta(l - h) + \sin 2\pi\beta(k - l)]. \quad (5)$$

Case 2.—When s is even (the indices are two odd and one even).

$$A = 2 \left[\bar{N} + \bar{N}a \cos \left(\frac{\pi}{2} \right) s + \bar{O} \left\{ \cos 2\pi\beta(h - k) + \cos 2\pi\beta(l - h) + \cos 2\pi\beta(k - l) \right\} \right].$$

$$B = 0.$$

I. In Case 1 only oxygen atoms would be expected to reflect. If two of the indices are equal to each other (as in $33\bar{5}$, $\bar{2}5\bar{2}$) this B term also becomes zero. This is true no matter what value is assigned to β so that the presence or absence of such points gives one excellent means of testing the truth of the general arrangement of atoms. No points of case 1 having two equal indices were found.

II. Points of case 1, except those where two indices are equal, can be used to place the oxygen atoms with accuracy. This can be done by plotting the variations in the value of the calculated amplitude with changes in the value of β . The various points of case 1 which appear in the photographs group themselves according to the values of $h - k$, $l - h$, $k - l$. In Fig. 5 curves 1, 2, 3, or 4 are due to groups of spots

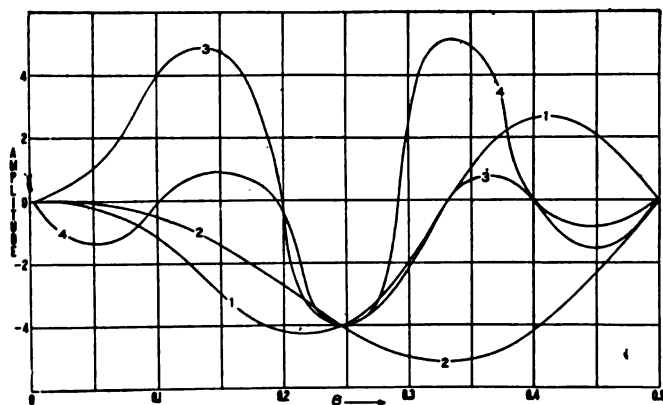


Fig. 5.

whose values for $h - k$, $l - h$, and $k - l$ are $(-1, -2, 3)$, $(-1, -1, 2)$, $(2, 3, \bar{5})$, and $(7, \bar{2}, \bar{5})$. These curves agree in all being intense only close to the point $\beta = 1/4$. The oxygen atom must lie, therefore, on the line

joining nitrogen atoms at a position near to one fourth of the distance from one nitrogen atom to the next.

III. If the value of β is exactly one fourth, planes fulfilling the condition of Case 1 (s is odd) fall into three classes.

(a) When all the indices are odd, the B term of expression (5) becomes zero.

(b) When the indices are two of them even and one of them odd and the difference between the even indices is divisible by 4, the B term of (5) becomes zero.

(c) When the indices are two of them even and one of them odd and the difference between the even indices is not divisible by 4, the B term of (5) is *not* zero.

An examination of three Laue photographs which averaged reflections from one hundred and fifty planes each, failed to show a single plane of either class (a) or of class (b). This leads to the conclusion that the oxygen atoms deviate very slightly, if at all, from the position $\beta = 1/4$.

As would be expected, since reflection is not then restricted to the oxygen atoms, spots from planes having two odd and one even indices (Case 2) are most numerous and intense. These planes also are of two sorts. If, on the one hand, s is divisible by 4, the effect due to the sodium atoms adds to that due to the nitrogen and oxygen atoms in building up the reflection from a plane; but if s can not be divided by 4 the amplitude contributed by the nitrogen atoms is to be subtracted from the effects of the other atoms. For the same wave-lengths of X-rays and equal spacing of like planes, a plane of the first sort should reflect more strongly than one of the second. This was found to be true for those planes which gave spots upon the photographs.

CONCLUSION.

The results of these experiments confirm the structure as deduced by W. L. Bragg.¹ Each sodium atom has arranged about it three equidistant oxygen atoms. Each of the NO_3 groups thus formed is surrounded by six equidistant sodium atoms. Each sodium atom has the centers of six NO_3 groups equally far from it. This structure may thus be thought of as similar to sodium chloride with NO_3 groups replacing the chlorine atoms.

The arrangement of the atoms within the true unit of structure is shown in Fig. 6. The coördinates of the atoms within this unit cell are (assuming the length of the side as 1):

N at (0, 0, 0) and at ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$),

¹ Bragg, W. L., *op. cit.*

N at $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and $(\frac{3}{4}, \frac{3}{4}, \frac{3}{4})$,

O at $(\beta, 1 - \beta, 0)$; $(0, \beta, 1 - \beta)$; $(1 - \beta, 0, \beta)$;

$(\frac{1}{2} - \beta, \beta - \frac{1}{2}, \frac{1}{2})$; $(\beta - \frac{1}{2}, \frac{1}{2}, \frac{1}{2} - \beta)$; $(\frac{1}{2}, \frac{1}{2} - \beta, \beta - \frac{1}{2})$,

where β has a value that is very close to $\frac{1}{4}$. The length of the side of

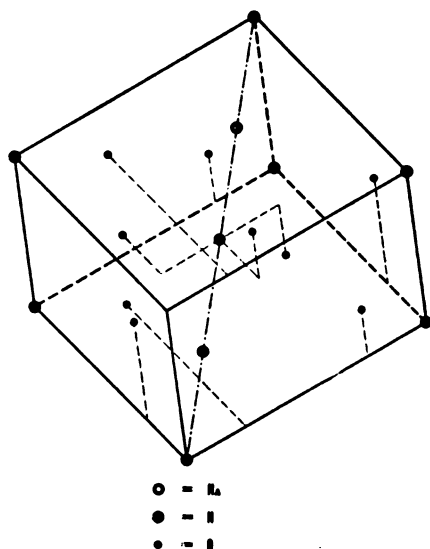


Fig. 6.

The unit rhombohedron of sodium nitrate.

this unit rhombohedron is 6.065×10^{-8} cm. and the angle between the axes is $47^\circ 14'$.

The writer wishes to express his thanks to the Department of Physics for the use of the apparatus necessary for carrying out these experiments. He is under deep obligation to Dr. S. Nishikawa without whose constant and kindly aid and criticism this work could not have been carried out.

DEPARTMENT OF CHEMISTRY,
CORNELL UNIVERSITY.

A NOTE ON THE CORRECTION OF CONTACT DIFFERENCE OF POTENTIAL DEVELOPED IN COMPTON'S MODIFICATION OF THE QUADRANT ELECTROMETER.

BY OTTO STUHLMAN, JR.

SYNOPSIS.

A method of eliminating contact difference of potential in Compton's modification of the quadrant electrometer is suggested. The quadrants must not be built up. Solder, fillers or finishers must be omitted. If the needle is of aluminium the quadrants must be of aluminium, milled out of a single piece of metal. The final curve shows the absence of contact difference of potential in such an instrument when reconstructed and possessing an aluminium needle and milled aluminium quadrants.

IN an article on "A Sensitive Modification of the Quadrant Electrometer"¹ it is said that "the most serious sources of difficulty appear to be irregularities in the needle, or quadrants, and contact difference of potential between the quadrants," and further that this effect is proportional to the first power of the needle potential. Incidentally it is also stated that "such contact differences of potential, if found troublesome, may be removed by cleaning (the quadrants) or compensated by a small potential permanently applied to the 'earthed' quadrants."

Unfortunately however, many conditions may arise when it is neither possible or desirable to have a compensating potential permanently applied to the "earthed" quadrants. In these circumstances the contact difference of potential existing in the quadrant system of the instrument must be removed by structural changes. The simplest and most obvious changes would be to make all the quadrants as well as the needle of the same metal. The quadrants should not be made of several pieces soldered together as in the "Compton Electrometer," but should be cut from the solid metal and then machined to the proper size and shape.

In Fig. 1, are shown three examples to indicate how, under electrostatic positive control, the scale deflections vary as the potential across the quadrants is increased. The scale deflections, proportional to θ , are plotted in Figs. 1, 2 and 3 so that, as in Fig. 1 for example, if the large deflections are to be strictly interpreted they must be read $\tan 2\theta = d/D$, where d is the deflection and D the distance of the scale

¹ A. H. Compton and K. T. Compton, *PHYS. REV.*, Vol. 14, Aug., 1919.

from the mirror. For the large deflections this correction is about one per cent. The potentials were measured by means of a standardized Simens and Halske millivoltmeter. The needle of the instrument was of aluminium foil and the quadrants of brass. Each of the latter were made of three pieces, top and bottom soldered to a circular vertical band.

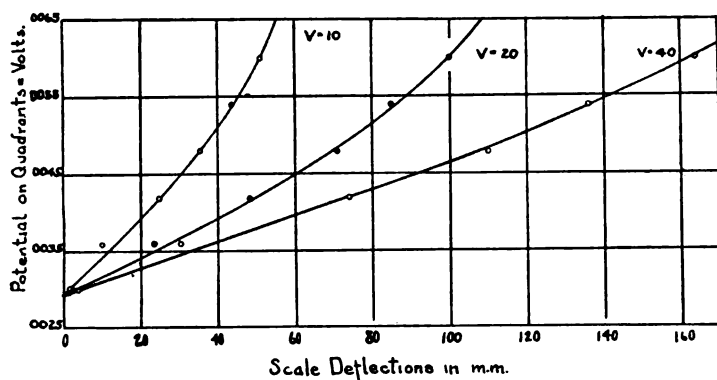


Fig. 1.

If now the needle is kept at a definite potential and the deflections observed as the potential on the quadrants are increased, it is found that the curve would not pass through the origin. Changes in the potential of the needle say from forty to twenty and finally to ten volts developed curves which possessed approximately equal intercepts on the voltage

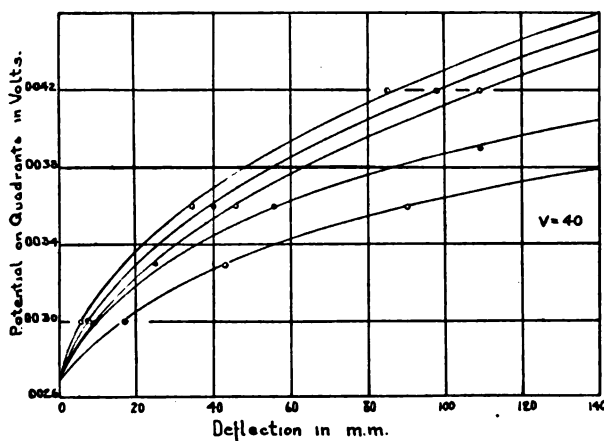


Fig. 2.

axis. This value was found to be 0.00292 volts. The quadrants were next adjusted to produce negative electrostatic control and the above observations repeated. The intercept of these curves had the same value as that of the above curve.

The quadrants were then removed, thoroughly cleaned in acid and scraped to remove all visible indications of solder. The observations were repeated and the voltage intercept was found to have dropped to 0.00264 volts. In Fig. 2 is shown a typical set of results obtained under these conditions. They represent the quadrants adjusted to negative electrostatic control with 40 volts on the needle. The successive curves show that the intercept is independent of the degree of negative control. The upper curves were obtained under less electrostatic control than each of the successive lower ones.

Hence variation in the degree of electrostatic control or potential on the needle does not change the value of the voltage intercept.

Further cleaning and scraping of the quadrants did not reduce this value below 0.00170 volts. A tilt of the needle about its minor axis may account for the fact that the curves do not pass through the origin.

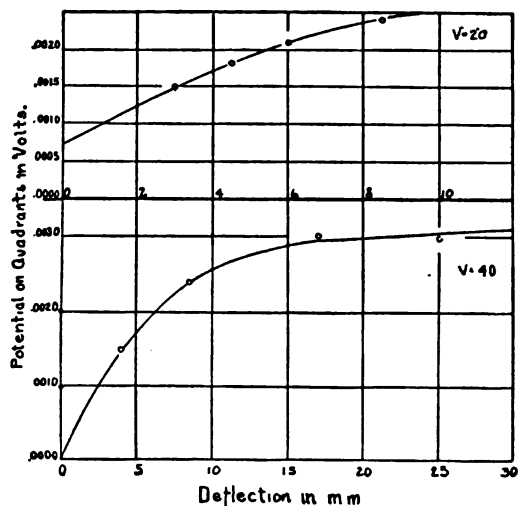


Fig. 3.

This explanation was eliminated because the same needle without variation in its tilt was used throughout the observations. A contact difference of potential in the system would account for it, especially in view of the successive changes in the value of the intercept, as the physical conditions of the inner surfaces of the quadrants were changed. In order to test this hypothesis, the quadrants were replaced by aluminium ones constructed as suggested above, the aluminium foil needle retaining its original tilt. This structural change produced the results shown in the upper curve of Fig. 3. The supposed contact difference of potential has been reduced to 0.00070 volts. An examination after twenty-

four hours and all later examinations showed no contact difference of potential and a typical curve developed under these conditions and under negative electrostatic control is shown in the lower half of Fig. 3. The curve passes through the origin. The subsequent drop from 0.00070 to zero volts is of course accounted for by the occlusion of air on the new surfaces of the aluminium quadrants, ageing¹ them to the same surface conditions as existed on the very much older aluminium foil needle.

A further point of interest in the above curves is the lack of uniformity in the sensitiveness for all values of the needle deflection. This has been pointed out by Compton, in that "the presence of the term $k_5 V^2 \theta^2$ in equation (5) shows that the sensitiveness is not the same for all values of θ , i.e., over all parts of the scale, though this term is not important, except at very high sensitiveness." Unfortunately no curves for this variation are shown in the above paper so that no comparison is possible. How close these curves approach to a straight line can be seen in the figures here presented. The best results were those obtained in Fig. 1, when the needle was charged to 40 volts and the quadrants adjusted to electrostatic positive, though nearly zero, control.

The data for this note was obtained while the writer was a member of the staff of the State University of Iowa.

WEST VIRGINIA UNIVERSITY,
MORGANTOWN, W. VA.

¹ See "On the Influence Contributing to the Variation of Contact Electromotive Force with Time," O. Stuhlman, *PHYS. REV.*, Vol. 8, p. 294, 1916.

NOTE ON THE ELECTROMAGNETIC FORCE BETWEEN TWO ATOMS.

BY G. A. SCHOTT.

SYNOPSIS.

Making use of results obtained in a previous paper giving the average radial repulsion exerted by a charge e on a charge e' when both charges are revolving in circular orbits at a great distance, the author obtains a formula for the force between atoms. He concludes, in reply to a criticism by A. C. Crehore,—

In a passive neutral atom the radial force vanishes completely.

When both atoms are ionised there is a residual force, negligible in comparison with an electrostatic force which may be taken to represent the chemical force between the ions.

When one atom is neutral and the other ionised, the electrostatic force vanishes, leaving a residual force, which is estimated to be too small to play anything but a very small part in chemical actions.

1. Crehore's preliminary reply¹ to my criticism² of his first paper³ has just come to my notice. Previously I purposely confined my attention as far as possible to the supposed gravitational attraction between two revolving *electrons*, but now Crehore has raised the question of the force between *neutral atoms*, so I will consider it briefly.

The problem is to calculate the average radial force, to the order of the inverse square of the distance, exerted by an atom of type 2 on an atom of type 1, both being supposed symmetrical about an axis, and the axes distributed uniformly in all directions on the average for a large number of atoms. My formula (50), p. 37, gives the average radial repulsion, F , exerted by a charge e , revolving in a circle with β times the speed of light, on another charge e' at a great distance r in the form

$$F = ee' \{1 + f(\beta)\} r^{-2}, \quad f(\beta) = \beta^2 \left(\frac{1}{1 - \beta^2} - \frac{1}{2\beta} \log \frac{1 + \beta}{1 - \beta} \right). \quad (1)$$

The formula (54), p. 456 of Crehore's earlier paper, when averaged for all directions of the axes of the orbits, is of the same form, but

$$f(\beta) = -\frac{1}{2}\beta^2. \quad (2)$$

The speed of the second charge e' does not occur at all, so that the law of action and reaction is violated, as Crehore objects. But this violation

¹ A. C. Crehore, *PHYS. REV.*, Vol. XIII., p. 89.

² G. A. Schott, *PHYS. REV.*, Vol. XII., p. 23.

³ A. C. Crehore, *PHYS. REV.*, Vol. IX., p. 445.

follows directly from the use of retarded potentials, which express the fact that electromagnetic force, momentum and energy require time for their propagation. Without it we could not even account for electromagnetic mass, which is a result of the unbalanced internal electromagnetic forces between the parts of the electron.

2. In applying (1) to the problem of the atoms we replace e, β, e' by $-e_2, \beta_2, -e_1$, or by $E_2, 0, E_1$, according as we seek the actions between the electrons, or the positive nuclei, of the atoms. The average radial repulsion is the sum of these four terms:

$$\left. \begin{aligned} (a) \text{ Electrons 2 acting on electrons 1: } & \Sigma e_1 \Sigma e_2 \{1 + f(\beta_2)\} r^{-2}, \\ (b) \text{ Electrons 2 acting on nucleus 1: } & -E_1 \Sigma e_2 \{1 + f(\beta_2)\} r^{-2}, \\ (c) \text{ Nucleus 2 acting on electrons 1: } & -\Sigma e_1 E_2 r^{-2}, \\ (d) \text{ Nucleus 2 acting on nucleus 1: } & E_1 E_2 r^{-2}, \end{aligned} \right\} \quad (3)$$

Hence we find

$$F = \{(\Sigma e_1 - E_1)(\Sigma e_2 - E_2) + (\Sigma e_1 - E_1) \Sigma e_2 f(\beta_2)\} r^{-2}, \quad (4)$$

where the summations are for all the electrons of the two atoms. The first term represents the electrostatic repulsion, the second a comparatively small residual force, which for low speeds is of the order β_2^2 with (2), but only of order β_2^4 with (1). There are three cases:

(1) *Passive atom neutral*; $\Sigma e_1 = E_1$.

The radial force F vanishes altogether.

In my opinion this disproves Crehore's objection to (1). He himself obtains a different result, because he retains the term $f(\beta_2)$ in (3a), but omits it in (3b); in other words he treats a passive nucleus differently from a passive electron, although the velocity of neither one nor the other occurs in (1) or (2). I cannot see any reason for this different treatment of passive negative and positive charges. It is different with Crehore's modified formula, which differs from (2) by the presence of an additional factor, viz., the square of β for the passive charge multiplied by a positive constant, chosen so as to account for ordinary gravitational attraction. Since (2) ought to be replaced by (1), the chief reason for this change disappears; besides, as it makes the formula agree with the law of action and reaction, it is difficult to reconcile it with the explanation of electromagnetic mass.

(2) *Both atoms ionized*; $\Sigma e_1 \neq E_1, \Sigma e_2 \neq E_2$.

The residual force may be neglected in comparison with the electrostatic force, which may be taken to represent the chemical force between the ions according to the usual interpretation.

(3) *Active atom neutral, passive atom ionized*; $\Sigma e_1 \neq E_1, \Sigma e_2 = E_2$.

The electrostatic force vanishes, and we are left with the residual force

$$F = (\Sigma e_1 - E_1)E_2f(\beta_2)r^{-2}.$$

Since $f(\beta_2)$ is essentially positive according to (1), this force is a repulsion, or an attraction, according as the passive ion is negative, or positive. Taking β_2 to be of the order 0.01 we see that the force is only of the order of one hundred-millionth of ordinary chemical forces. Thus it is not likely to play anything but a very small part in chemical actions, though it might conceivably be influential in solution phenomena and others of like nature. I think that I was quite justified in my former paper in expressing a doubt as to the possibility of detecting its existence by experiment.

THE PHYSICAL REVIEW.

CHARCOAL ACTIVATION.

BY H. HORTON SHELDON.

SYNOPSIS.

Adsorption of Gases by Charcoal—Heat Effect.—The first part of this paper comprises a study of the heat effect on the adsorption of hydrogen and nitrogen by charcoal, and is a continuation of the work of Dr. Harvey B. Lemon¹ in which air was used. The charcoal is heated for periods varying from one hour to twenty hours at temperatures from 400° C. to 1000° C., and at the same time the gases given off are pumped out with a mercury condensation pump. The activity of the charcoal is tested after each treatment by its adsorption of gas at liquid air temperature, and the results are shown by curves where logarithm of time in minutes, is plotted against logarithm of pressure in centimeters. The agreement with predictions from the hydrocarbon theory of adsorption are pointed out. That activation takes place by slow oxidation at room temperature is also shown.

A New Form of Charcoal—Selective Adsorption.—By heating to 1000° C. for 3½ hours, a sample of charcoal was put into a condition such that it adsorbed hydrogen more readily than nitrogen. Theory is advanced to account for this.

THE results obtained at Ryerson Laboratory some time ago by Dr. Harvey B. Lemon¹ on the activation of charcoal, and published recently, led at the time to two obvious suggestions; the first, that the activation was due to the oxidation from the air used—the hydrocarbon hypothesis; the second, that it was due to a modification of the charcoal structure, brought about by the heat treatment.

Of these two, the hydrocarbon hypothesis met with the greater favor, and in articles published since the war² has been enlarged into a more or less complete theory. The observations on which this theory was based, however, were apparently made from experiments in which the charcoal was tested under field conditions, such as existed in gas masks, and no attempt was made to free the charcoal from air, moisture, etc. Further, since no data was published in support of this theory, it was decided to perform some experiments here with pure gases to satisfy

¹ PHYSICAL REVIEW, Vol. 14, No. 4, Oct., 1919.

² J. of Ind. and Eng. Chem., Vol. II., No. 5, May, 1919. N. K. Chaney, paper presented at Thirty-sixth General Meeting of the American Electro-Chemical Soc., Chicago, 1919.

ourselves of the conclusions drawn. Accordingly work was begun on pure nitrogen and pure hydrogen.

In the course of the work, which was similar to that done by Dr. Lemon with air, it was observed that charcoal could be activated or deactivated so far as the adsorption of nitrogen was concerned, much more easily than was the case with hydrogen. This at once suggested the possibility of putting charcoal into such a condition that it would adsorb relatively a large quantity of hydrogen compared to nitrogen, thus being the reverse of all previously known charcoals.

This was successfully carried out as noted in *Science* for December 19, 1919; and in a paper by the author before the American Physical Society in November, in which the method was described. So little has as yet been done on this, that it is at present impossible to predict its utility in commerce.

The apparatus which was identical with Dr. Lemon's, consisted of a central volume, connected on the one side to a McLeod gauge and on the other to a mercury condensation pump. From the central volume branched three tubes which contained the charcoal, and each of which could be closed from the rest of the system by stopcocks. The tubes were made of iron and were connected to the glass system by ground joints. The gas in these tubes could be tested by the use of Geissler tubes attached to their sides, which were also used to roughly gauge the vacuum. The purity of the hydrogen used was tested by using the Geissler tubes with a disruptive discharge, but in the case of the nitrogen the test was made by observing the afterglow in an electrodeless discharge tube.

Each charcoal tube contained 25.7 gms. of charcoal, which was dried by heating and weighed while still hot. The first tube contained U. S. Government 600-minute charcoal, and the other two Sample No. 16, which was carbonized at 670° C. for three hours. This latter was part of the same sample No. 16 used by Dr. Lemon and later by Dr. Lemon and Miss Blodgett.¹ Only one of these latter tubes will be recorded, the second being used as a check only.

The method used was to enclose the three tubes in a furnace and to heat the charcoal to various temperatures, and for various lengths of time as specified throughout the paper. All gases given off from the charcoal when heated, were pumped out by the condensation pump which was kept running, until, when the furnace was allowed to cool; it was closed off by a stopcock, at about 400° C. Below this temperature an auxiliary

¹ PHYSICAL REVIEW, Vol. XIV., No. 5, Nov., 1919.

charcoal bulb immersed in liquid air was used to insure a good vacuum. This process has been called "outgassing."

The central volume was then filled with the gas whose adsorption it was desired to measure, and readings taken on the individual charcoal tubes at liquid air temperature.

These readings are shown on the graphs where the logarithm of the time in minutes is plotted against the logarithm of the pressure in centimeters. With the exception of the curves in Figs. 5 and 6, which were taken with an initial volume of 1,780 c.c. (at normal temperature and pressure) all the curves were taken with an initial volume of 926 c.c. normal.

PART I.

The curves of Fig. 1 are the adsorption curves of hydrogen. Curve 1 was taken after a preliminary outgassing of 15 hrs. at 400°C .; curve 2 after 8 hrs. at 400°C .; and curve 3 after 20 hrs. at 400°C .

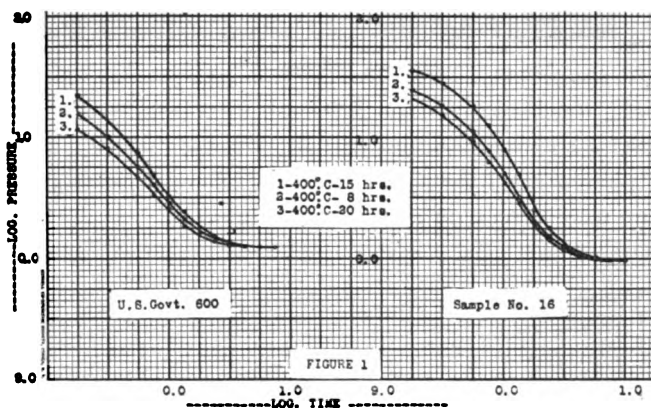


Fig. 1.

It will be observed here that the final pressure is unchanged throughout, while at this temperature when air was used the charcoal was greatly activated. This agrees with the hydrocarbon theory that the activation at this temperature is due to the removal of inactive hydrocarbons by oxidation. The fact, however, that the rate of adsorption is greatly increased as shown by the drop in the initial end of the curve may be of significance and is discussed later.

In Fig. 2, curve 1 is curve 3 re-drawn from Fig. 1. Curve 2 was taken after 16 hrs. at 600°C .; curve 3 after 15 hrs. at 700°C .; curve 4 after a second 15 hrs. at 700°C .; curve 5 after 10 hrs. at 800°C . and curve 6 after 9 hrs. at 800°C .

From the figure it will be seen that an outgassing at 600°C . had no

effect, and so from this it may be concluded that any temperature below this is incapable of driving off hydrocarbons when only hydrogen is present. The first outgassing at 700°C . gives a decided improvement, showing that, according to the hydrocarbon theory, the boiling point of some of the hydrocarbons had been exceeded. The activity attained

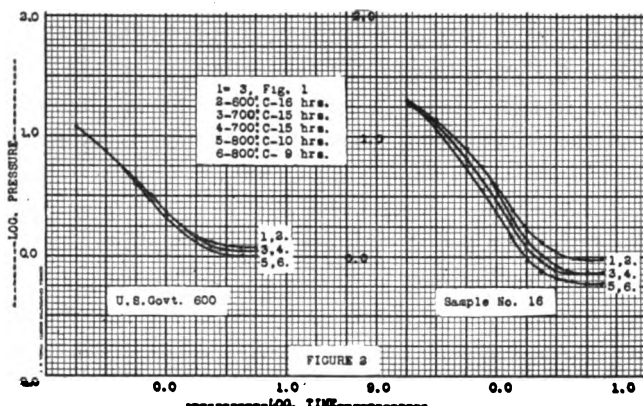


Fig. 2.

after 15 hrs. of this temperature was not improved after a second run of 15 hrs. Apparently all the hydrocarbons that could be driven off at this temperature were driven off in the first outgassing.

Ten hours at 800°C ., however, again gave improvement, showing

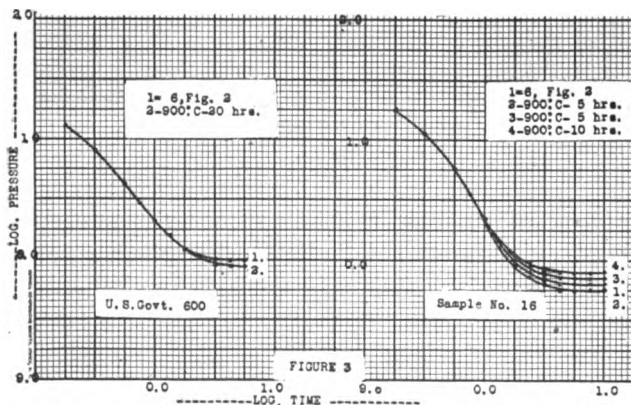


Fig. 3.

that other hydrocarbons had been driven off. A second 9 hours showed that no further improvement could be made at this temperature. Thus this set of curves shows three slightly different stages of activity.

In Fig. 3, curve 1 is curve 6 of Fig. 2 re-drawn. Curve 2 is taken after 5 hrs. at 900°C .; curve 3 after a second 5 hrs. at 900°C .; and curve 4 after 10 hrs. at this same temperature.

In the case of Sample 16, we find a slight activation after the first outgassing due no doubt to hydrocarbon removal, while subsequent outgassings deactivate. The government charcoal, however, shows only

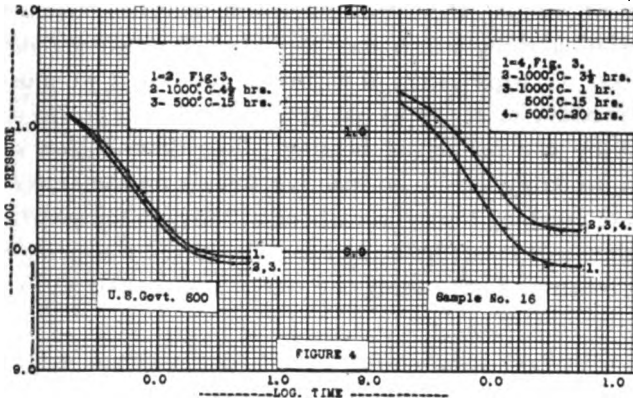


Fig. 4.

a slight improvement. In Fig. 4, we note a continued deactivation for Sample 16 and activation for U. S. Government 600. This is not a disagreement, however, but is an essential difference in the charcoals used, resulting from the method of manufacture. In the case of Sample

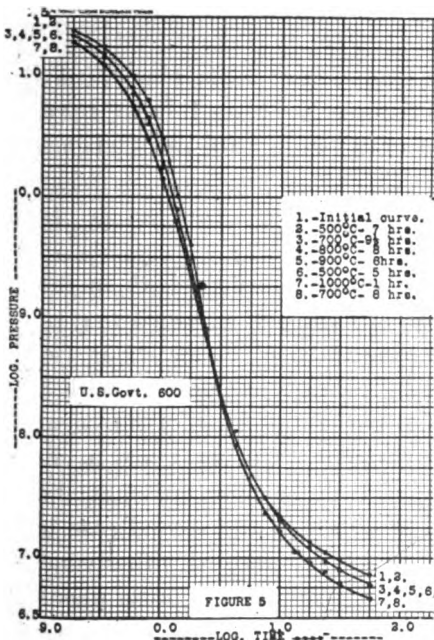


Fig. 5.

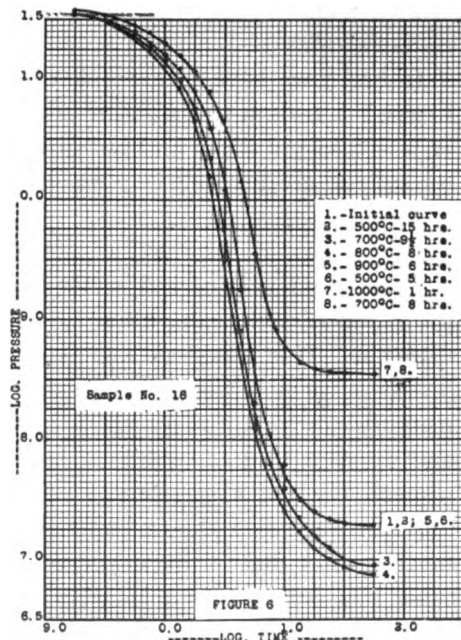


Fig. 6.

16, there are present active carbon, hydrocarbons, and inactive carbon. In the government sample, the hydrocarbons have supposedly all been removed, leaving only active and inactive carbon. If we are to deactivate the government sample, then, we must form inactive carbon from the active material, a thing which has not been done when temperatures as high as 1600° C. were used¹; if we are to activate it we must remove some of the inactive material, as has apparently been done to some very slight extent by the prolonged heating. The deactivation shown for Sample 16 is probably due to an inactive deposit on the active material as the result of cracking the hydrocarbons at the high temperatures used, or possibly the basic structure or active carbon itself was being removed.

After this treatment outgassings at 500° C. were tried and still found to be ineffective.

The work with hydrogen having completely satisfied the hydrocarbon theory, work was begun on nitrogen, and from Figs. 5 and 6 it will be seen that there is agreement throughout with predictions made from the hydrocarbon theory.

In this case curve 1 is the adsorption curve after a preliminary outgassing only; curve 2 after 15 hrs. at 500° C.; curve 3 after 9½ hrs. at 700° C.; curve 4 after 8 hrs. at 800° C.; curve 5 after 6 hrs. at 900° C.; curve 6 after 5 hrs. at 500° C.; curve 7 after 1 hr. at 1000° C.; and curve 8 after 8 hrs. at 700° C. That curves 5 and 6 fall on 2 in Fig. 6, is of course accidental.

In Fig. 7 we have an example of slow oxidation at room temperature. Curve 1 is from a run with air taken on Sample No. 9, on July 10, 1915. It was then put away and not disturbed until October 10, 1919, when a second run given by curve 2 was made.

PART II.

The second part of this work consisted, as previously stated, in the formation of a charcoal whose adsorption of hydrogen exceeded that of nitrogen. This was done merely by heating to 1000° C. for about 3½ hrs. in a vacuum which was kept down by continuous pumping with a mercury condensation pump. In Fig. 8, curve 1 is that of nitrogen, and curve 2 that of hydrogen before treatment. Curves 3 and 4 are respectively the same after treatment. It will be observed that the difference between the amounts of gas, adsorbed in the second case is much greater than in the first case. This work was verified with several other samples, but, as anticipated, the nature of the U. S. Government charcoal made it impossible with this type.

¹ N. K. Chaney, *ibid.*

It is obvious then, that there must be a decided difference between the adsorption by charcoal of hydrogen and of nitrogen.

In an article by McBain¹ it is shown that hydrogen adsorption, or "sorption" as he calls it, takes place in two stages: the first, surface condensation; the second, solid solution. In a later paper by Miss Homfray²

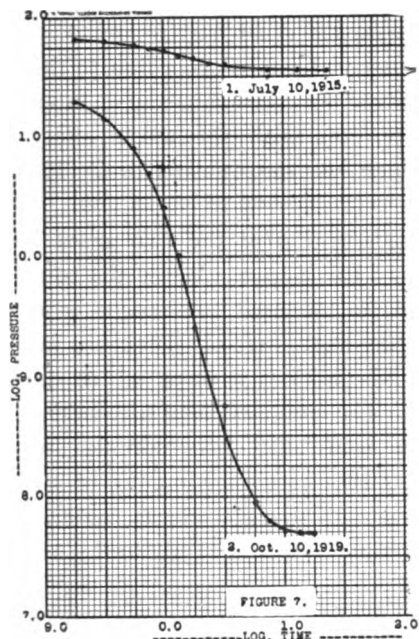


Fig. 7.

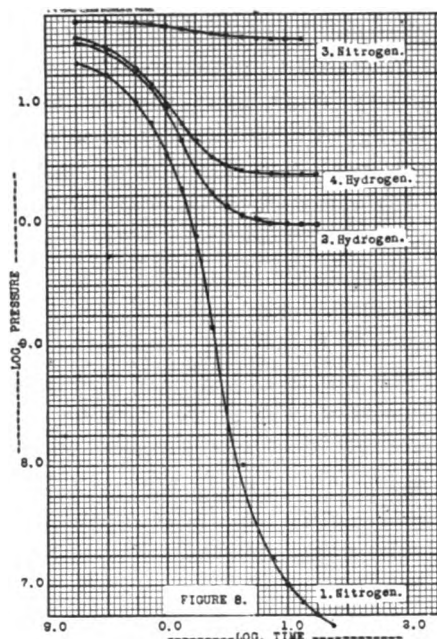


Fig. 8.

the adsorption of several gases and mixtures are studied, with the conclusion that the whole action is that of solid solution, and that two distinct stages as found by McBain do not exist. She did not, however, work with hydrogen at liquid air temperature. These papers then indicate a dual action in the case of hydrogen which does not seem to exist for other gases, and no contradiction has been found.

Since the generally accepted theory of adsorption is, at present, the surface condensation theory, it seems reasonable to assume this as the principal action in the case of both gases, and we might explain the apparent difference between hydrogen and nitrogen adsorption as due to a secondary action in the first case, which is either a chemical action, or as McBain believes, solid solution.

While it is perhaps difficult to draw a distinct line between solution and chemical action, the nature of the substances involved immediately

¹ Phil. Mag., 6, 18, 916, 1909.

² Z. für Phys. Chem., 74, 129, 1910.

suggests a loose reversible chemical combination between the hydrogen and some of the hydrocarbons present, to form hydrocarbons more stable at this liquid air temperature. Such an action in the case of nitrogen would not be expected. That such an action cannot be with the active carbon is evident from the fact that the U. S. Government 600-minute charcoal could not be put into this condition, and with this charcoal the hydrocarbons are supposedly for the most part removed.

The deposit of inactive carbons on the active base is, according to this theory, much more effective in deactivating for nitrogen, since, interfering with surface condensation, it interferes with the whole effect, this not being the case for hydrogen which has the dual nature. The observations bear this out.

A second apparent difference in the adsorption of the two gases is shown in the hydrogen curves of Part I. of this paper. While the total amount of gas adsorbed was unchanged by the heat treatments from 400° C. to 600° C., it will be noted that the beginnings of the curves fall very rapidly, showing a speeding up of the action, a thing which did not take place in the case of nitrogen. The magnitude of the action at once dismisses the possibility of the iron tubes used having any appreciable part in this. It is suggested that this speeding up of the action might be due to an increased fineness of division of the material with which the hydrogen supposedly combines, brought about by the heating.

Further work of this sort will be begun shortly and experiments on nitrogen and hydrogen mixtures similar to those of Dr. Lemon and Miss Blodgett are already in progress by Dr. Lemon and Miss Hepburn.

The author is greatly indebted to Dr. Lemon for advice and valuable suggestions in connection with this work.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO.

THE EFFECT OF TEMPERATURE UPON THE TRANSMISSION OF INFRA-RED RADIATION THROUGH VARIOUS GLASSES.

BY GEORGE ROSENGARTEN.

SYNOPSIS.

Measurements of Infra-red Transmission.—The use of colored glasses as absorption screens for transmitting bands of monochromatic radiation depends upon the variation in the transmission of the radiation with change in temperature. The region investigated extends from 1μ to 5μ . The temperature was varied from 40 to 460 degrees centigrade.

Apparatus; Tungsten Lamp Source, Quartz Spectrometer, and Galvanometer.—The radiation from a tungsten lamp was passed through glasses placed in an electric heater. The transmission of radiation of various wave-lengths was observed by means of a spectrometer using a pair of large rock salt prisms which could be rotated through any desired angle and the radiation of varying wave-length made to fall on the slit of a thermo-pile. The intensity of radiation was determined by the deflection of a highly sensitive galvanometer.

Conclusions; Transmission Independent of Temperature.—The results appear to indicate that to within $8 \pm$ per cent. the variation in temperature produces no change in the transmission of infra-red radiation through the colored glasses tested.

THE transmission of infra-red radiation through various glasses has been investigated by Dr. W. W. Coblentz¹ and others. The effect of temperature upon the transmission of the radiation through colored glasses has been carried out by K. S. Gibson² for the visible portion of the spectrum.

The present investigation was undertaken for the purpose of measuring the change in the transmission for the radiations in the infra-red region, extending to about 5μ , as the temperature of the glass was raised. The glasses examined were obtained from the Corning Glass Works through the kindness of Dr. H. P. Gage. They are marked with the maker's numbers and are similar to those examined by Dr. W. W. Coblentz, the transmission curves of which are published in Sci. Paper No. 325 of the Bureau of Standards. The size of the glasses ranged from 3 to 5 mm. in thickness and about 25 mm. in diameter.

APPARATUS.

The spectrometer used in the investigation was built in the laboratory and is shown in Fig. 1. The main feature of the instrument is the use

¹ Bulletin 325, Bureau of Stds.

² PHYS. REV., Vol. VII., 1916.

of two large rock salt prisms 8 cm. by 11 cm. having a refracting angle of 35 degrees 15 minutes. The prisms are mounted upon a large table so as to make angles of 45 degrees with the silvered mirror M_3 . The prism table is caused to rotate about a vertical axis by means of an accurate screw which acts upon an arm connected with the table. The

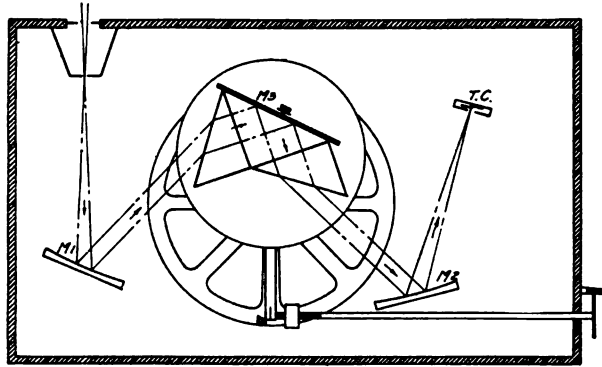


Fig. 1.

rotation is measured on a graduated head outside of the box enclosing the spectrometer. The radiation enters the spectrometer through a slit about $\frac{1}{2}$ mm. in width and falls upon a silvered mirror M_1 placed at a distance of 25 cm. from the slit. The focal length of this mirror being 25 cm., the rays leaving its surface are parallel and, falling upon the rock salt prism, suffer refraction. When the prism table is properly set, the rays enter the prism at minimum deviation and pass through the prism perpendicular to the second surface. At the surface of the mirror M_3 they are reflected and pass through the second prism emerging at an angle of 90 degrees with the direction they had when entering the

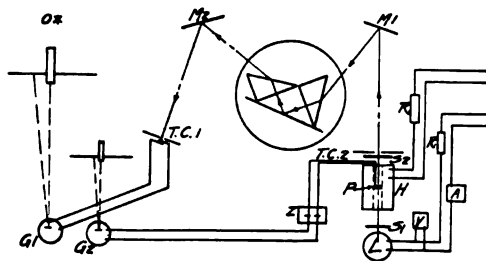


Fig. 2.

first prism. The rays are brought to a focus by another silvered mirror M_2 upon the slit of a thermopile placed at 25 cm. distance.

The slit, mirrors, spectrometer, and thermopile are all mounted upon

the same base and placed in a box which was kept closed during each set of observations. By turning the screw from the outside the table carrying the prism was turned through any desired angle, all other parts of the apparatus remaining fixed. At each position of the prisms a radiation of a different wave-length was brought to a focus on the thermopile *T.C. 1*. The thermopile consisted of 16 junction of bismuth and silver¹ mounted in back of a slit about $\frac{1}{2}$ mm. in width and connected with a highly sensitive d'Arsonval galvanometer manufactured by Leeds & Northrup Co. The sensitivity of this instrument is rated at 33 mm. per microvolt and appeared to remain constant during any set of observations. Readings were made by means of a telescope and scale at a distance of 2 meters.

The general arrangement of the apparatus is shown in Fig. 2. A constant potential of 105 volts was applied to a tungsten lamp which served as a source of radiation. The glass *P* to be examined was mounted in a vertical position on the end of a rod and was introduced into the cylindrical tube forming the heater (*H*). The glass was heated to different temperatures by varying the current passing through a coil of wire wrapped around the outside of this cylindrical tube. The heater was covered by a cylinder of fire clay. The temperature was determined by means of an iron-constantan thermo-couple *T.C. 2* introduced into the opening alongside of the glass. The cold ends of the couple were placed in an ice bath (*Z*) and connected by copper leads to a galvanometer *G 2*. The reading of this galvanometer was at all times made visible by reflecting a beam of light from the galvanometer mirror to a transparent scale. The apparatus was so arranged that all observations and adjustments could be made from the position *O*.

Calibration.—The spectrometer was calibrated by observing the position of certain well-known transmission and emission bands, *i.e.*, the maximum emission band of the bunsen flame at $4.4\ \mu$, the absorption band of a film of water at $3\ \mu$ and the position of the sodium line. Thermo-couple used in determining the temperature of the heater was calibrated by comparison with several high temperature thermometers placed in the heater and readings taken simultaneously upon the thermometer and the galvanometer deflection. From these data a curve was plotted and the temperature obtained from a reading on the transparent scale giving the deflection of the galvanometer.

MEASUREMENTS.

The method of mapping the spectrum was to cause different radiations to fall in succession upon the thermopile *T.C. 1* and observing the

¹ B. S. Sci. Paper No. 229.

deflection produced on the galvanometer *G* 1. With the asbestos shutter *S* 1 removed, the radiation from the lamp passed through the opening in the electric heater and was interrupted by the shutter *S* 2. The position of the spectrometer table was read on the graduated head of the operating screw. By pulling aside the shutter *S* 2 the radiation passed through the slit of the spectrometer, and its intensity was determined by the deflection of the galvanometer *G* 1. Several readings were made for each position of the prisms with variations of about $\frac{1}{2}$ mm. The prisms were rotated through a small angle and the new positions determined by a reading on the graduated head.

The spectrum of the source was first mapped for radiations extending from $1\ \mu$ to $5\ \mu$, the temperature of the heater being the same as that of the room. The glass specimen *P* was then introduced and the transmission at room temperature determined for each position in the spectrum. By sending a current through the electric heater the temperature of the glass was raised to any desired degree. When a steady state was reached the transmission was observed at this increased temperature.

The radiation of the heated parts of the apparatus was considerable and in order to determine the transmission of the radiation from the source *L* it was necessary to deduct the effect of the radiation of the heated parts. The shutter *S* 1 was introduced making it possible to cut off the radiation from the lamp. For each setting in the spectrum the deflections of the galvanometer were observed for the following conditions. First, the radiation from the lamp interrupted by the shutter *S* 1 and only the radiation from the heated parts of the apparatus allowed to enter the spectrometer. Second, by removing the shutter *S* 1 the radiations from the lamp and the heated parts of the apparatus allowed to enter the spectrometer at the same time. Third, same conditions as the first. Fourth, same conditions as the second. By subtracting the average of the deflections produced by the heated parts of the apparatus from the average of the deflections produced by the lamp and heated parts jointly it was possible to determine the intensity of the radiation from the lamp transmitted by the glass.

Measurements were made in this manner for such glasses as the blue-purple *G* 585, blue-green *G* 584, *G* 171 *IZ* and others for temperatures up to 500 degrees centigrade. The measurements have been repeated a number of times and the change in the percentage transmission with the increase in temperature for radiations from $1\ \mu$ to $5\ \mu$, if any exists, must have been of the same order of magnitude as the instrumental errors. Owing to the difficulties involved they were as large as $8 \pm$ per cent. The results of a typical set of observations are given in Table I.

TABLE I.
Transmission of the Blue-purple Glass G 585.

Scale.	Deflections for Temperature.					Approx. Wave-length.
	Source.	40 Deg.	260 Deg.	460 Deg.	Per Cent.	
6.80	13.5 mm.	3.0 mm.	3.2 mm.	2.8 mm.	- 6.7	5.6 μ
7.00	16.0	3.0	3.0	2.8	- 6.7	5.0
7.20	20.5	3.8	4.5	3.7	- 3.3	4.4
7.40	26.2	5.5	5.0	6.0	+ 9.1	4.0
7.60	32.8	7.0	7.5	8.0	+14.3	3.6
7.80	36.5	8.2	8.5	9.2	+12.2	3.2
8.00	41.0	11.0	11.00	11.2	+ 1.8	2.8
8.20	50.0	17.5	17.0	18.2	+ 4.0	2.6
8.40	61.5	22.5	22.5	21.8	- 3.1	2.4
8.60	62.0	17.0	16.8	17.5	+ 2.9	2.2
8.80	56.5	11.0	11.7	11.5	+ 4.5	2.0
9.00	48.0	8.0	9.0	8.8	+10.0	1.8
9.20	35.2	7.2	7.5	7.8	+ 8.3	1.6
9.40	32.5	6.5	7.5	8.2	+26.0	1.4
9.60	25.8	7.8	8.0	7.7	- 1.3	1.3
9.80	21.0	7.5	7.6	7.0	- 6.6	1.2
10.00	18.5	7.2	7.0	7.5	+ 4.2	1.1

The first column indicates the reading on the graduated head of the operating screw, the second column is the deflection obtained for the radiation from the source *L*, the third column gives the deflection of the galvanometer produced by the transmission of the radiation through the glass at the temperature of 40 degrees centigrade, the fourth and fifth columns give the deflection at 260 and 460 degrees respectively, the sixth column gives the percentage change between 40 and 460 degrees, the seventh column gives the approximate wave-length as obtained from the calibration curve.

Observations were also made upon various specimens for a single wave-length. The spectrometer was set in position and the transmission observed for various temperatures. In this manner no part of the apparatus was moved after the observations were started. The transmission obtained in this manner may be seen in Table II.

Summary.—The results of the above experiment seem to show that to within $8 \pm$ per cent. there is no change in the transmission of the infra-red radiation through the glasses observed for the region 1μ to 5μ as indicated in Table I. The results obtained in Table II. indicate the degree of accuracy of the galvanometer readings under similar conditions. The change in the transmission for observations at this wave-length is much smaller.

The present investigation was suggested by Dr. W. W. Coblentz who

TABLE II.

Transmission of Blue-green glass G 584, Approx. Wave-Length 2.6μ .

Temp.	Deflections.				
	Heater.	Total.	Lamp.	Mean.	% Change.
40° C.....	0	29.0	29.0	29.8	
	0	30.0	30.0		
	0	30.0	30.0		
	0	30.0	30.0		
	0	30.0	30.0		
135.....	0	30.0	30.0	30.5	+2.3
	0	31.0	31.0		
	0	30.5	30.5		
	0	31.0	31.0		
	0	30.0	30.0		
190.....	0	30.0	30.0	30.3	+1.7
	0	30.0	30.0		
	0	30.0	30.0		
	0	30.5	30.5		
	0	31.0	31.0		
290.....	2.5	33.0	30.5	30.2	+1.3
	3.0	33.5	30.5		
	3.0	33.0	30.0		
	3.5	34.0	30.5		
	4.0	33.5	29.5		
360.....	9.0	39.0	30.0	29.5	-1.0
	9.5	39.0	29.5		
	8.5	38.0	29.5		
	8.5	37.0	28.5		
	8.5	37.0	28.5		
	7.0	37.0	30.0		
480.....	29.0	57.0	28.0	29.3	-1.7
	29.0	57.0	28.0		
	28.0	59.0	31.0		
	29.0	58.5	29.5		
	29.0	59.0	30.0		

has shown continued interest in the problem. The apparatus was placed at my disposal and set up in the Randal Morgan Laboratory of Physics at the University of Pennsylvania. Suggestions have been made during the progress of the work by Dr. Goodspeed and Dr. Richards for which I wish to express my appreciation.

THE THEORY OF LINEAR-SINOIDAL OSCILLATIONS.

BY HENRY G. CORDES.

SYNOPSIS.

Constant frequency arc discharges; theory assuming direct current supply and arc potential drop to remain constant.—A vacuum arc is shunted with inductance, capacitance and resistance in series which constitutes a discharge circuit. The arc is supplied with direct current through a large inductance. The arc vapor is ionized and de-ionized at a constant frequency. The variations of potential and current in the discharge circuit are termed *linear-sinoidal oscillations*. Mathematical expressions are derived for the following quantities in terms of the *priming angle* and *damping angle*, viz., the logarithmic decrement, ratio of natural frequency to discharge frequency, *impedance factor* and ratio of the effective discharge circuit current to the direct current. These quantities are expressed in terms of transcendental functions of the angles, hence relations between them are determined by means of a chart and a table. Expressions for potentials at the instants of priming (critical ionization) and unpriming are given. The relation between the values of the direct current and effective currents through the arc and in the discharge circuit is established. The actual fluctuation of the direct current supply is approximately determined. The counter E.M.F. of an oscillator having a coupled secondary oscillatory circuit varies in a manner similar to the counter E.M.F. of a motor.

De-ionization of arc vapor; experimental determination of the rate.—An arc is extinguished by means of initiating a series of linear-sinoidal oscillations of decreasing frequency. The values of the priming angle, ratio of frequencies, discharge frequency, unprimed interval, re-ignition potential and other quantities are approximately determined for the initial oscillations. An expression for the numerical value of the rate of de-ionization is given.

Stability of an arc oscillator.—The values of series resistance to produce stability depends upon discharge circuit resistance, rate of de-ionization and rate of change of the rate of de-ionization. Auxiliary means to produce a constant discharge frequency are required to stabilize an efficient arc oscillator for *radio signalling*.

A PARTICULAR type of oscillation exists, the theory of which has been little developed although it forms the basis for direct current impact sustention of oscillating current. No general term has been applied to an oscillation which consists of two partial sinoidal oscillations produced by a direct current and a sinoidal current. During the first partial oscillation the potential of the capacitance varies practically as a linear function of the time and during the second partial oscillation the potential varies as a sinoidal function of the time, hence the term "linear-sinoidal" will be used to distinguish this particular type of oscillation.

The theory developed in the following pages cannot be completely

realized or verified by experiment but it is nevertheless useful as the basis for explaining actual phenomena. As an analogous instance the law of falling bodies in a vacuum may be mentioned. This law applied to an actual body falling in a vacuum is of little value, but it is a very important factor in investigating any phenomena involving the law of gravitation.

The Carnot's cycle of a heat engine is another analogous instance. No heat engine can be constructed without friction and heat radiation losses, yet the theory of this cycle forms an important factor in the study of the heat engine. The theory of linear-sinoidal oscillations will therefore be presented for consideration as an important factor in explaining actual phenomena. An experiment will be described and the results interpreted by means of the theory developed.

Figure 1 shows an arrangement of circuits which, with certain assumptions, forms the basis for the principal theoretical deductions.

In Fig. 1 the condenser C is discharged at a constant frequency N by

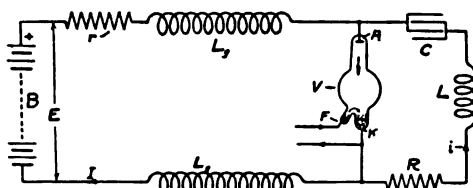


Fig. 1.

ionizing the path from the anode A to the cathode K in a vacuum tube V which contains a gas at low pressure.

The path from A to K is initially ionized at the frequency N by a spark passing from the auxiliary anode F to the cathode K . The vacuum tube V is an electrical check valve which will not allow current to pass from K to A although an E.M.F. tending to produce an inverse current is impressed upon these electrodes. The potential drop through V will be considered a constant and its value will be represented by E_v . The valve V is therefore assumed to be a device which discharges C at a constant frequency, has a constant potential drop for any value of current greater than zero and acts as an electrical check valve.

The arrangement of Fig. 1 consists of three circuits which will be called the charging circuit, the discharge circuit and the arc circuit. The charging circuit consists of a source B of direct current, a line resistance r , a line inductance, L_0 , a capacitance C , an inductance L , and a resistance R . The inductance L_0 is large compared to L . The discharge circuit is an oscillatory circuit having a capacitance C , a discharger V , an

inductance L , and resistance R . The arc circuit is the direct current circuit $B - r - L_g - V$ short-circuited momentarily by the arc V . The period T of a complete linear-sinoidal oscillation consists of two time intervals. During the first interval T_1 , the arc and discharge circuits are closed and during the second interval T_2 current flows only in the charging circuit. The first interval will be referred to as the primed period and the second as the unprimed period.

Figure 2 shows the variations of potential and current in the discharge circuit during a complete cycle of a linear-sinoidal oscillation. The amplitude of the potential of C and of the current through L and R

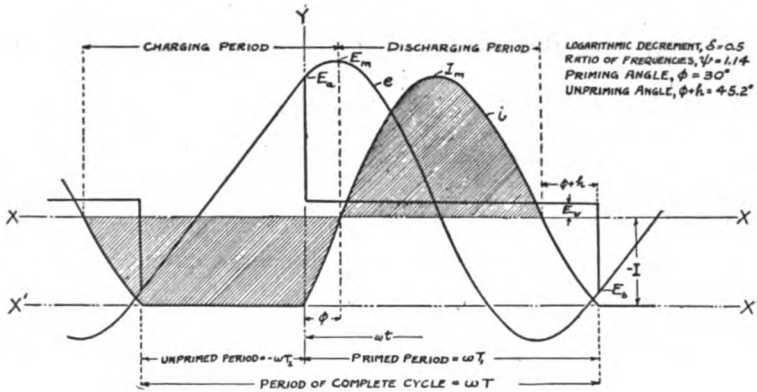


Fig. 2.

are measured by distances from the X -axis and time is plotted proportional to the distance from the Y -axis. The current through V is measured by distance from the X' -axis and is the resultant of the currents in the arc and discharge circuits. Curve e represents the potential of C and curve i represents the current through L and R . A direct current I flows into C until the potential of $C = E_a$, at which instant the valve V becomes conductive and the potential between the electrodes of the valve drops instantaneously to the value E_v . The quantity of electricity flowing into C must equal the quantity flowing out of C therefore the shaded area above the X -axis which represents the quantity flowing during the discharging period must equal the shaded area below the X -axis which represents the quantity flowing during the charging period.

It is assumed that L_g is infinitely large so that the direct current I will be constant during a complete cycle. When the amplitude of the negative current in the discharge circuit equals I the current through V becomes zero. It is assumed that when the current through V becomes zero that V becomes instantly and automatically a non-conductor.

MATHEMATICAL ANALYSIS.

During the period T_1 when the energy in the discharge circuit is free to oscillate the following potential relations exist,

$$L \frac{di}{dt} + Ri + \int \frac{idt}{C} + E_v = 0. \quad (1)$$

Let

$$\alpha = \frac{R}{2L}. \quad (2)$$

Let

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 2\pi f, \quad (3)$$

where f = the natural frequency of the discharge circuit. The solution of (1) is

$$i = I_0 e^{-\alpha t} \sin(\omega t - \phi) \quad (4)$$

where I_0 is the maximum value of i when R is considered equal to zero. The angle ϕ is shown in Fig. 2.

At the instant $t = 0$ when the discharge circuit becomes free to oscillate a negative current I flows in the discharge circuit. Substituting these values in (4),

$$I = I_0 \sin \phi. \quad (5)$$

At the instant $t = T_1$ the discharge circuit ceases its free sinoidal oscillation while a negative current I is flowing. Again substituting in (4),

$$-I = I_0 e^{-\alpha T_1} \sin(\omega T_1 - \phi). \quad (6)$$

Equation (6) can be written in a more convenient form by letting

$$\omega T_1 = 2\phi + h + \pi, \quad (7)$$

which substituted in (6) gives

$$I = I_0 e^{-\alpha T_1} \sin(\phi + h). \quad (8)$$

Equating potentials in the discharge circuit when $t = 0$ gives

$$\left[L \frac{di}{dt} \right]^{t=0} + [Ri]^{t=0} - E_a + E_v = 0. \quad (9)$$

From (2), (5) and (9)

$$E_a = \omega LI \cot \phi - \alpha LI + E_v. \quad (10)$$

Equating potentials in the discharge circuit when $t = T_1$ gives

$$\left[L \frac{di}{dt} \right]^{t=T_1} + [Ri]^{t=T_1} + E_b + E_v = 0. \quad (11)$$

From (2), (7), (8) and (11),

$$E_b = \omega LI \cot(\phi + h) + \alpha LI - E_v. \quad (12)$$

Equating the quantities of electricity flowing during the primed and unprimed periods will give the equations

$$\int_0^{\pi} i dt = (E_a + E_b)C \quad (13)$$

and

$$(E_a + E_b)C = IT_2. \quad (14)$$

Adding (10) and (12),

$$E_a + E_b = \omega LI [\cot \phi + \cot (\phi + h)]. \quad (15)$$

From (14) and (15)

$$T_2 = \frac{\cot \phi + \cot (\phi + h)}{\omega} \cdot \frac{I}{I + (\delta/2\pi)^2}, \quad (16)$$

where the logarithmic decrement

$$\delta = \frac{2\pi\alpha}{\omega}. \quad (17)$$

From (7) and (16),

$$T = T_1 + T_2 = \frac{\cot \phi + \cot (\phi + h)}{\omega} \cdot \frac{I}{I + (\delta/2\pi)^2} + \frac{2\phi + h + \pi}{\omega}. \quad (18)$$

Let

$$\psi = \frac{f}{N} = \frac{\omega T}{2\pi}, \quad (19)$$

where N = the discharge frequency, or frequency of the linear-sinoidal oscillations. Then,

$$\psi = \frac{\cot \phi + \cot (\phi + h)}{2\pi} \cdot \frac{I}{I + (\delta/2\pi)^2} + \frac{2\phi + h + \pi}{2\pi}. \quad (20)$$

In order to evaluate the angles ϕ and h the quantities ψ and δ must be known. The value of δ can be expressed in terms of ϕ and h .

From (5) and (8),

$$e^{-\alpha\pi} = \frac{\sin \phi}{\sin (\phi + h)}. \quad (21)$$

From (7) and (17),

$$\alpha T_1 = \left(\frac{2\phi + h + \pi}{2\pi} \right) \delta. \quad (22)$$

From (21) and (22),

$$\delta = \frac{\log \sin (\phi + h) - \log \sin \phi}{\frac{2\phi + h}{2\pi} + \frac{1}{2}}. \quad (23)$$

The values of ϕ and h can be determined from the values of ψ and δ by means of a chart. Figs. 3 and 4 are charts in which the ordinates represent the values of ϕ and the abscissas represent the values of h .

A number of δ -curves are plotted. Values were assigned to ϕ and h in (23) and a series of values of δ were calculated. This series was used to obtain, by interpolation, values of ϕ and h for given values of δ which are 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.16, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.12, 1.16, 2.0 and 3.0.

A similar series of curves are plotted for values of ψ calculated from equation (20). Curves representing values of ψ for 1.001, 1.003, 1.005, 1.01, 1.02, 1.03, 1.05, 1.07, 1.10, 1.14, 1.2, 1.3, 1.5 and 2.0 are shown.

Figures 3 and 4 will be referred to as the $\phi \cdot h$ chart. Every point

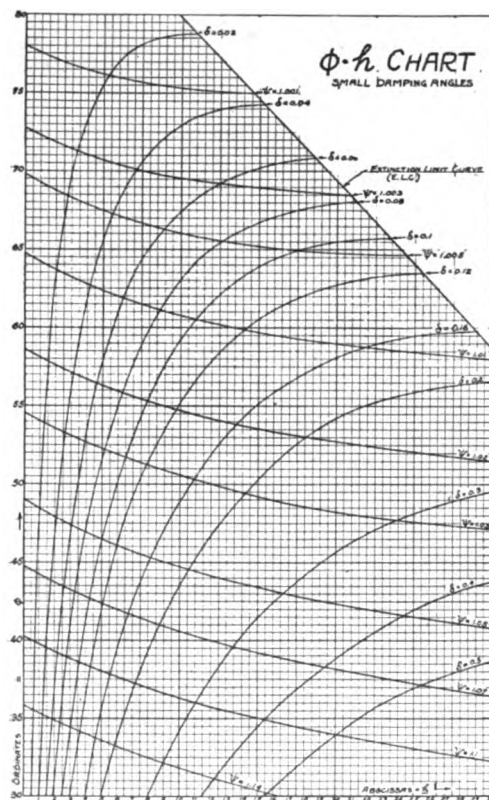


Fig. 3.

on this chart represents one value of ϕ , h , δ and ψ . Any two of these quantities will determine a point on the chart therefore when δ and ψ are known the corresponding values of ϕ and h can be found. The quantities L , C , R and N determine the values of ϕ and h .

The variation of h as a function of the damping of the discharge circuit current is illustrated in Fig. 5. The current of equation (4) is represented

by means of polar coördinates. The curve representing the current approaches a circle as the damping approaches zero. The outer spiral represents the current in a discharge circuit whose logarithmic decrement = 0.4 and the inner spiral is a current whose decrement = 0.83. The

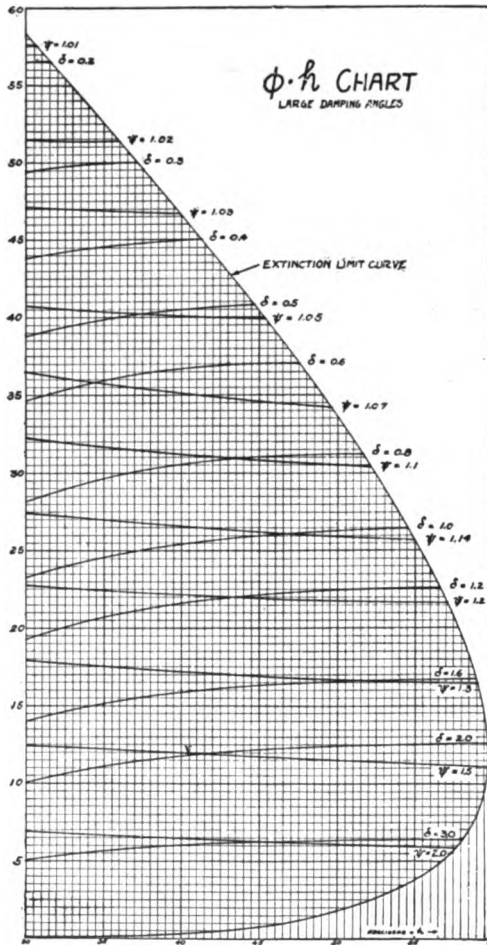


Fig. 4.

line $X - X$ is parallel to the polar axis and its distance from the polar axis represents the amplitude of the direct current. The priming angle ϕ is the angle between the polar axis and a line drawn from the pole to the intersection of the line $X - X$ and the circle. The damping angle h is the angle between the lines drawn from the pole to the intersection of the line $X - X$ with the circle and with the spiral. The unprimed period cannot conveniently be illustrated by means of polar coördinates.

The polar axis and line $X - X$ must be rotated positively about the pole through an angle of $\omega T - 2\pi$ radians between the instants of extinction and re-ignition to represent the unprimed period.

The largest value that δ and h can have for a given value of ϕ is deter-

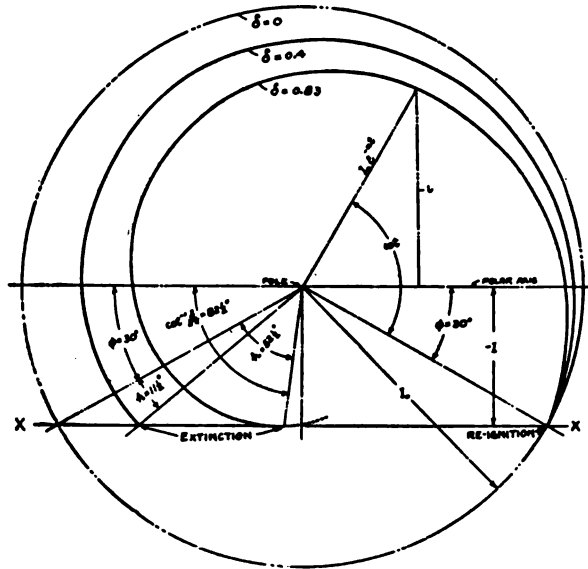


Fig. 5.

mined by the maximum negative amplitude of i in equation (4). This occurs when

$$\delta_1 = 2\pi \cot(\phi_1 + h). \quad (24)$$

If δ is greater than this value the valve V will not unprime and the capacitance C cannot be charged. The valve V will then practically form a permanent short-circuit on the line.

The extinction limit curve on the $\phi \cdot h$ chart is the locus of points obtained by substituting equation (24) in (23). Any one of the three variables may be eliminated. Table I. gives values of these variables from which the extinction limit curve was drawn. The angle $\phi + h$ was eliminated, values were assigned to δ_1 and values of ϕ_1 were easily found by interpolation.

Given the quantities L, C, R, N, I and E_s , the value of E_m and E_a can be determined with the aid of the $\phi \cdot h$ chart from equations (10) and (12).

The maximum potential E_m impressed upon the capacitance C can also be calculated by the given quantities by the relation expressed by

TABLE I.

δ_1	$\phi_1 + h$	ϕ_1	δ_1	$\phi_1 + h$	ϕ_1	δ_1	$\phi_1 + h$	ϕ_1
0.01	89° 55'	82° 1'	0.7	83° 38'	33° 52'	2.4	69° 6'	9° 35'
0.02	89° 49'	78° 46'	0.8	82° 45'	31° 5'	2.5	68° 18'	8° 57'
0.03	89° 44'	76° 17'	0.9	81° 51'	28° 37'	2.6	67° 31'	8° 22'
0.04	89° 38'	74° 16'	1.0	80° 57'	26° 24'	2.7	66° 45'	7° 50'
0.05	89° 33'	72° 29'	1.1	80° 4'	24° 24'	2.8	65° 59'	7° 19'
0.06	89° 27'	70° 53'	1.2	79° 11'	22° 35'	2.9	65° 13'	6° 51'
0.07	89° 22'	69° 26'	1.3	78° 19'	20° 56'	3.0	64° 29'	6° 24'
0.08	89° 16'	68° 5'	1.4	77° 26'	19° 25'	3.1	63° 44'	6° 0'
0.09	89° 11'	66° 50'	1.5	76° 34'	18° 3'	3.2	63° 1'	5° 37'
0.10	89° 5'	65° 42'	1.6	75° 43'	16° 47'	3.3	62° 17'	5° 15'
0.12	88° 55'	63° 26'	1.7	74° 52'	15° 37'	3.4	61° 35'	4° 55'
0.16	88° 33'	59° 46'	1.8	74° 1'	14° 32'	3.5	60° 53'	4° 36'
0.20	88° 11'	56° 34'	1.9	73° 11'	13° 33'	3.7	59° 24'	4° 5'
0.30	87° 16'	50° 4'	2.0	72° 21'	12° 38'	4.0	57° 27'	3° 22'
0.40	86° 21'	44° 56'	2.1	71° 31'	11° 47'	5.0	51° 29'	1° 45'
0.50	85° 27'	40° 40'	2.2	70° 42'	11° 0'	7.0	41° 55'	0° 30'
0.60	84° 34'	37° 2'	2.3	69° 54'	10° 16'	10.0	32° 8'	0° 5'

equation (1) when $t = \phi/\omega$ which is

$$\left[L \frac{di}{dt} \right]^{t=\frac{\phi}{\omega}} + [Ri]^{t=\frac{\phi}{\omega}} - E_m + E_v = 0. \quad (25)$$

Solving (25) for E_m gives

$$E_m = \frac{\omega LI}{\sin \phi} e^{-\frac{\phi}{2\pi}} + E_v, \quad (26)$$

which shows that for given values of L , C , I and E_v the value of E_m varies practically inversely as $\sin \phi$ which depends upon the discharge frequency N .

When RI and E_v are small compared to E_m and E_a the ratio of E_a to E_m varies practically as $\cos \phi$ which shows that E_m becomes very large compared to E_a as ϕ approaches $\pi/2$.

The maximum value of current i in equation (4) is

$$I_m = \frac{I}{\sin \phi} e^{-\frac{\delta}{2\pi} (\cot^{-1} \frac{\delta}{2\pi} + \phi)} \sin \left(\cot^{-1} \frac{\delta}{2\pi} \right). \quad (27)$$

DIRECT CURRENT IMPEDANCE.

The average counter E.M.F. of the capacitance C during the interval of time T_2 is represented by the ordinate of the middle point of the straight line connecting the points E_b and E_a in Fig. 2. From (10), (12) and (17),

$$\frac{E_a - E_b}{2} = \frac{\omega LI}{2} \left[\cot \phi - \cot (\phi + h) - \frac{\delta}{\pi} \right] + E_v. \quad (28)$$

The potential drop due to current I flowing through resistance R during T_2 is

$$IR = 2\alpha LI = \omega LI \frac{\delta}{\pi}. \quad (29)$$

The total average E.M.F. consumed by the oscillator during T_2 is

$$\frac{E_a - E_b}{2} + IR = \frac{\omega LI}{2} \left[\cot \phi - \cot (\phi + h) + \frac{\delta}{\pi} \right] + E_v. \quad (30)$$

The E.M.F. consumed by the oscillator during T_1 is E_v . The E.M.F. impressed upon the terminals A and K of the valve V is equal to the average E.M.F. consumed by the linear-sinoidal oscillation, or,

$$E - Ir = \left\{ \left[\frac{E_a - E_b}{2} + IR \right] T_2 + E_v T_1 \right\} \frac{1}{T}. \quad (31)$$

From (30) and (31)

$$E - Ir = \frac{\omega LI}{2} \left[\cot \phi - \cot (\phi + h) + \frac{\delta}{\pi} \right] \frac{T_2}{T} + E_v. \quad (32)$$

From (16) and (32)

$$\begin{aligned} & \frac{E - E_v - Ir}{\omega LI} \\ &= \left[\cot \phi - \cot (\phi + h) + \frac{\delta}{\pi} \right] \frac{\cot \phi + \cot (\phi + h)}{4\pi\psi} \cdot \frac{1}{1 + (\delta/2\pi)^2}. \end{aligned} \quad (33)$$

Let

$$\beta = \frac{E - E_v - Ir}{\omega LI}. \quad (34)$$

Then

$$\beta = \left[\cot \phi - \cot (\phi + h) + \frac{\delta}{\pi} \right] \frac{\cot \phi + \cot (\phi + h)}{4\pi\psi} \cdot \frac{1}{1 + (\delta/2\pi)^2}. \quad (35)$$

Equation (34) may be written

$$\frac{E - E_v}{I} = \beta \omega L + r. \quad (36)$$

The term $\beta \omega L$ will be referred to as "impedance" since it impedes the flow of direct current to the oscillator. The impedance depends upon the quantities L , C , R and the discharge frequency. For a given value of L , C and R the impedance increases as N decreases. For a given value of L , C and N the impedance will increase as R is increased. Decreasing either L or C while the other three quantities remain constant will increase the impedance.

During each cycle the energy delivered to the terminals of the valve

equals the energy stored or dissipated in the valve and discharge circuit, or,

$$(E - Ir)IT + \frac{1}{2}CE_0^2 = \frac{1}{2}CE_a^2 + I^2RT_2 + IE_aT_1. \quad (37)$$

The relation expressed by (37) can also be derived from (14) and (31). The discharge circuit capacitance C returns energy to the charging circuit from the instant the arc is extinguished until the potential of C becomes zero.

EFFECTIVE CURRENT.

The effective current of a linear-sinoidal oscillation can be expressed by two equations. The first equates the energy delivered to the linear-sinoidal oscillations and the energy dissipated by the oscillations, which is

$$(E - Ir - E_a)I = I_a^2R, \quad (38)$$

where I_a is the effective linear-sinoidal current which may be measured with a hot-wire ammeter placed in series with the discharge circuit capacitance.

The second equation expresses the fact that the square of the effective current is equal to the average value of the square of the instantaneous current during a complete cycle, which is

$$I_a^2 = [I^2T_2 + \int_0^{T_1} i^2 dt] \frac{1}{T}, \quad (39)$$

$$= I^2 \frac{T_2}{T} - \frac{I_0^2}{4\alpha T} [\epsilon^{-2\alpha t}]_0^{T_1} - \frac{I_0^2}{4T} \left[\epsilon^{-2\alpha t} \frac{\omega \sin 2(\omega t - \phi) - \alpha \cos 2(\omega t - \phi)}{\alpha^2 + \omega^2} \right]_0^{T_1}. \quad (40)$$

From (16)

$$\frac{I^2T_2}{T} = \frac{I^2}{T} \left[\frac{\cot \phi + \cot (\phi + h)}{\omega} \cdot \frac{1}{1 + (\delta/2\pi)^2} \right]. \quad (41)$$

From (5), (7) and (21)

$$\frac{I_0^2}{4\alpha T} [\epsilon^{-2\alpha t}]_0^{T_1} = \frac{I^2}{4\alpha T} [\cot^2 (\phi + h) - \cot^2 \phi]. \quad (42)$$

From (5), (7) and (21)

$$\begin{aligned} & \frac{I_0^2}{4T} \left[\epsilon^{-2\alpha T_1} \frac{\omega \sin 2(\omega T_1 - \phi) - \alpha \cos 2(\omega T_1 - \phi)}{\alpha^2 + \omega^2} \right] \\ &= \frac{I^2}{4T(\alpha^2 + \omega^2)} [2\omega \cot (\phi + h) - \alpha \cot^2 (\phi + h) + \alpha]. \end{aligned} \quad (43)$$

From (5),

$$\frac{I_0^2 \omega \sin(-2\phi) - \alpha \cos(-2\phi)}{4T(\alpha^2 + \omega^2)} = \frac{I^2}{4T(\alpha^2 + \omega^2)} [\alpha - 2\omega \cot \phi - \alpha \cot^2 \phi]. \quad (44)$$

Substitute (41), (42), (43) and (44) in (40) and let

$$\gamma = \frac{I_a}{I}, \quad (45)$$

then the result reduces to

$$\gamma^2 = \left[\frac{\cot \phi - \cot(\phi + h)}{\delta} + \frac{1}{\pi} \right] \frac{\cot \phi + \cot(\phi + h)}{4\psi} \cdot \frac{1}{1 + (\delta/2\pi)^2}. \quad (46)$$

A very interesting and important relation exists between γ and β . From (35) and (46)

$$\gamma^2 = \frac{\pi}{\delta} \cdot \beta. \quad (47)$$

A simpler and easier method of deriving equation (46) is as follows: Equation (38) may be written

$$\frac{E - E_v}{I} = \gamma^2 R + r. \quad (48)$$

Then (47) can easily be derived from (36) and (48). Substitute (35) in (47) and the result will be equation (46).

Each of the quantities ψ , δ , β and γ have been expressed as functions of the angles ϕ and h . Any two of these six quantities will theoretically determine the remaining four. The angles ϕ and h are useful only in plotting curves and in establishing relations among the other four quantities. A complete $\phi \cdot h$ chart should also have β -curves and γ -curves plotted upon it.

The γ -curves have been found to be practically parallel to the ψ -curves, that is, the members of equation (47) remain nearly constant for a given value of ψ when R is varied. The consequence of this relationship is that ϕ , h , δ and β cannot be determined from values of ψ and γ but that practically only one value of γ exists for a given value of ψ over a large range of discharge circuit decrements. Hence the natural frequency f can be determined from values of the direct current I , the effective current I_a and the frequency N of the oscillations.

Table II. ($\psi \cdot \gamma$ table) shows values of ψ and the corresponding values of γ . For each value of ψ two values of γ are given; one value, γ_0 , is given for zero damping and another value, γ_1 , for maximum damping, i.e., at the intersection of the ψ -curve with the extinction limit curve.

TABLE II.

 $\psi \cdot \gamma$ Table.

ψ .	γ_0 .	γ_1 .	ψ .	γ_0 .	γ_1 .
1.000	0.71	0.71	1.14	1.04	1.01
1.001	0.72	0.72	1.2	1.13	1.10
1.003	0.74	0.73	1.3	1.25	1.21
1.005	0.75	0.74	1.4	1.35	1.32
1.01	0.77	0.76	1.5	1.44	1.42
1.02	0.81	0.79	1.6	1.53	1.51
1.03	0.84	0.82	1.7	1.61	1.60
1.05	0.88	0.86	1.8	1.69	1.68
1.07	0.93	0.90	1.9	1.77	1.76
1.10	0.98	0.95	2.0	1.84	1.83

When $h = 0$ in equation (46) the value of the resulting indeterminate expression is

$$\gamma_0^2 = \frac{1}{2\psi_0} \left[\left(\frac{\phi}{\pi} + \frac{1}{2} \right) \csc^2 \phi + \frac{\cot \phi}{\pi} \right], \quad (49)$$

which value γ approaches as δ approaches zero. As ψ approaches unity γ^2 approaches the value 0.5 which value is obtained by substituting $\pi/2$ for ϕ in (49). The impedance $\gamma^2 R$ varies directly as R provided the discharge frequency is constant. For values of γ less than unity an ohm of resistance in the discharge circuit will impede the flow of direct current less than an ohm in the direct current circuit while for values of γ greater than unity resistance is more effective in the discharge circuit. The value of ψ corresponding to $\gamma = 1$ is about 1.12.

The simple relation expressed by equation (47) and the fact that the γ -curves are nearly parallel to the ψ -curves render β -curves and γ -curves unnecessary upon the $\phi \cdot h$ chart.

The average current through the valve V is I and the effective current through V can be calculated from the values of I and I_a . Let I_b = the effective current through V , then

$$I_b^2 = \frac{1}{T} \int_0^{T_1} (i + I)^2 dt. \quad (50)$$

From (13) and (14)

$$\int_0^{T_1} i dt = IT_1. \quad (51)$$

Substituting (39) and (51) in (50) gives

$$I_b^2 = I_a^2 + I^2. \quad (52)$$

VALVE POTENTIALS.

The maximum potential impressed upon the valve V is at the instant of re-ignition, *i.e.*, at the beginning of the period T_1 . Let E_1 represent the value of this potential, then

$$E_1 = E_a + IR. \quad (53)$$

From (10) and (53)

$$E_1 = \omega LI \cot \phi + \alpha LI + E_v. \quad (54)$$

Let E_2 = the maximum potential tending to produce inverse current through the valve, then

$$E_2 = E_b - IR. \quad (55)$$

From (12) and (55)

$$E_2 = \omega LI \cot (\phi + h) - \alpha LI - E_v. \quad (56)$$

Equations (10), (12), (54) and (56) may be written in the form

$$E_a = \omega L \left[\cot \phi - \frac{\delta}{2\pi} \right] I + E_v, \quad (57)$$

$$E_b = \omega L \left[\cot (\phi + h) + \frac{\delta}{2\pi} \right] I - E_v, \quad (58)$$

$$E_1 = \omega L \left[\cot \phi + \frac{\delta}{2\pi} \right] I + E_v, \quad (59)$$

$$E_2 = \omega L \left[\cot (\phi + h) - \frac{\delta}{2\pi} \right] I - E_v. \quad (60)$$

The coefficients of I in these equations depend upon L , C , R and N . Assuming E_v comparatively small, these equations show that the potentials E_a , E_b , E_1 and E_2 vary practically directly as the direct current I . The values of these potentials can be found when any two of the six quantities ψ , δ , β , γ , ϕ and h are known except ψ and γ . Equations (57), (58), (59) and (60) considered in connection with equation (36) or (48) show that these potentials vary practically directly as the average potential E provided L , C , R and N remain constant and E_v is comparatively small.

DIRECT CURRENT FLUCTUATION.

The theory of linear-sinoidal oscillations is based upon the assumption that the line inductance L_o is infinitely large which is equivalent to assuming that the period $2\pi\sqrt{L_o C}$ is infinitely long. The charging circuit is an oscillatory circuit in which a partial sinoidal oscillation takes place during the unprimed period T_2 of each oscillation. The current flowing through B , r and L_o is the resultant of a direct current of constant amplitude and a linear-sinoidal oscillating current of comparatively small amplitude.

Figure 6 shows a curve representing the oscillating component of the current. This component is represented by a nearly straight line during the time interval T_1 and a sinoidal curve during the interval T_2 . During the primed period T_1 the rate of change of the current in the arc circuit is practically a constant provided r is small. During the unprimed interval the rate of change of current depends largely upon the potential of the capacitance C . A suggestive name for the oscillating

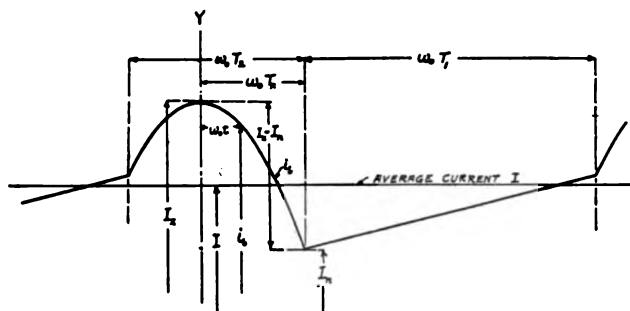


Fig. 6.

component is "sinoidal-linear" oscillation since the periods of linear and sinoidal variations of current in L_0 and L are interchanged.

Let $L_0 = L_0 + L$ = the inductance of the charging circuit. Let $r_0 = R + r$ = the resistance and α_1 = the damping factor. Let $\omega_0/2\pi$ = the natural frequency of the circuit.

In Fig. 6 let the current

$$i_0 = I_x e^{-\alpha_1 t} \cos \omega_0 t, \quad (61)$$

where I_x is the maximum amplitude of the current in the charging circuit. Let T_s = the time between maximum value I_x and the minimum value I_n of the current, i_0 , then

$$I_n = I_x e^{-\alpha_1 T_s} \cos \omega_0 T_s. \quad (62)$$

The equation of potentials in the charging circuit when $t = T_s$ is

$$E - \left[L_0 \frac{di_0}{dt} \right]^{t=T_s} - I_n r_0 - E_a = 0. \quad (63)$$

From (62) and (63)

$$E + \omega_0 L_0 \sqrt{I_x^2 e^{-2\alpha_1 T_s} - I_n^2} - \frac{I_n r_0}{2} - E_a = 0. \quad (64)$$

For large L_0 and small r , $e^{-2\alpha_1 T_s} = 1$ and $I_x + I_n = 2I$ approximately. Let ΔI represent the current fluctuation.

Then $\Delta I = I_x - I_n$ which is negligible when compared to I . Under

these conditions equation (64) becomes

$$E + \omega_0 L_c \sqrt{2I \cdot \Delta I} - \frac{Ir_c}{2} - E_a = 0 \quad (65)$$

or

$$\Delta I = \left[E_a - E + \frac{Ir_c}{2} \right]^2 \cdot \frac{C}{2IL_c}. \quad (66)$$

The current curve of Fig. 6 is similar to the potential curve of Fig. 2 while the potential variations between the terminals of L_c are similar to those between A and K of V shown in Fig. 2. The ordinates of Fig. 6 are drawn to a much larger scale than the ordinates of Fig. 2.

EXPERIMENTS.

A few experiments were made with available apparatus at the University of Washington by Mr. T. M. Libby to experimentally verify

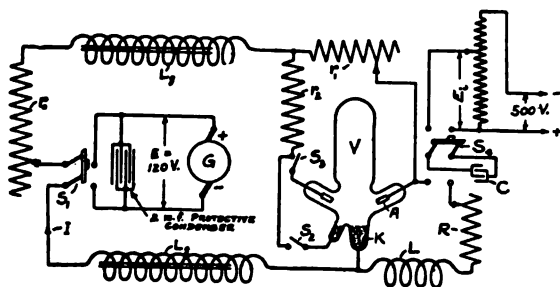


Fig. 7.

phenomena predicted from the theory of linear-sinoidal oscillations and to determine the rate of de-ionization of mercury arc vapor.

Figure 7 is a diagram of connections of the experiments. A ten-ampere arc rectifier was used for a discharge valve V . Current was started from the anodes to the cathode by means of switch S_2 and tilting V . The mercury vapor in V was raised to the proper temperature by passing about five amperes from each anode to the cathode for several minutes. The switch S_3 was then also opened. The line inductance L_1 consisted of the secondaries of two 220-2200 v. - 10 K.W. iron-core transformers. The discharge circuit condenser C was charged to a potential E_i and was then discharged through R and L by means of switch S_4 . This discharge of C extinguished the arc provided R was small enough, and the potential E_i was sufficiently high.

The extinction of the arc is explained in terms of the theory as follows. The discharge of C unprimes the valve for an interval T_2 . The length of T_2 depends upon I and the potential e_1 between anode A and cathode K

required to re-ignite the arc, that is, e_1 initiates a primed interval T_1 . The arc is therefore extinguished by initiating a train of linear-sinoidal oscillations of decreasing frequency. The length of the train depends upon the electromagnetic energy stored in the line inductance L_g .

In Fig. 7 let $r_0 + r_1 = r$, then

$$E - E_v = Ir \quad (67)$$

when a constant direct current I is flowing.

By unpriming the valve with the discharge of C a direct current impedance $\gamma^2 R$ is introduced into the d.-c. circuit so that for an instant equation (67) becomes

$$-L_g \frac{dI}{dt} + E - E_v = \gamma^2 RI + rI. \quad (68)$$

From (67) and (68)

$$-\frac{dI}{dt} = \frac{\gamma^2 RI}{L_g}. \quad (69)$$

Equation (69) shows the rate at which the direct current starts to decrease when the energy consumed by R per cycle is small compared to $(L_g I^2)/2$, that is, when the period of a linear-sinoidal oscillation is short compared to the duration of a train of oscillations.

Assume, contrary to fact, that e_1 remains constant during a train of oscillations. The decrease in I due to the introduction of $\gamma^2 R$ increases the ratio T_2/T and the time T_2 required to re-ignite the arc. Since γ increases with this ratio it is seen that the introduction of impedance decreases I and the decrease of I in turn increases the impedance and so on until the arc is extinguished.

In fact e_1 increases rapidly as I decreases because the potential of C rises more slowly which increases T_2 and this gives the arc vapor more time to de-ionize so that the final re-igniting potential e_1 of a train of oscillation is very high.

The value of the re-igniting potential e_1 in the experiment rose during a train of oscillations to the potential required to disrupt about an eighth inch air gap. The distributive capacitance of inductance L_g which is equivalent to a capacitance in parallel with L_g required the presence of the protective condenser shown. The maximum and final value of e_1 varies with L_g and with the rate of de-ionization of the arc vapor.

For a given value of I , E_v , C and L in Fig. 7 there is a maximum value of R ($= R_1$) which will allow the discharge of C to extinguish the arc. Assume, contrary to fact, that the potential difference between A and K remains constant until the arc current is reduced to zero then the minimum value of E_v which will reduce the current to zero is determined by

the energy stored in the condenser minus the energy dissipated by transforming the electrostatic energy into electromagnetic energy which gives

$$E_i (\text{min.}) = I \sqrt{\frac{L}{C}} \cdot e^{\frac{\delta_1}{2\pi} (\cot^{-1} \frac{\delta_1}{2\pi})} \csc \left(\cot^{-1} \frac{\delta_1}{2\pi} \right), \quad (70)$$

where δ_1 corresponds to R_1 . Since the potential drop does not remain constant between A and K for small currents the actual value of E_i will be less than the value necessary to satisfy equation (70).

In the experiment Fig. 7 the quantities E , E_v and E_i were measured with a voltmeter, I with an ammeter and L and C with a wavemeter. The resistance R consisted of fine wire whose high frequency value was known. The values of these quantities were: $C = 0.01$ m.f., $f = 316,000$, $\omega L = 50.4$ ohms, $R_1 = 18$ ohms, $\delta_1 = 1.14$, $I = 5$ amperes, E_i (+ leakage) = 450 volts and $E_v = 13$ volts at from 2 to 10 amperes of steady arc current.

From the above data all the quantities of the initial linear-oscillations of a train can be approximately determined. The initial value of the re-igniting potential e_1 of a train of oscillations can be found by substituting e_1 for E_1 in (59) which gives

$$e_1 = \omega L \left(\cot \phi_1 + \frac{\delta_1}{2\pi} \right) I + E_v, \quad (71)$$

where ϕ_1 is the value of ϕ at the point where the δ -curve intersects the extinction limit curve. The value of ϕ_1 on the $\phi \cdot h$ chart at the point where $\delta = 1.14$ is about $23\frac{1}{2}$ degrees. Substituting this value in (71) and solving for e_1 gives $e_1 = 623$ volts. On the $\phi \cdot h$ chart when $\phi_1 = 23\frac{1}{2}^\circ$, then $\psi_1 = 1.17$ and from the $\psi \cdot \gamma$ table $\gamma_1 = 1.05$. Let n = the variable discharge frequency, then the initial discharge frequency n_0 substituted in equation (19) for N gives $n_0 = 370,000$ cycles per second. From equations (16) and (24) $T_2 = 1.18$ micro-seconds. The values of variables e_a , e_b and e_2 corresponding to E_a , E_b and E_2 in equations (57), (58) and (60) are $e_a = 530$ volts, $e_b = 78$ volts and $e_2 = -13$ volts for the initial oscillations.

The ratio e_1/T_2 may be considered a measure of the rate of de-ionization of the arc vapor under given conditions. This ratio will be influenced by the manner and time required to reduce the arc current to zero before the potential between A and K starts to rise practically as a linear function of the time. Both the magnitude of e_2 and the duration of a negative potential upon valve V will affect the value of the re-igniting potential e_1 .

When C was increased from 0.01 to 0.04 m.f. and R was small the minimum value of E_i (+ leakage) required to extinguish the arc was

reduced from 450 to 240 volts or less. The value of R_1 became more critical as the capacitance C was increased. An increase of E_i resulted in requiring a larger R_1 to extinguish the arc. This result was expected from the fact that the initial T_1 was increased by increasing E_i which gave the arc vapor more time to de-ionize and thereby increased the initial value of e_1 and decreased the initial discharge frequency.

The results of this experiment are only approximate due to the fact that (1) the potential drop between anode and cathode does not remain constant for small values of arc current which gives too large values for R_1 ; (2) the value of R_1 depends upon E_i whose value depends upon leakage during the time required to throw switch S_4 ; and, (3) the presence of high-high frequency oscillating current transients which flow in the oscillatory circuit formed by the capacitance between, and the inductance of, the leads to the anode and cathode or which flow through the distributive capacitance of L_0 and protective condenser in series. The dielectric of a 0.01 M.F. condenser must have a resistance of 2,400 megohms in order that its charge be reduced only twenty per cent. in one-tenth second of isolation. No special precautions to reduce leakage were taken. It is proposed to overcome this unknown leakage by permanently connecting the source of E_i to C through high inductance and high resistance and extinguishing the arc by closing a S.P.-S.T. switch in the discharge circuit.

An attempt was made to measure the initial value of n with a wave-meter but the impulse of the final oscillations of a train destroyed the cumulative effect of the initial oscillations. The effect of intensity above a limited value must be eliminated and the cumulative effect must be retained. A special arrangement is required to secure this result. It may be noted that extinction of the arc developed a high potential between turns of the discharge circuit inductance L ; this was attributed to the final discharges of a train.

STABILITY OF LINEAR-SINOIDAL OSCILLATIONS.

Stability of linear-sinoidal oscillations can be produced by either providing a properly timed priming spark or by increasing the potential E of the d.c. source and the series resistance r to a sufficiently high value.

The later method will be considered. In order to have stability in any mechanical, electrical or other system in which energy is supplied and consumed, the requirement for stability is that *the rate of change of energy consumption with respect to an independent variable must be greater than the rate of change of energy supply with respect to the same variable.* This criterion will be applied to the arrangement of Fig. 7.

Differentiate equation (38) with respect to I and from the resulting differential equation and (38) the critical series resistance is

$$r = R \left(\gamma^2 - 2\gamma \frac{dI_a}{dI} \right). \quad (72)$$

A resistance greater than this value is required to produce stability. The I^2r loss may be considered as either decreasing the energy supply or increasing the energy consumption. In equation (72) the value of dI_a/dI depends upon the rate of change of the rate of de-ionization. The value of r increases as the rate of change decreases. The rate of change probably depends upon the rate of de-ionization. The rate of change will be negative when I_a increases as I decreases and positive when I_a decreases as I decreases. The initial value of e_1 of a train varies with the rate of de-ionization and as this rate is decreased ϕ increases and γ and R must be less, hence critical r varies with rate of de-ionization.

The values of ϕ and h of a train are determined by a δ -curve. In the experiment ϕ and h are determined by the locus of points on the $\phi \cdot h$ chart at which $\delta = 1.14$. The duration of a train is decreased as R and γ are increased and as the rate of change is decreased which results in increasing the rate of energy consumption and decreasing the total energy consumed per train.

These interpretations of equation (72) appear to explain the cause for the greater instability of an arc when the temperature of the arc vapor is low and the rate of de-ionization is high. There is always a small capacitance and inductance externally between the anode and cathode of an arc due to the connecting leads. The slightly drooping volt-ampere characteristic and the low resistance of such an oscillatory circuit will produce sinoidal oscillations of sufficient amplitude to reduce the arc current to zero and thus initiate a train of unstable linear-sinoidal oscillations.

INDUCTIVELY COUPLED SECONDARY CIRCUIT.

Energy can be withdrawn from the discharge circuit by coupling a secondary oscillatory circuit to the inductance L , to the capacitance C or to the valve V . Figure 8 is the same as figure 1 with the addition of a secondary circuit $L_2 - C_2 - r_2$ coupled to the inductance L . The presence of the secondary circuit reduces the effective inductance, and increases the effective resistance, of the discharge circuit. The direct current impedance is increased by either increasing ω or R therefore current flowing in the secondary circuit increases the impedance. The $\psi \cdot \gamma$ table shows that the impedance $\gamma^2 R$ increases with R when ψ and N

remain constant. When R and N are constant the table also shows that an increase in ψ due to an increase in ω correspondingly increases γ .

The arrangement of Fig. 8 can be compared to a direct current motor. Let the energy consumed in the secondary circuit be compared to the load placed upon the motor shaft. When the motor shaft cannot turn no power is delivered to the shaft and a heavy direct current flows through the armature resistance. A similar condition exists in Fig. 8

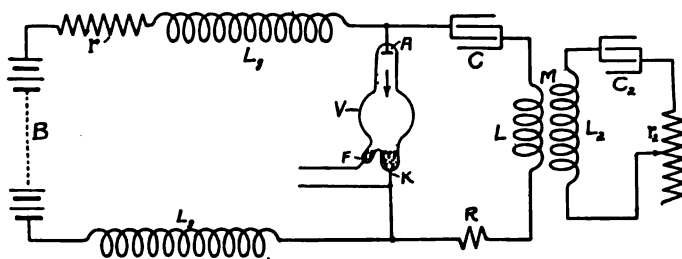


Fig. 8.

when the resistance r_2 is infinitely large and the resistance R is similar to the armature resistance. As r_2 is reduced, current flows in the secondary circuit, the effective value of ω becomes greater and the impedance becomes greater with a corresponding decrease in the direct current. As the motor shaft begins to turn a counter E.M.F. is generated which decreases the current. If r_2 be made zero the impedance will become large and little current will flow which is analogous to the d.c. motor with no load.

In Fig. 1 if L is placed in series with the valve V instead of in series with C and R the theory developed in the preceding pages will apply except the fluctuation of the direct current. The maximum potential producing fluctuation would be increased from E_1 to E_m .

REFERENCES.

The theory developed to explain the phenomena of arc oscillators described in the *PHYSICAL REVIEW* (Vol. VI., No. 6, pages 450-469) by B. Liebowitz and in the *Proceedings of the Institute of Radio Engineers* (Vol. 5, pages 255-316) by P. O. Pedersen is based upon experimental results.

The vacuum-arc oscillator proposed by Liebowitz depends upon a large series resistance and a high potential source of direct current to produce stable oscillations. This arrangement is inefficient and high potential is undesirable.

The Poulsen arc oscillator is stabilized by means of an arc whose

volt-ampere characteristic has a steep slope, the value of the angle ϕ is large and the rate of de-ionization is small compared to a vacuum-arc. This oscillator requires a comparatively high potential direct current supply and the arc loss is large.

The theory of linear-sinoidal oscillations indicates that efficient arc oscillations can be produced by providing a constant frequency*priming spark. The series resistance or arc loss required to secure stability can thereby be avoided.

The rate of de-ionization of arc vapor when an arc has been extinguished depends upon the velocity with which ions recombine and probably also upon the rate of cessation of the ionizing effect of the incandescent cathode and vapor. Reduction of gas pressure facilitates recombination of ions. Lowering the temperature of the vacuum tube decreases the gas pressure and probably reduces the ionizing effect due to incandescence of the gas.

Little appears to be known regarding the law governing the rate of de-ionization of arc vapor as influenced by the composition and pressure of the vapor or gas. The law of this phenomenon has practical application in radio signalling and may extend knowledge of the electron theory. In addition to showing an application of the theory it is believed that the experiment described in this paper is the first attempt to measure the rate of de-ionization of arc vapor.

BREMERTON, WASH.,
April, 1920.

LIST OF PRINCIPAL SYMBOLS.

- C Capacitance of discharge circuit condenser.
- L Inductance of discharge circuit inductance.
- R Resistance of discharge circuit.
- L_0 Inductance in series with d.-c. source.
- r Resistance in series with d.-c. source.
- f Natural frequency of discharge circuit.
- N Discharge frequency.
- I Direct current.
- I_a Effective discharge circuit current.
- E E.M.F. of d.-c. source.
- E_a Potential of C at instant of re-ignition.
- E_b Potential of C at instant of extinction.
- E_1 Potential between anode and cathode at instant of re-ignition.
- E_2 Potential between anode and cathode at instant of extinction.
- E_v Potential drop of current through arc.
- T Time of one complete oscillation.

- T_1 Time of primed period.
- T_2 Time of unprimed period.
- ϕ Priming angle.
- h Damping angle.
- δ Logarithmic decrement of discharge circuit.
- α Damping factor of discharge circuit.
- ψ Ratio of frequencies.
- β Impedance factor.
- γ Ratio of currents.
- ω $2\pi f$.

THE K-CHARACTERISTIC ABSORPTION FREQUENCIES FOR THE CHEMICAL ELEMENTS MAGNESIUM TO CHROMIUM.

BY HUGO FRICKE.

SYNOPSIS.

Absorption of X-Rays.—This paper contains an account of an experimental investigation concerning the discontinuity in the X-ray absorption corresponding to the K-series for the chemical elements from magnesium to chromium inclusively. The method followed was the same as that devised and employed by de Broglie. A specially designed vacuum spectrograph was used.

Fine Structure of Absorption.—The spectrograms show that the discontinuity has a rather complex structure, a result in advance of those obtained by earlier investigators. A photometric study of the plates was made in order to obtain a more accurate knowledge of the detailed structure of the absorption limits.

Results. These are recorded in tables which give for each element the wave-lengths of the different remarkable points in the structure of the discontinuities. The theoretical bearing of the new observations is briefly discussed.

DURING the last few years, an accurate method for determining the longer wave-lengths of X-rays has been worked out by M. Siegbahn and his assistants, employing a vacuum spectrograph designed by him.¹ An interesting feature of the method is that the wave-length differences which can be determined correspond to an amount of energy of the order of 1 volt times the charge on the electron, that is of the same order as arises in processes performed in the exterior of the atom. For the frequency ν , the potential E , and the glancing angle ϕ , we find

$$\frac{\Delta\nu}{\nu} = \frac{\Delta E}{E} = \frac{\Delta\phi}{\tan\phi}.$$

$\Delta\phi$ can be determined by Siegbahn's method and apparatus to within 10^{-4} . For the longest waves with which one can work, E is about 1,000 volts, that is $\Delta E < 1$ volt. It may therefore be expected that Siegbahn's method will lead to the detection of many effects which are caused by the action of the outermost electrons from which effects a calculation of the arrangement of the electrons can be made. For instance, it may be found that the position of certain lines of the X-ray

¹ M. Siegbahn, Phil. Mag. (37), p. 601, 1919, and Ann. d. Phys. (4), 59, p. 56, 1919. W. Stenström, Ann. d. Phys. (4), 57, p. 347, 1918 and Diss. Lund., 1919.

spectrum of a given substance depends on the chemical combination of the radiating atom. It is also to be expected that the variation in the wave-lengths of a given line for the different elements in the periodic table will not be continuous, but a little irregular, corresponding to the discontinuities in the arrangement of the outermost electrons as we progress in the periodic table. Effects of these sorts ought to be most strongly pronounced for the limiting frequencies of the series, because one of the two energies, on the difference of which the frequency depends, is directly associated with the conditions in the outermost part of the atom.

Up to the present time measurements of the longer wave-lengths with the precision stated above have been made for only a few lines.¹

The following is a contribution to the completion of this work. It consists of a series of measurements of the limiting frequencies in the K-series for the chemical elements from magnesium to chromium. As will be seen a rather complex structure of the limit is found.

The measurements were all performed in 1918 at the physical laboratory of the University of Lund (Sweden), using the vacuum spectrograph of M. Siegbahn. It gives me great pleasure to express here my gratitude to Prof. M. Siegbahn for putting at my disposal the vacuum spectrograph and the other resources of his X-ray laboratory, and for the interest he has taken in my work.

As regards the construction and operation of the vacuum spectrograph the reader is referred to the above cited papers of M. Siegbahn and W. Stenström.

The method of investigation is that first employed by de Broglie.¹ A suitable part of the continuous X-ray spectrum from the target in an X-ray tube, with an absorbing substance introduced in the path of the rays, is photographed.

Care must be taken to use the proper quantity of absorbing substance. Too great or too small a quantity effaces the details of the structure of the limit.

The quantity should be so chosen that the ratio between the intensities of the emergent rays on opposite sides of the limiting frequency has a convenient value which is neither too great nor too small.

Glocker² has given two simple formulas, which express the absorption coefficient of the X-rays for the two sides of the limit as a function of the wave-lengths of the absorbed radiation and the atomic number of the absorbing substance. Using these formulæ the quantities of the sub-

¹ M. de Broglie, C. R., 158, p. 1493, 1914.

² Glocker, Phys. Zeits., 19, p. 66, 1918.

stances to be investigated were calculated so that the ratio in question was about 1 : 6.

For most substances the pure element or one of its salts was spread in a thin layer upon a thin sheet of paper. This was placed between the slit and the crystal.

Argon was procured from the air by absorption of the oxygen and nitrogen. This was done by the well known method of leading the air over glowing copper and magnesium. Most of the oxygen was previously absorbed by pyrogallol. The argon was drawn at a pressure of 6 cm. of mercury into a container. This was made of a brass tube 8 cm. long closed at both ends with plates, which, to allow for the passage of the X-rays, were provided with slits closed with gold-beater's skin. A test before filling the tube showed that it could support a vacuum sufficiently well. The tube when under exposure, was placed between the slit and the crystal.

In the case of chlorine, since in this work rock-salt was used as the crystal, no particular absorbing substance was needed.

In the case of aluminium and magnesium some difficulties were encountered, owing to the fact that the absorption of organic substances for the long wave-lengths here employed is very considerable. The proper quantities of Al and Mg to be used are about 0.25 mg. per cm.² (or foils about 1 μ thick). The most convenient manner of manipulating such small quantities would be perhaps to employ the same method as is used for most of the other substances (see above), that is to spread suitable salts of Mg and Al (Mg O, Al₂ O₃) in a thin layer on a thin sheet of paper. As stated in the cited papers of M. Siegbahn and W. Stenström, it is necessary to place a suitable foil in the slit in order to attain the high vacuum in the discharge tube; the best method for that purpose would then be to use gold-beater's skin in the case of Al and a thin foil of Al in the case of Mg (the limiting wave-length of Mg being longer than the limiting wave-length of Al, its absorption in this substance will be comparatively small). This arrangement will, however, owing to the great absorption of the gold-beater's skin, and of the paper demand too long a time of exposure. For this reason it was decided to employ another method. Thin foils of Al and Mg were placed over the slit, acting both to seal the tube and to absorb the rays. Owing to the comparatively small absorption of these substances for wave-lengths longer than the limiting ones, we can with this arrangement reduce the time of exposure very considerably, but in employing this method it is necessary to use fairly thick foils, and therefore we will find only the sudden change in the absorption. No details of the limit can be found. The Al foil

had a thickness of $7\ \mu$, the Mg foil $10\ \mu$. The latter was made from ordinary mg. wire by rolling. Foil as thin as $6\ \mu$ could be made in this way. This thin foil was found, however, to be unable to maintain the high vacuum in the X-ray tube, because of small holes which could not be avoided at these small thicknesses; nothing thinner than $10\ \mu$ was found adequate for the purpose. Calculation shows that these two foils absorb practically all the radiation of a wave-length smaller than the limiting one; in accordance with this, the plates for Mg and Al, as we shall see, do not show any structure of the limit.

The discharge tube is a metal one of the Coolidge type. This is a great advantage in this work, owing to the fact that the potential can immediately be given the desired value. This is important because it is necessary, especially for the longest wave-lengths, to be able to use such a low potential that the spectra of higher orders are not produced. Otherwise these spectra, their smaller absorption compensating for their smaller reflection from the crystal, will be strong enough to mask completely the structure of the limit. Care of this must especially be taken when working with sugar as a crystal, for here we have a third order spectrum which is just as strong as, or even stronger than, the 1st order. An unpleasant consequence of the low potential is the comparatively slight intensity of the radiation. For the lightest substances the low potential causes further difficulty, in that here the space charge prevents the passage through the tube of the great current usually used, and particularly desirable in this case. For magnesium, in order to obtain a current of the proper magnitude, it was necessary to use a potential rather greater than that indicated by the above-mentioned considerations; in consequence we have here a distinct superposition of the third order spectrum (see below). Tungsten was used for the anticathode. It gives a very strong continuous radiation, and allows the passage of a large current. The same metal was used for the incandescent spirals. The current through the tube was about 40 milliamperes, the time of exposure varying from 3 to 16 hours. Only one plate of each substance was taken; in order to get the right time of exposure at once, a rough calculation of the absorption of the wave-length in question was made before the exposure, and the time for this chosen in accordance with the result obtained. So far as possible the chemical compound of the absorbing substance was chosen so as to contain only very light atoms. This, however, was not always possible.

A sugar crystal was used for the lighter substances examined. For the others a rock salt crystal was employed. The reflecting power of both these crystals is very good. The grating constant used for sugar is

$$\log 2d = 1.32503$$

and for rock salt

$$\log 2d = 0.750354.$$

The first was determined by photographing the CuK_α line using for the wave-length of this line the value given by Siegbahn.¹ The second is the generally adopted value.

The width of the slit (between the anticathode and the crystal) was 0.10 mm.

Under exposure the crystal was turned through a certain angle so that a proper wave-length interval around the wave-length in question is obtained on the plate; this angle was $1^\circ.5-2^\circ.0$.

For the calculation of the desired wave-lengths from the photographic plate, it is necessary to photograph a known spectral line on the same plate. For this purpose lines only are used on which precision measurements had already been made at Lund.²

To determine the different details in the limit a photometric investigation of the plates was made. The arrangement used for this was the following: Through a microscope objective a narrow ray of light (.02 mm. wide) from an incandescent lamp run by a storage battery, was thrown on the plate; after emerging it was suitably enlarged and thrown upon a thermopile; the deflection of a galvanometer connected to this gave a measure of the blackening of the plate. The plate was mounted on a slide; the galvanometer mirror formed an image of the filament of an incandescent lamp upon a photographic paper wrapped around a cylinder. Automatic registration could then be obtained by coupling (by toothed wheels) the cylinder and the slide. The slide was moved by a motor at a speed determined by the oscillation constants of the aperiodic system composed of the thermopile and the galvanometer. These constants were previously determined by a simple experiment.

The ratio between the distances on the plate and on the photographic paper must be determined for each plate, as the dimensions of the paper are altered slightly when it is developed. For this purpose, before the photometric investigation, two fine lines are drawn on each plate at a convenient distance from the limit. They are recorded very distinctly on the curve of blackening. By measuring their distance on this and on the plate the desired ratio can be obtained, and is found to be about 17.46 : 1. The measurement on the plate was made with a comparator with better than .01 mm. accuracy.

By means of a microscope the correct orientation of the light ray on

¹ M. Siegbahn, loc. cit.

² M. Siegbahn, loc. cit.

the plate is obtained. Owing to the short length (1 mm.) of the ray three investigations are made, with the ray at the upper, middle and lower part of the plate.

Only the part of the plate around the limit is investigated; the distance of the reference line from one of the drawn lines is measured with the comparator with an accuracy of about .01 mm.

By the photometric method here described the position of the points on the photographic plates can be determined with an accuracy of a few hundredths of a mm. A difficulty encountered in this method is the above mentioned alteration of the photographic paper when developed; this is not always exactly the same in every part.

No limit was found for silicon. First a plate was taken with the above mentioned arrangement. As absorbing substance pure silicon was used, spread in a layer of .70 mg. per cm.² on a sheet of paper. During the exposure the crystal was turned through an angle corresponding to the wave-length interval

$$\lambda = 6.46 - 7.09\text{\AA} \text{ (Si } K_{\beta 1}\lambda = 6.76).^1$$

No limit could be detected on the plate. The reason may be that the limit was covered by the $WM_{\beta 1}$ line; this has a wave-length of 6.75\AA, and is recorded very strongly on the plate.

A new plate was taken with platinum as anticathode; this time J used 1.7 mg. of silicon per cm.² and examined the wave-length interval $\lambda = 6.35-7.09\text{\AA}$. No limit could be detected; still the $WM_{\beta 1}$ line was rather strong owing to the deposition of tungsten from the incandescent spiral on the target. Later, spirals of molybdenum were tried. These were found to be impracticable because of their great evaporation, which causes so poor a vacuum that a sufficiently strong current could not be obtained, and also causes the spiral to be rapidly consumed. Tantalum, which probably would have been found very suitable, was unfortunately not procurable.

As has already been mentioned above, the plates show that the limit is not, as has been commonly thought up to this time, a simple discontinuity in the blackening of the plate, which covers a wave-length interval equal to about the width of a spectralline; but it is found, that the limit presents a rather complex structure. As a rule we can state: we have on the plate two parts of uniform, but unequal blackenings; the limit is the transition between these two; the stronger of the two blackenings has always on the side of the shorter wave-lengths a very distinct boundary (K_m), on the other side of this we have a very bright

¹ M. Siegbahn and W. Stenström, *Phys. Zeits.*, 15, 1916.

line or band (L); after this we frequently have a dark line and then again sometimes a bright band.

On the plates for chromium, vanadium and titanium, a characteristic narrow dark line is seen in the bright line L close up to the boundary K_m ; the phenomenon is most pronounced for chromium and vanadium; for titanium the fine bright boundary line between K_m and the dark line is less distinct.

No structure is found for the limits of magnesium, aluminium and argon. As regards the first two substances, this was to be expected for the reason mentioned above. As regards argon the reason perhaps may be that the plate is rather underexposed.

In the following for each substance tables will be given containing the wave-length (λ , unit $\text{\AA}$) of K_m and the wave-length difference ($\Delta\lambda$, unit $\text{\AA}$) between K_m and the different remarkable points in the structure of the limit, also the distances (Δx unit mm.) on the photographic plate, the frequency differences ($\Delta\nu$ unit : 10^{15}) and energy differences ($\Delta\epsilon$ unit: volt times the charge of the electron) corresponding to the latter are given. $\Delta\lambda$ is determined by:

$$\Delta\lambda = \frac{\lambda}{2\gamma \tan \varphi} \Delta x,$$

λ : Wave-length of K_m ,

φ : glancing angle,

γ : Distance from the axis of the vacuum spectrograph to the photographic plate. In addition a description of each plate is given. Owing to different well known optical illusions, it is always specially emphasized when features observed in the plates are not shown in the photometric curves.

12 MAGNESIUM (FIG. 1).

Absorbing screen: pure Mg : 1.74 mg. per cm^2 ; reference line:

$$\text{SnL}\alpha_1 \lambda = 3.5929.$$

Crystal: Sugar.

Spectrum—1st order.

$$K_m : \lambda = 9.5112 \text{ \AA.},$$

$$K_m - K_l : \Delta x = 0.26 \text{ mm.},$$

$$\Delta\lambda = 0.019 \text{ \AA.},$$

$$\Delta\nu = 0.64 \times 10^{15},$$

$$\Delta\epsilon = 2.7 \text{ volt.}$$

Description of the plate: No structure of the limit is seen; this appears only as a distinct discontinuity $K_m - K_l$ in the blackening of the plate.

13 ALUMINIUM (FIG. 1).

Absorbing screen: pure Al, 1.89 mg. per cm.²; reference line:

$$\text{WLa}_1 : \lambda = 1.4735 \text{ and } \text{SnLa}_1 : \lambda = 3.5929.$$

Crystal: Sugar.*Spectrum:* 1st order.

$$K_m : \lambda = 7.9470 \text{ \AA.},$$

$$K_m - K_i : \Delta x = 0.32 \text{ mm.},$$

$$\Delta \lambda = 0.025 \text{ \AA.},$$

$$\Delta \nu = 1.18 \times 10^{15},$$

$$\Delta \epsilon = 4.9 \text{ volt.}$$

Description of the plate: What is stated for magnesium will apply here also.

15 PHOSPHORUS (FIG. 2).

Absorbing screen: H₃PO₄, 0.80 mg. P per cm.²; reference line:

$$\text{Sn } \mathcal{L}\alpha_1 \lambda = 3.5929.$$

Crystal: Sugar.*Spectrum:* 3d order.

$$K_m : \lambda = 5.7580 \text{ \AA.}$$

	$K_m - \mathcal{L}.$	$K_m - K_i.$
Δx	0.52 mm.	0.86 mm.
$\Delta \lambda$	0.0084 \AA.	0.0139 \AA.
$\Delta \nu$	$0.76.10^{15}$	$1.26.10^{15}$
$\Delta \epsilon$	3.16 volt	5.25 volt

Description of the plate: Two parts of unequal blackening (I - K_m and K_i - II), separated by a bright line ($K_m \mathcal{L} K_i$).

16 SULPHUR (FIG. 3 AND FIG. 9).

Absorbing screen: S, 0.90 mg. per cm.²; reference line:

$$\text{Sn } \mathcal{L}\alpha_1 \lambda = 3.5929.$$

Crystal: Sugar.*Spectrum:* 3d order.

$$K_m : \lambda = 5.0123 \text{ \AA.}$$

	$K_m - \mathcal{L}.$	$K_m - \mathcal{M}.$	$K_m - N.$
Δx	0.23 mm.	0.64	1.29
$\Delta \lambda$	0.0045 \AA.	0.0125	0.0253
$\Delta \nu$	$0.54.10^{15}$	1.49	3.02
$\Delta \epsilon$	2.3 volt	6.2	12.6

Description of the plate: Starting from the boundary' (K_m) of the stronger uniform blackening ($I - K_m$), we have first a distinct bright line ($K_m - \mathcal{L}M$, 0.3 mm.), then a distinct dark line ($\mathcal{M}$) and then at last a very faint bright band (MN , 0.7 mm.), which, with an apparently distinct limit, borders on the fainter uniform blackening ($N - II$).

Whether there is in reality a distinct limit between the bright band (MN) and the uniform blackening (NII) cannot, owing to the faintness of the bright band, be decided with certainty from the photometric curves.

17 CHLORINE [FIG. 3].

No absorbing screen. Reference line: $ClK\alpha_1 \lambda = 4.7187$.

Crystal: Rocksalt.

Spectrum: 1st order.

$$K_m = 4.3844.$$

	$K_m - \mathcal{L}$.	$K_m - \mathcal{M}$.
Δx	0.43 mm.	0.95 —
$\Delta \lambda$	0.0060 Å.	0.0132 —
$\Delta \nu$	$0.93 \cdot 10^{15}$	2.06 —
$\Delta \epsilon$	3.9 volt	8.6 —

Description of the plate: Looks exactly like that for sulphur; the bright line apparently is 0.6 mm., the bright band 0.9 mm. wide.

18 ARGON (FIG. 1).

Absorbing substance: A., 1.5 mg. per $cm.^2$; reference line:

$$Sn \mathcal{L}\alpha_1 \lambda = 3.5929.$$

Crystal: Sugar.

Spectrum: 3d order.

$$K_m \lambda = 3.8657.$$

$K_m - K_1$	
Δx	0/14 mm.
$\Delta \lambda$	0.0033 Å.
$\Delta \nu$	$0.67 \cdot 10^{15}$
$\Delta \epsilon$	2.8 volt.

Description of the plate: The same as for those of magnesium and aluminium.

19 POTASSIUM (FIG. 4).

Absorbing substance: K_2CO_3 , 2.00 mg. K per $cm.^2$; reference line:

$$Sn \mathcal{L}\alpha : \lambda = 3.5929.$$

Crystal: Sugar.

Spectrum: 1st order.

$$K_m : \lambda = 3.4345.$$

	$K_m - \mathcal{L}.$	$K_m - \mathcal{M}.$	$K_m - \mathcal{L}'.$
$\Delta\chi$	0.16 mm.	0.35—	0.47—
$\Delta\lambda$	0.013 Å.	0.027—	0.039—
$\Delta\nu$	$3.4.10^{15}$	6.9—	9.9—
$\Delta\epsilon$	14. volt	29.—	41.—

Description of the plate: Looks about the same as the plates for sulphur and chlorine; a bright line ($\mathcal{L}$), a dark line ($\mathcal{M}$) and a fainter bright band ($\mathcal{L}'$); no distinct limit of this however can be seen here; still it must be remarked, that, $\text{Sn}\mathcal{L}\beta$, which is presented very strongly, perhaps covers the last part of the bright band.

The photometric curves seem to indicate the same structure for the bright band as for the bright line $\mathcal{L}$.

20 CALCIUM (FIG. 2).

Absorbing substance: CaCO_3 , 1 mg. Ca. per cm^2 ; reference line:

$$\text{CuK}\alpha_1 \lambda = 1.5374.$$

$$\text{W}\mathcal{L}\alpha_1 \lambda = 1.4735.$$

Crystal: Rocksalt.

Spectrum: 1st order.

$$K_m : \lambda = 3.0633 \text{ Å.}$$

	$K_m - \mathcal{L}.$	$K_m - K_1.$
$\Delta\chi$	0.41 mm.	1.2—
$\Delta\lambda$	0.0078 Å.	0.023—
$\Delta\nu$	$2.48.10^{15}$	7.4—
$\Delta\epsilon$	10.3 volt	31—

Description of the plate: Looks about the same as the plate for phosphorus.

21 SCANDIUM (FIG. 5.)

Absorbing substance: $\text{Sc}_2(\text{SO}_4)_3$, 1.7 mg. Sc. per cm^2 ; reference line:

$$\text{VK}\alpha_1 \lambda = 2.4987 \text{ Å.}$$

Crystal: Rocksalt.

Spectrum: 1st order.

$$K_m : \lambda = 2.7517 \text{ Å.}$$

	$K_m - \mathcal{L}.$	$K_m - \mathcal{M}.$	$K_m - \mathcal{N}.$
$\Delta\chi$	0.47 mm.	1.14—	2.30—
$\Delta\lambda$	0.0091 Å.	0.0222—	0.0446—
$\Delta\nu$	$3.62.10^{15}$	8.79—	17.67—
$\Delta\epsilon$	15.1 volt	36.7—	73.7—

Description of the plate: Here and also on the following plates for titanium and vanadium two dark, rather broad and diffuse lines (M_1 and M_2) are seen in the fainter uniform blackening; their distances from K_m are about 1 and 3 mm.; the first is the stronger; it forms the boundary between $\mathcal{L}$ and the fainter uniform blackening. The existence of the second is perhaps questionable; it cannot be shown with certainty on the photometric curves.

22 TITANIUM (FIG. 6 and Fig. 10.)

Absorbing substance: TiO_2 , 2.0 mg. Ti per $cm.^2$; reference line:

$$Cr K\alpha_1 \lambda = 2.2852.$$

Crystal: Rocksalt.

Spectrum: 1st order.

$$K_m, \lambda = 2.4937.$$

	$K_m - S'$	$[K_m - S'']$	$K_m - m$
Δx	0.63 mm.	1.07 —	0.14 —
$\Delta \lambda$	0.0124 Å.	0.0214 —	0.0028 —
$\Delta \nu$	$6.0 \cdot 10^{15}$	10.3 —	1.3 —
$\Delta \epsilon$	25.0 volt	43.0 —	5.6 —

Description of the plate: M_1 (see under scandium) has here a very distinct boundary on the side toward the bright line $\mathcal{L}$; this has a very well defined breadth ($\mathcal{L}' - \mathcal{L}''$). In the middle of $\mathcal{L}$ is seen a very faint dark line M' ; where this is shown on the photometric curves cannot with certainty be determined. As already mentioned above we have a very dark distinct line (here not so pronounced as in the case of vanadium and chromium) in $\mathcal{L}$ close up to K_m ; it is shown in the photometric curves at m .

23 VANADIUM (FIG. 7).

Absorbing substance: V_2O_5 , 2mg. V per $cm.^2$; reference line:

$$Fe K\alpha_1 : \lambda = 1.9324.$$

Crystal: Rocksalt.

Spectrum: 1st order.

$$K_m : \lambda = 2.2653.$$

	$K_m - l$	$K_m - m$	$K_m - \mathcal{L}$	$K_m - M'$	$K_m - M$
Δx	0.17 mm.	0.26 —	0.60 —	1.18 —	1.60 —
$\Delta \lambda$	0.0035 Å.	0.0053 —	0.0123 —	0.0242 —	0.0327 —
$\Delta \nu$	$2.0 \cdot 10^{15}$	3.1 —	7.2 —	14.2 —	19.1 —
$\Delta \epsilon$	8.5 volt	12.9 —	30.0 —	59.0 —	79.8 —



Fig. 1.
Aluminium.

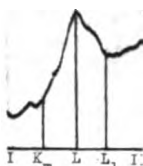


Fig. 2.
Phosphorus.

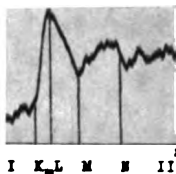


Fig. 3.
Sulphur.



Fig. 4.
Potassium.

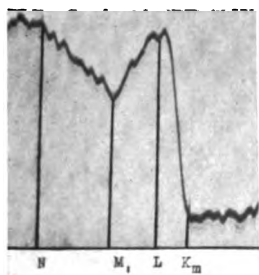


Fig. 5.
Scandium.

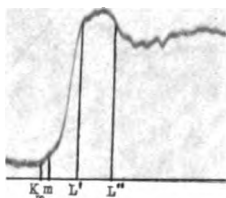


Fig. 6.
Titanium.

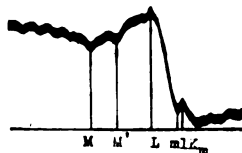


Fig. 7.
Vanadium.



Fig. 8.
Chromium.

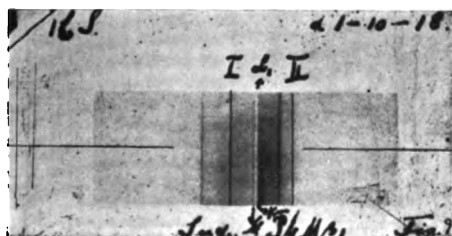


Fig. 9.
Sulphur.



Fig. 10.
Titanium.

HUGO FRICKE.

Description of the plate: Like the plate for titanium; the dark line m close to K_m is here seen very distinctly; the bright line between m and K_m is shown on the photometric curves by 1.

24 CHROMIUM (FIG. 8).

Absorbing substance: K_2CrO_4 , 1 mg. Cr. per $cm.^2$; reference line:

$$W\mathcal{L}_{\alpha_1} \lambda = 1.4735.$$

Crystal: Rocksalt.

Spectrum: 1st order.

$$K_m : \lambda = 2.0675 \text{ \AA}.$$

	$K_m - l.$	$K_m - m.$	$K_m - \mathcal{L}.$	$K_m - M'.$	$K_m - M.$
$\Delta\chi$	0.13 mm.	0.20 —	0.47 —	0.74 —	1.05 —
$\Delta\lambda$	0.0026 Å.	0.0040 —	0.0098 —	0.0153 —	0.0218 —
$\Delta\nu$	$1.8 \cdot 10^{16}$	2.8 —	6.8 —	10.7 —	15.3 —
$\Delta\epsilon$	7.6 volt	11.8 —	28.6 —	45.0 —	64.0 —

Description of the plate: The two dark lines M_1 and M_2 are not seen here; the bright line $\mathcal{L}$ has very distinct boundaries on both sides ($\mathcal{L}'$ and $\mathcal{L}''$); as in the case of titanium and vanadium a very faint dark line M' is seen in the middle of $\mathcal{L}$. What is stated for vanadium about the dark line m will apply here also.

No satisfactory theory for the limiting frequency has as yet been published. Kossel,¹ on the basis of Bohr's theory, regards the limiting frequency of a certain series as corresponding to the passing of the electron from the ring in the atom, corresponding to this series, to the space outside all the rings of electrons in the atom. On the basis of this theory, a theory for the structure of the limit here discovered would be, that different orbits exist outside the atom, to all of which the electron (with different probabilities) can go starting from the K-ring. The different passages corresponding to these orbits will give a series of absorption lines. Further, the electron from the K-ring can pass to infinity with all kinds of velocities (still with a varying probability); this will give a broad absorption band succeeding the above-mentioned absorption lines. The last type of passage is used by Bohr² to explain the continuous absorption band, which is observed by R. W. Wood at the head of the principal series of sodium, extending to the extreme ultra-violet.

It ought, however, to be mentioned that this theory does not seem to explain the more special formation of the structure observed for the different substances.

¹ W. Kossel, Verh. d. D. Phys. Ges. (16), p. 953, 1914.

² N. Bohr, Phil. Mag. (26), p. 17, 1913.

TABLE FOR K_{α} .

Atomic Number.	Element.	$K_{\alpha} : \lambda.$	$\sqrt{\frac{1}{\lambda}}$	Difference.
12	Mg	9.5112	0.32425	
13	Al	7.9470	0.35475	.3050
14	Si	—	—	(.6199)
15	P	5.7580	0.41674	
16	S	5.0123	0.44666	.2992
17	Cl	4.3844	0.47759	.3093
18	A	3.8657	0.50861	.3102
19	K	3.4345	0.53960	.3099
20	Ca	3.0633	0.57135	.3175
21	Sc	2.7517	0.60284	.3149
22	Ti	2.4937	0.63326	.3042
23	V	2.2653	0.66442	.3116
24	Cr.	2.0675	0.69547	.3105

For none of the substances, with the exception of Mg, are the wave-lengths of the limit found, with certainty, longer than any of the K -lines. For Mg the wave-length in question is found considerably longer than the wave-length of the $K_{\beta 1}$ line.¹ The reason for this perhaps may be illustrated by the following simple consideration: The K_{β} line corresponds to a transition of the electron from one ring of electrons (the M-ring) to another ring situated nearer the nucleus (the K-ring). A simple calculation shows however, that more energy is not always required to move the electron from the innermost ring to infinity than to move it between two such rings. The latter passage can require the greater amount of energy if the M-ring is situated in the exterior of the atom.

Limiting frequencies have previously been determined by de Broglie,² Wagner³ and Siegbahn-Jonsson⁴ using the photographic method and by

¹ M. Siegbahn and W. Stenström, loc. cit.

² M. de Broglie, C. R., 163, p. 87, 1916, and Jour. de Phys., 5, p. 161, 1916.

³ E. Wagner, Bayr. Akad. d. Wiss., 1916.

⁴ M. Siegbahn and E. Jonsson, Phys. Zeits., 20, p. 251, 1919.

Blake¹-Duane, Duane²-Kang-Fuh-Hu and Duane-Takeo Shimizu³ using the ionization method.⁴

No structure of the limit has been found by any of these authors. As regards the smaller wave-lengths the reason for that is certainly that the structure here covers so small an interval of wave-length, that it could not be observed with the accuracy obtainable in the investigations concerned; this agrees with the theory here given. As regards the longer wave-lengths, the reason may be that in order to get sufficiently high intensity and a distinct limit the investigators have used comparatively broad slits, high potentials and large quantities of the absorbing substances.⁵

COPENHAGEN, DENMARK.

¹ F. C. Blake and W. Duane, *PHYS. REV.*, 10, p. 697, 1917.

² W. Duane and Kang-Fuh-Hu, *PHYS. REV.*, 14, p. 516, 1919.

³ W. Duane and Takeo Shimizu, *PHYS. REV.*, 14, p. 522, 1919.

⁴ After the completion of this work, W. Stenström (Diss. Lund. Sweden 1919), using the photographic method, has found three limits in the M-series. These present a structure about the same as is found here; however, the whole complex is, as to be expected, rather faint.

⁵ Wagner² states, that in his investigation of the limit of Iron he has used a foil of Iron 0.02 mm. thick. A calculation shows that this foil will absorb practically the whole radiation of shorter wave-lengths than the limiting wave-length. Furthermore he has used a slit 0.4 mm. wide.

THE AUDION OSCILLATOR.

BY R. A. HEISING.

SYNOPSIS.

Conditions Bordering Free Oscillations in Audion Generator Circuits.—Conditions required for the production of free oscillations in audion oscillator circuits can be mathematically predetermined. It necessitates an assumption of a straight line characteristic for the tube, which assumption becomes exact for small oscillations. The required conditions can be expressed in terms of the constants of the circuit elements and the constants of the tube. The equations are set up as for ordinary circuits making use of Nichols' explanation of the behavior of the audion in which the amplifying action is ascribed to a fictitious generator located between the plate and filament inside the tube and having a voltage μ times the voltage applied to the grid. Two simple circuits are treated in detail and results plotted in several sets of curves.

I. INTRODUCTION.

THE audion generator of sustained high frequency currents has been the subject of much investigation during the past few years and has given rise to numerous papers. It has been discussed by Hazeltine (I.R.E., April, 1918), Vallauri (L'Elettrotecnica, January, 1917), and in Bureau of Standards Circular No. 74. The writer has done considerable work along this line, both theoretical and experimental, and many of the results are in the course of publication. Analytic studies have been made by Hazeltine, Vallauri and others. In this article duplication of their work will be avoided where possible.

2. APPROXIMATIONS AND ASSUMPTIONS.

The difficulty of securing an exact solution for audion circuits has been evident to all investigators. The primary reason is found in the nature of its characteristic curve. According to Van der Bijl, the equation expressing its behavior is

$$i = B(E_b + \mu E_c + k)^2, \quad (1)$$

where E_b is the plate potential, E_c is the grid potential, μ is the amplification constant and B and k are constants. This curve is a parabola which makes any circuit equations derived from it extremely complex. Vallauri assumes a straight line characteristic at the outset as does Hazeltine by his definition of mutual conductance g . The straight line equation will be assumed in this work for several reasons. They are:

(1) the solution is simplified (2) according to Van der Bijl (PHYS. REV., Sept., 1918, p. 182) the output impedance of the audion is independent of a variable potential impressed on the grid (3) the above parabolic equation ceases to hold when E_b becomes negative, E_c becomes positive, or $E_b + \mu E_c + k$ is negative. From the second reason we deduce that the equations derived hold not only for small oscillations over such a small part of the curve that it is sensibly a straight line, but they hold for large oscillations up to the value that will make E_b negative, E_c positive or $E_b + \mu E_c + k$ negative. Under these conditions (reason 3)

equation 1 is no longer the characteristic curve as the latter takes the form shown by the full line in Fig. 1. The parabolic equation fits the actual curve over only part of its length. For positive values of E_c two things occur in the tube which are not contained in this equation—(a) current flows from the grid to the filament, (b) voltage saturation of the filament occurs. These things cause the actual

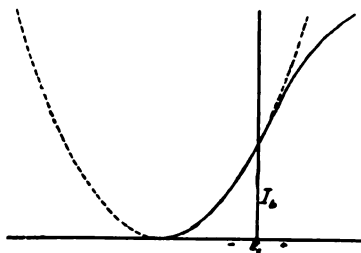


Fig. 1.
Characteristic curve.

curve to depart from the right-hand limb of the parabola as shown in Fig. 1. For negative values of $E_b + \mu E_c + k$ the left-hand limb of the parabola disappears entirely. For negative values of E_b regardless of the value of $E_b + \mu E_c + k$ the equation fails. These departures make it useless to consider the parabolic equation in any oscillator solution. The actual function containing all variables is exceedingly complicated.

The assumption of the straight line equation together with reason 2 give equations which hold for the oscillation frequency up to the limits mentioned and give circuit conditions required for oscillating as well as most of the transient conditions correctly.

3. EQUATION ASSUMED.

The equation assumed is

$$i = B(E_b + \mu E_c + k). \quad (2)$$

The output impedance (or internal impedance as it is sometimes called) is

$$Z = \frac{dE_b}{di} = \frac{1}{B} \quad (3)$$

and is constant at all times. In setting up the differential equations of the circuit advantage is taken of the theorem stated by Nichols in the

PHYSICAL REVIEW, Aug., 1917, that an element whose impedance value varies can be considered as having a constant value but with a variable E.M.F. located within. In the case of the audion, the constant resistance is the output impedance Z in equation (3) and the contained E.M.F. is μE_c . Referring to Fig. 3, which is a diagrammatic representation of Fig. 2, the fictitious generator G is located within the audion and has

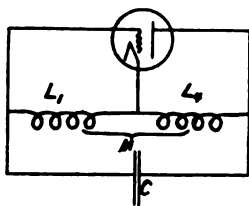


Fig. 2.

Hartley oscillator circuit.

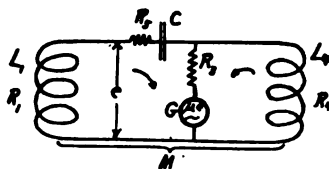


Fig. 3.

Electrical equivalent of Hartley circuit.

an E.M.F. μe , where e is the E.M.F. across L_1 (between grid and filament). If instead we set up the differential equations in the regular manner and substitute equation (2) identical results are secured. For simplicity and clearness, Nichols' method is the better.

4. THE OSCILLATORY CIRCUIT.

The simplest type of oscillatory circuit consists of an inductance, a resistance, and a capacity in series. The differential equation of the circuit is

$$L \frac{di}{dt} + Ri + \int \frac{idt}{c} = E, \quad (4)$$

where E is constant. The solution of this circuit is secured by making the assumption that

$$i = Ae^{at}. \quad (5)$$

Differentiating (4) and substituting (5)

$$Ae^{at} \left(La^2 + Ra + \frac{1}{C} \right) = 0. \quad (6)$$

Solving for (a)

$$a = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}. \quad (7)$$

Giving as the solution

$$i = Ae^{\left(-\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t} + Be^{\left(-\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t}. \quad (8)$$

The values of A and B are secured by substituting the boundary conditions in a given case. The part with which we are concerned, however,

is the exponential constant a . Under certain conditions, viz: $(R^2/4L^2) < (1/LC)$ the radical term is imaginary. The exponential term then has the general form

$$a = -\frac{R}{2L} \pm j\omega, \quad (9)$$

which means that the circuit is oscillatory, that the frequency of oscillation is $\omega/2\pi$, and that the oscillations are damped according to $e^{(-R/2L)t}$. The value of ω is given by $\sqrt{(1/LC) - (R^2/4L^2)}$ and is largely dependent upon the inductance and capacity.

The length of time the oscillations last depends upon the value of $-(R/2L)$. If this is large numerically, the oscillations die out rapidly; if it is very small, the oscillations will continue for a considerable time. If we can make it zero, the damping term $e^{(-R/2L)t}$ reduces to unity for all values of time, and we have the condition of oscillations occurring without a driving E.M.F. which oscillations neither increase nor decrease with time. If, however, we can make the exponent positive, the damping term becomes an "undamping" term $e^{(+R/2L)t}$, and the oscillations will increase with time. If we insert a carbon arc in a series circuit containing inductance, capacity and low resistance, the combination produces the condition just referred to—a positive damping constant—and oscillations occur which increase with time. Such oscillations once started increase continuously until some characteristic of the arc or circuit is altered. The alteration occurring reduces the damping constant to zero after which the oscillations continue indefinitely with unchanged amplitude.

5. PRODUCTION OF OSCILLATIONS BY AN AMPLIFIER AND ITS CIRCUIT.

Continuous oscillations can also be produced by amplifiers. An amplifier is a device which takes power in some form but converts part of it into a different form due to the operation of a control member. A relay is an amplifier. The magnet is the control member and the power conversion occurs by the making and breaking of the armature contact points. A relay provided with a spring, bell, and clapper forms an electric bell and can be used to produce oscillations. The bell takes power in the form of unidirectional current and converts it into mechanical and electrical oscillations by virtue of the control elements (magnets) operating on the armature and varying the electrical constants in some part of the circuit (breaks a contact). The audion acts in a similar manner and produces oscillations. Potential changes on the grid (instead of current through a magnet) cause the plate to filament resistance to change and power is delivered to a circuit. Equation for systems like these are discussed by Nichols (PHYSICAL REVIEW, August, 1917) and

are solved in the same general manner as are those for the simple cases mentioned previously.

6. CIRCUIT EQUATIONS.

The simpler oscillator circuits only will be considered. The more complex circuits can be more clearly explained by applying to them the deductions made from the simple circuits than can be done by using the complicated equations of the complex circuits themselves. The circuit shown in Fig. 2, called the Hartley circuit on account of its inventor, is shown diagrammatically in Fig. 3. The equations, assuming the current directions indicated by the arrows as positive, are

$$\left\{ \begin{aligned} R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_4}{dt} + R_3(i_1 + i_4) + \int \frac{i_1 dt}{C} + i_1 R_5 \\ \qquad \qquad \qquad = -\mu \left(R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_4}{dt} \right), \quad (10) \\ -M \frac{di_1}{dt} + R_3(i_1 + i_4) + R_4 i_4 + L_4 \frac{di_4}{dt} = -\mu \left(R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_4}{dt} \right). \end{aligned} \right.$$

The right-hand terms stand for the voltage generated by the fictitious generator G . The voltage is opposite in direction to the drop across the inductance L_1 and is therefore given an opposite sign. M is the mutual inductance between L_1 and L_4 . In these equations, the continuous potentials and currents are omitted as they cancel out. The equations stand for the varying values only.

The assumption is now made that the current in any part, such as in L_1 has the form (as in equation 5)

$$i_1 = I_1 e^{at}. \quad (11)$$

On substituting in the above equations and dividing out the e^{at} term we have

$$\left\{ \begin{aligned} I_1(R_1 + aL_1) + I_1 \left(R_5 + \frac{1}{aC} \right) + I_1 R_3 - I_4 Ma + I_4 R_3 \\ \qquad \qquad \qquad = -\mu(I_1(R_1 + aL_1) - I_4 Ma), \quad (12) \\ -I_1 Ma + I_1 R_3 + I_4 R_3 + I_4(R_4 + aL_4) = -\mu(I_1(R_1 + aL_1) - I_4 Ma). \end{aligned} \right.$$

If the exponent a is a pure imaginary number such as $j\omega$, the equations reduce to the usual complex form for alternating current circuits. We are then justified in writing in a more simple manner the above as

$$\left. \begin{aligned} I_1(Z_1 + Z_5 + Z_3 + \mu Z_1) + I_4(Z_3 - W - \mu W) &= 0, \\ I_1(Z_3 - W + \mu Z_1) + I_4(Z_3 + Z_4 - \mu W) &= 0. \end{aligned} \right\} \quad (13)$$

Where Z_1, Z_5 , etc., are the complex impedances of the circuit elements.

Heaviside (Electrical Papers) and Campbell (A. I. E. E., Apr., 1911) have shown that such equations also hold where a is complex—that is with damped or expanding oscillations. Equations 13 thus hold for any value of a and in this case the impedances are:

$$\left. \begin{aligned} Z_1 &= R_1 + aL_1, \\ Z_4 &= R_4 + aL_4, \\ Z_5 &= R_5 + \frac{1}{aC}, \\ W &= aM, \\ Z_3 &= R_3. \end{aligned} \right\} \quad (14)$$

These are the values for the particular circuit considered (Fig. 2) but can be given different values if the elements of the oscillator are constructed in other ways than that shown.

On solving this equation for I_1 we get

$$I_1 = \frac{o(W + Z_4)}{(Z_3 + Z_4 - \mu W)(Z_1 + \mu Z_1 + Z_3 + Z_5) - (Z_3 - W - \mu W)(Z_3 + \mu Z_1 - W)}. \quad (15)$$

This equation has the dimensions of a zero E.M.F. divided by an impedance of the value

$$\frac{(Z_3 + Z_4 - \mu W)(Z_1 + \mu Z_1 + Z_3 + Z_5) - (Z_3 - W - \mu W)(Z_3 + \mu Z_1 - W)}{W + Z_4} \quad (16)$$

and giving a finite current. Such a condition obtains only if the impedance is also zero. We can thus write the determining equation for free oscillation

$$(Z_3 + Z_4 - \mu W)(Z_1 + \mu Z_1 + Z_3 + Z_5) - (Z_3 - W - \mu W)(Z_3 + \mu Z_1 - W) = 0. \quad (17)$$

This equation tells nothing about the value of I_1 . It indicates only the conditions obtaining under which a current can flow without an applied E.M.F. It is the usual equation giving the frequency and damping constants of transient currents in a circuit. In this type of circuit, however, the transients may not be damped, but may expand. The equation determines the value of the exponent a in equation 11, just as equation 6 gives its value for the ordinary oscillatory circuit. The actual value of I_1 is determined by the boundary conditions imposed by the oscillator circuit when the switch is closed, and the logarithmic term e^{at} as affected by the value of a and the lapse of time t .

In the actual circuit, the latter mentioned increase of current with time does not continue indefinitely due to the limitations of the audion characteristics and the steady state conditions resulting cannot be determined by these equations.

On collecting the terms in a different way

$$Z_3(Z_1 + Z_4 + Z_5 + 2W) + (\mu + 1)(Z_1Z_4 - W^2 - Z_5W) + Z_5W + Z_4Z_5 = 0. \quad (18)$$

This equation holds for any circuit having impedance in the places of L_1 , L_4 or C . For applying it to them it is only necessary to substitute their impedances in a form similar to those in equations 14.

For the Hartley circuit substitute (14) in (18)

$$\begin{aligned} Z_3 \left[R_1 + R_4 + R_5 + aL_1 + aL_4 + \frac{1}{aC} + 2aM \right] \\ + (\mu + 1) \left[R_1R_4 + a^2L_1L_4 + aR_1L_4 + aR_4L_1 - a^2M^2 - aMR_5 - \frac{M}{C} \right] \\ + aMR_5 + \frac{M}{C} + R_4R_5 + \frac{L_4}{C} + \frac{R_4}{aC} + aR_5L_4 = 0. \end{aligned} \quad (19)$$

Multiplying through by a and collecting in terms of its descending powers

$$\begin{aligned} a^3(\mu + 1)[L_1L_4 - M^2] + a^2[Z_3(L_1 + L_4 + 2M) + (\mu + 1)(R_1L_4 \\ + R_4L_1 - MR_5) - MR_5 + R_5L_4] \\ + a \left[Z_3(R_1 + R_4 + R_5) + (\mu + 1) \left(R_1R_4 - \frac{M}{C} \right) + R_4R_5 \right. \\ \left. + \frac{M}{C} + \frac{L_4}{C} \right] + \frac{Z_3 + R_4}{C} = 0, \end{aligned} \quad (20)$$

which has the general form

$$a^3 + a^2x + ay + z = 0, \quad (21)$$

where

$$\left. \begin{aligned} x &= \frac{Z_3(L_1 + L_4 + 2M) + (\mu + 1)(R_1L_4 + R_4L_1 - MR_5) + R_5(L_4 - M)}{(\mu + 1)(L_1L_4 - M^2)}, \\ y &= \frac{Z_3(R_1 + R_4 + R_5) + (\mu + 1) \left(R_1R_4 - \frac{M}{C} \right) + R_4R_5 + \frac{M + L_4}{C}}{(\mu + 1)(L_1L_4 - M^2)}, \\ z &= \frac{Z_3 + R_4}{(\mu + 1)(L_1L_4 - M^2)C}. \end{aligned} \right\} \quad (22)$$

An equation of this type has three roots. If these roots are P_1 , P_2 and P_3 the coefficients in the equation are:

$$\left. \begin{aligned} x &= -(P_1 + P_2 + P_3), \\ y &= P_1P_2 + P_2P_3 + P_3P_1, \\ z &= -P_1P_2P_3. \end{aligned} \right\} \quad (23)$$

As we are expecting a solution giving a freely oscillatory condition, we know that a is complex and that the above coefficients are all real. The roots must have the form

$$\left. \begin{aligned} P_1 &= d_1 + j\omega, \\ P_2 &= d_1 - j\omega, \\ P_3 &= d_2 \end{aligned} \right\} \quad (24)$$

to satisfy these conditions. Combining with (23) we get

$$\left. \begin{aligned} x &= -d_2 - 2d_1, \\ y &= d_1^2 + \omega^2 + 2d_1d_2, \\ z &= -d_2(d_1^2 + \omega^2), \end{aligned} \right\} \quad (25)$$

where d_1 and d_2 are the damping constants and $\omega = 2\pi f$.

As stated before, if d_1 is zero, the oscillations when once started will persist indefinitely, neither decreasing nor increasing. If d_1 is negative, they will, as in all ordinary tuned circuits, die out. The condition under which a circuit will just oscillate is that of $d_1 = 0$ but if it is the slightest bit greater, oscillations will increase with time. The condition of most interest to us is that condition of the circuit at which oscillations just will, or will not occur as we are then enabled to determine the circuit requirements under which free oscillations will, or will not take place. It marks the change from the normal stable circuit to the freely oscillatory circuit and the conditions of the circuit influencing this change are of the utmost importance.

The equation (21) as it is, admits of no simple exact solution accurate under all conditions but does admit of an exact solution at and near the condition of $d_1 = 0$. If we take the case of d_1 almost zero, we can neglect it in comparison with d_2 or ω in equations (25). If we do not consider it exactly zero, the term containing the product of d_1 and d_2 must be left in, and it enables us to determine the value of d_1 at and around zero with extreme accuracy. Under this condition we can rewrite equations (25) neglecting the second powers of d_1 and the first power in comparison with d_2 giving

$$\left. \begin{aligned} x &= -d_2, \\ y &= \omega^2 + 2d_1d_2, \\ z &= -d_2\omega^2, \end{aligned} \right\} \text{ from which } \left\{ \begin{aligned} d_2 &= -x, \\ \omega^2 &= \frac{z}{x}, \\ d_1 &= \frac{1}{2x} \left(\frac{z}{x} - y \right). \end{aligned} \right\} \quad (26)$$

7. DIRECT CURRENT TRANSIENT TERM.

From equations 22 and 26 we can write

$$d_2 = - \frac{Z_3(L_1 + L_4 + 2M) + (\mu + 1)(R_1L_4 + R_4L_1 - MR_5) + R_5(L_4 - M)}{(\mu + 1)(L_1L_4 - M^2)} . \quad (27)$$

This factor is always negative under conditions even approximating those of free oscillation and indicates that the current term multiplied by e^{d_2t} decreases with time under all conditions in the circuit. It is the space current damping term. This transient term is not of much importance to us as it multiplies no periodic or A.C. term and therefore is not concerned with the oscillation current in the circuit but can only be concerned with a direct current term. It indicates the rapidity with which the space current builds up to its final value. If we consider R_1 , R_4 , and R_5 as zero, C_5 as infinite and M as zero, and look at the circuit as a resistance Z_3 in series with two parallel inductances L_1 and L_4 this damping term reduces to

$$d_2 = - \frac{Z_3}{(\mu + 1) \frac{L_1L_4}{L_1 + L_4}} . \quad (28)$$

In this, $(L_1L_4)/(L_1 + L_4)$ is the effective inductance of the two inductances in parallel. If $\mu = 0$ which is the case of a pure resistance for Z_3 with no amplifying characteristic, the equation is

$$d_2 = - \frac{Z_3}{L_e} , \quad (29)$$

where L_e is the effective inductance and Z_3 is the resistance. This is the damping constant in a circuit containing only inductance and resistance.

The direct current solution for this circuit is therefore,

$$i = \frac{E_b + \mu E_c + k}{Z_3 + R_4} (1 - e^{d_2t}) , \quad (30)$$

where E_c is whatever constant negative potential is applied to the grid, K is a constant and d_2 is the value given in equation 27. It will be observed that if μ is large, the damping constant d_2 is small, and that it will take a longer time for the space current to rise to the final value than in the case of a low μ audion.

8. FREQUENCY TERM.

From equations 22 and 26 we get the frequency term

$$\omega^2 = \frac{Z_3 + R_4}{C(\mu + 1)(L_1L_4 - M^2)} \times \frac{(\mu + 1)(L_1L_4 - M^2)}{Z_3(L_1 + L_4 + 2M) + (\mu + 1)(R_1L_4 + R_4L_1 - MR_5) + R_5(L_4 - M)} , \quad (31)$$

which, on collecting terms and calling $L_1 + L_4 + 2M = L_0$ (the total inductance in the oscillation circuit), gives

$$\omega^2 = \frac{1}{L_4 C} \times \frac{1 + \frac{R_4}{Z_3}}{1 + \frac{(\mu + 1)(R_1 L_4 + R_4 L_1 - M R_5) + R_5(L_4 - M)}{Z_3 L_0}}. \quad (32)$$

This shows the frequency to be determined largely by the inductance and capacity of the circuit. If all resistances R_1 , R_4 , and R_5 are zero, the equation reduces to

$$\omega^2 = \frac{1}{L_0 C}. \quad (33)$$

These resistances, however, never are zero but are always of such a value as to affect the frequency somewhat. The fraction multiplying $1/L_0 C$ is always less than unity as no values can be given to R_4 , Z_3 , L_1 , and L_0 which will produce a freely oscillatory circuit and make it greater than unity. The denominator contains

$$1 + \frac{R_4}{Z_3} \frac{L_1}{L_0} (\mu + 1), \quad \text{and} \quad \frac{L_1}{L_0} (\mu + 1) \quad \text{and}$$

will always be equal to or greater than unity. This will always make the fraction less than unity. The adding of resistance to any part of the oscillator circuit will therefore increase the value of the denominator and reduce the computed value for the frequency.

The effect of resistance in the circuit is therefore always to make the circuit oscillate at a frequency lower than the resonant frequency of the inductance and capacity. The greater the resistances, the greater this difference. The effect of the amplifying constant is to reduce the frequency still further.

9. OSCILLATION "DAMPING" OR "EXPANDING" TERM.

Returning to equations 22 and 26 we can write the damping constant

$$d_1 = \left[\frac{(\mu + 1)(L_1 L_4 - M^2)}{2 \{ Z_3 L_0 + (\mu + 1)(R_1 L_4 + R_4 L_1 - M R_5) + R_5(L_4 - M) \}} \right] \times \left[\omega^2 - \frac{Z_3(R_1 + R_4 + R_5) + (\mu + 1) \left(R_1 R_4 - \frac{M}{C} \right) + R_4 R_5 + \frac{M + L_4}{C}}{(\mu + 1)(L_1 L_4 - M^2)} \right], \quad (34)$$

where ω^2 is written for its equivalent value Z/X and L_4 is written for $L_1 + L_4 + 2M$.

This equation contains two terms multiplied together. In order that d_1 may become zero or positive, one of these terms must be zero or

positive under certain conditions. The first term in brackets will always be positive. M^2 is always less than L_1L_4 as it is the mutual inductance between L_1 and L_4 and the positive terms in the denominator preponderate. The second term of the equation can be either positive or negative. If it is positive or zero, d_1 will be positive or zero and our condition of free oscillations will occur. We can therefore say that in order to have the condition of free oscillations, that the circuit be constructed such that

$$\omega^2 - \frac{Z_3(R_1 + R_4 + R_5) + (\mu + 1) \left(R_1R_4 - \frac{M}{C} \right) + R_4R_5 + \frac{M + L_4}{C}}{(\mu + 1)(L_1L_4 - M^2)} \geq 0. \quad (35)$$

This equation can be put in a somewhat different form

$$\mu + 1 \geq \frac{Z_3(R_1 + R_4 + R_5) + R_4R_5 + \frac{M}{C} + \frac{L_4}{C}}{\omega^2(L_1L_4 - M^2) - R_1R_4 + \frac{M}{C}} \quad (36)$$

as the condition required in order that free oscillations will occur. The term ω^2 is left in this equation although an exact solution would require the substitution of its value from equation 32. However, the solution for $(\mu + 1)$ is then very much more complicated. At the most, ω varies from the resonant value for the circuit not more than a per cent. or so, and in practical work the error is of no consequence.

Inspection of this equation shows that increasing any resistance in the circuit, tends to stop the oscillations. Also, that an audion with high output impedance Z_3 requires a higher value of μ in order to oscillate in a given circuit and that the high μ audion will oscillate where low μ ones will not, other things being the same.

If we put in the approximate value of $\omega^2 = 1/L_0C$ the equation becomes

$$\mu + 1 \geq \frac{Z_3(R_1 + R_4 + R_5)C + M + L_4 + R_4R_5C}{\frac{L_1L_4 - M^2}{C} - R_1R_4C + M}. \quad (37)$$

This shows that, other things being constant, increasing the capacity C not only lowers the frequency, but increases the value μ must have in order that the circuit will continue to oscillate. In a given circuit, with a given audion if C is increased, one of the resistances must be reduced, or vice versa.

If we assume a circuit has no resistance whatever, a certain proportionment must still be made or free oscillations will not occur. If $R_1 = R_4 = R_5 = 0$, the equation reduces to

$$\mu + 1 \geq \frac{(L_4 + ML_0)}{L_1L_4 - M^2 + ML_0}, \quad (38)$$

which gives

$$\mu \cong \frac{L_4 + M}{L_1 + M}. \quad (39)$$

This means that regardless of there being zero resistance in the oscillation circuit elements, the amplification constant must bear the above relation to the circuit or it will not oscillate. If there is any ohmic resistance, the amplification constant must be greater. If the mutual inductance is zero

$$\mu \cong \frac{L_4}{L_1}, \quad (40)$$

which shows that the ratio of the plate to grid inductances, and therefore the ratio of the reactive voltages on plate and grid must be equal to or less than μ . This relation can also be derived in a simpler way. In an oscillator circuit, the alternating voltage applied on the grid by the oscillation current will necessarily be

$$E_g = \omega L_1 I_0 \quad (41)$$

and on the plate

$$E_p = \omega L_4 I_0, \quad (42)$$

where I_0 is the oscillation current. The driving voltage of the fictitious generator in the tube is then

$$E_G = \mu \omega L_1 I_0 \quad (43)$$

and it must be equal to or greater than the voltage drops in the plate circuit.

$$\mu \omega L_1 I_0 \cong \omega L_4 I_0 + Z_b I_b, \quad (44)$$

where I_b is the alternating space current. This gives the condition

$$\mu \cong \frac{L_4}{L_1} + \frac{Z_b I_b}{\omega L_1 I_0} \quad (45)$$

as the required proportion which agrees with our earlier results, for, if the resistance of the oscillation circuit is zero, the impedance attached to the plate is infinite and I_b must be zero. We have left then the same requirement for free oscillations as is given by equation 40—

$$\mu \cong \frac{L_4}{L_1}.$$

If the mutual inductance between L_1 and L_4 is made zero, equation 37 is further simplified. Experience has shown that the terms $R_4 R_b C$ and $R_1 R_4 C$ are less than .1 per cent. of the other terms and we can further simplify by neglecting them. The equation then becomes

$$\mu + 1 \cong \frac{Z_b(R_1 + R_4 + R_b)C + L_4}{L_1 L_4} L_0, \quad (46)$$

where $L_0 = L_1 + L_4$. This is also equal to

$$\mu \cong Z_3(R_1 + R_4 + R_5) \frac{CL_0}{L_1 L_4} + \frac{L_4}{L_1}. \quad (47)$$

This shows that with all resistance in the circuit zero, the amplification

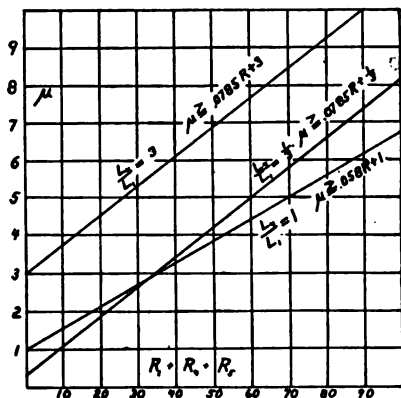


Fig. 4.

Required amplification constant as a function of resistance. Circuit constants in Fig. 5.

constant must still be equal to or greater than L_4/L_1 and that as the resistance is increased, the required value is a straight line function of the total resistance. Curves are plotted in Fig. 4 for the required amplification constant as a function of the total resistance. These curves are for various ratios of L_4/L_1 with $L_1 + L_4$ the same in all cases. It will be observed that the curves for $L_4/L_1 = 3$ and $L_4/L_1 = \frac{1}{3}$ have the same slopes. This means that the amplifying constant required increases with resistance at the same rate in the two cases al-

though it begins at different values. For the condition of $L_4/L_1 = 1$ the rate of increase required is a minimum. This is because $L_1 \times L_4$ is a maximum under this condition.

Equation 47 also shows that in a given case with the ratio between L_4 and L_1 kept constant, the ratio of L_0 to L_4 kept constant, and the product $L_0 \times C$ kept constant, we can increase the total resistance in a circuit and not change the oscillatory conditions provided we increase L_1/C and therefore L_0/C in the same ratio. If therefore in a given case, it is necessary to vary the resistance and it is desired to have the same degree of oscillation, this can be accomplished by changing L_0 and C in such a manner that their ratio changes in the same proportion as the resistance, keeping, however, the product $L_0 \times C$ and the quotient L_4/L_1 the same.

In Fig. 5 are two curves showing the resistance required to stop oscillators having similar circuit and audion constants but different amplifying constants. If $\mu = 10$, it requires a much larger resistance in a given case than if $\mu = 2$. The curves are plotted for required resistance as a function of that part of the total inductance that is in the plate circuit. This shows that with L_1 and L_4 approximately the same in value, the circuit will oscillate with a greater resistance than with other adjustments. When the constants of an audion are known, the

value of L_4 which can produce oscillations with a maximum resistance can be secured from the maximum value of equation 47. Solving equation 47 for R

$$R_1 + R_4 + R_5 = \frac{\mu L_0 L_4 - L_4^2(\mu + 1)}{Z_s C L_0}, \quad (48)$$

where $L_0 - L_4$ is substituted for L_1 . The curves in Fig. 5 are plotted

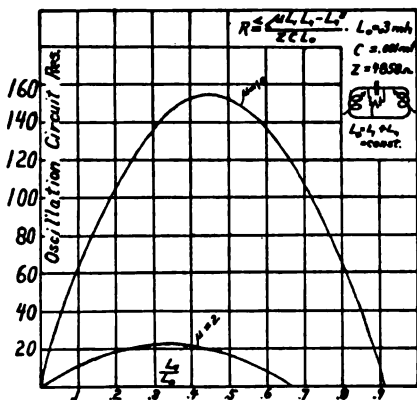


Fig. 5.

Resistance required to stop oscillations.

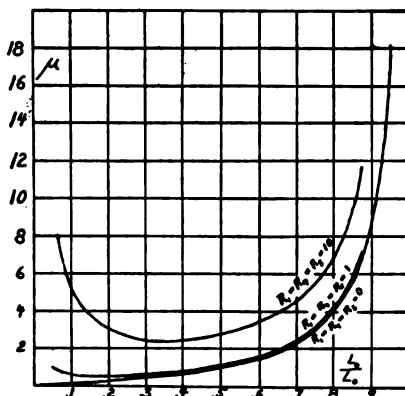


Fig. 6.

Amplification constant required as a function of L_4/L_0 for free oscillations.

from the above equation with L_4 plotted in percentage of L_0 . On differentiating with respect to L_4 and equating to zero we get

$$\frac{L_4}{L_0} = \frac{\mu}{2(\mu + 1)}. \quad (49)$$

This equation enables us to get the best coupling for working into high resistance circuits where we wish to have the greatest reliability in oscillation.

This does not mean, however, that under these conditions the audion will deliver the maximum power. On the contrary, this is not the case as other things enter into that. It only means that oscillations will occur in a circuit of higher resistance with this relation occurring than with any other arrangement.

Figure 6 contains curves indicating the values of μ required for various ratios of L_4/L_0 for three cases of R_1 , R_4 , and R_5 each equal to 0, 1 and 10 ohms in the three cases respectively. As indicated by the curves, a minimum value of μ is required in a given case if the value of L_4 is chosen properly. The value of L_4 at which the minimum μ is required can be

secured from equation 50, which is secured by differentiating 47 with respect to L_4

$$L_4 = Z_3(R_1 + R_4 + R_3)C \left[\sqrt{1 + \frac{L_0}{Z_3(R_1 + R_4 + R_3)C}} - 1 \right]. \quad (50)$$

10. FORM OF EXPANDING CONSTANT d_1 .

In Fig. 7 a Hartley circuit is given with audion and circuit constants such as can occur in practice. The resistance of the condenser R_s is

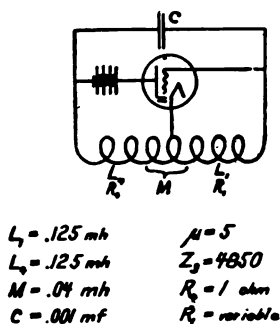


Fig. 7.

Circuit and constants for Figs. 8, 9 and 10.

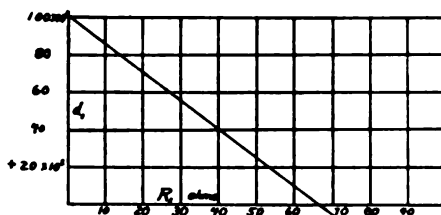


Fig. 8.

Variation of "undamping" constant d_1 with resistance.

assumed as zero as it usually is small compared to other resistances, and the value of R_4 is given as 1 ohm. Curves are given in Figs. 8, 9, and 10 of d_1 , ω^2 , and d_2 as a function of R_1 . R_1 and R_4 enter into the equation under these conditions in exactly similar ways and the curves can be assumed as a function of R_4 with R_1 as one ohm just as well.

Figure 8 gives d_1 as a function of R_1 and shows it to be almost a straight-line function. In fact the variation is too small to see on a curve. If we neglect most of the resistance terms in equation 34 and write

$$d_1 = \frac{(\mu + 1)(L_1 L_4 - M^2)}{2Z_3 L_0} \left[\omega^2 - \frac{Z_3(R_1 + R_4) + \frac{L_4}{C} + \frac{M}{C} - (\mu + 1)\frac{M}{C}}{(\mu + 1)(L_1 L_4 - M^2)} \right], \quad (51)$$

we have an equation which is a straight line with R_1 or R_4 as the variable. The error in the above is about 4 per cent. in this case with $R_1 = 70$ ohms and proportionally smaller if R_1 is smaller. The curve as computed and plotted is from equation (34). The curve shows that 66.5 ohms for R_1 is the critical resistance of the circuit. For values below 66.5 ohms, d_1 is positive and free oscillations occur. For values above 66.5 ohms, d_1 is negative and oscillations die out. The circuit is then said to be stable or not freely oscillatory. If R_1 is of the order of zero or 1 ohm, the ex-

panding constant d_1 is 80,000. In such a case in one eighty-thousandth of a second, the oscillations will increase to e times their value and keep on increasing in the same manner. In one forty-thousandth of a second, they will increase to e^2 and in one ten-thousandth of a second to e^4 ($= 3,000$) times the starting value. This increasing does not continue indefinitely as the characteristic curve departs from a straight line. Increasing R_1 reduces d_1 until it changes sign at $R_1 = 66.5$ ohms. It is possible to give R_1 such a value, close to the critical value and above it, that the oscillations will be only slightly damped and the initial oscillation produced by closing the switch will persist for several seconds. The

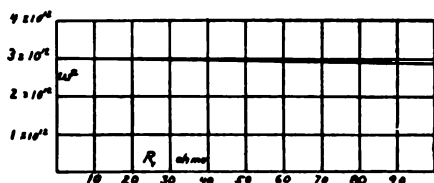


Fig. 9.

Variation of ω^2 with resistance.

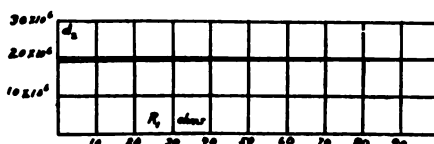


Fig. 10.

Variation of damping constant d_2 with resistance.

sensitivity of the circuit then to a little resistance such as can be introduced by bringing a coin or other piece of metal near to the circuit, is sharply marked.

11. FORM OF THE FREQUENCY TERM.

The value of ω^2 is shown in Fig. 9. At $R_1 = 0$ it is very close to $1/L_0C$ and would have that value if R_4 equaled zero also. Increasing R_1 lowers the frequency and at the critical value it is 1.6 per cent. lower.

12. D.C. DAMPING CONSTANT.

Figure 10 shows the damping constant d_2 as a function of R_1 . The predominating resistance term in this equation (27) is Z_3 , and R_1 and R_4 affect it but slightly. Their influence, however, is to raise the value a few per cent. (2.5 per cent.) between $R_1 = 0$ and the critical value. This constant is always negative when conditions are such as to produce oscillations and is of such a large value that it has ceased to influence the circuit long before the oscillations have changed appreciably.

13. RANGE FOR ACCURATE USE.

In securing equations 26 from equations 25 it was assumed that $2d_1$ was small in comparison to d_2 , and that d_1^2 was small compared to ω^2 , and to $\omega^2 + 2d_1d_2$. The order of magnitude of these values for $R_1 = 1$

in the circuit in Fig. 7 are

$$\left. \begin{aligned} d_2 &= -19. \quad \times 10^6, \\ d_1 &= +.1 \quad \times 10^6, \\ \omega^2 &= +3.027 \times 10^{12}. \end{aligned} \right\} \quad (52)$$

The first assumption that d_1 was negligible in comparison to d_2 , produces an error in d_2 of 1 in 190 for the most highly oscillatory condition and less than that for other conditions in the oscillatory range. The error produced by neglecting d_1^2 in comparison with ω^2 is 1 in 300 under the same conditions and less toward the critical condition. The error produced by neglecting d_1^2 in comparison with $\omega^2 + 2d_1d_2$ is 1 in 490 under the same conditions. We thus see that our assumptions were justified and that the solutions secured are not only accurate at and around the critical point, but that the errors introduced when using the solutions over the rest of the oscillatory range covered by the parabolic equation are comparable with the errors of measurement of the circuit constants.

14. STARTING OF OSCILLATIONS.

The manner in which an oscillator starts may be described as follows: On throwing on the plate voltage, the space current builds up according to the form $I_p (1 - e^{d_1 t})$ and a transient oscillation is started in the oscillation circuit. This transient, instead of dying out as in ordinary oscillatory circuits, builds up, on account of the circuit damping constant d_1 being positive.

In a damped oscillatory circuit, the oscillations, theoretically, never become zero. In a circuit of this kind capable of oscillating, it is theoretically possible to start the plate current through the audion and have no oscillations, and attach the circuit afterward and make it take an infinitely long time for the oscillations to build up. As long as no transient starts in the circuit, no oscillations will result. The condition is equivalent to that of balancing an object on a sharp point. Such an object will never fall until something starts it. Similarly the audion oscillator circuit will never oscillate until something starts it. Practically this condition is as impossible to attain as is the mechanical analogue given because there are all kinds of electrical disturbances traveling in the ether which will start it off the same as external mechanical disturbances would start off the unstable mechanical analogue. The oscillations once started, no matter how small, will increase with time until the stage of full oscillation is reached.

The manner in which the current builds up in an oscillator is shown in Figs. 11 and 12.

Figure 11 shows the space current building up with the first closing of the switch and the subsequent alternating current as the oscillations increase. At the place where the saturation point of the filament, and the bottom of the curve come into play due to the oscillations having built up, the logarithmic increase according to these equations ceases and the space alternating current begins to assume a constant value with

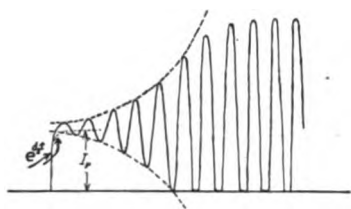


Fig. 11.

Space current of starting oscillator.

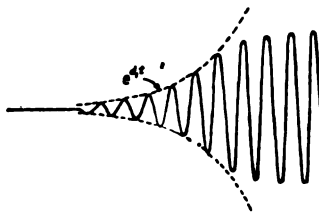


Fig. 12.

Oscillation current of starting oscillator.

the peaks flattened. At the same time the oscillation current in Fig. 12 also stops increasing logarithmically and approaches a constant value which is the steady state condition.

15. OTHER REQUIREMENTS INDICATED.

If we take equation 18, which gives the requirements for oscillation, and make the mutual reactance between plate and grid circuits zero ($W = 0$) we have

$$Z_3(Z_1 + Z_4 + Z_5) + Z_4(Z_5 + (\mu + 1)Z_1) = 0. \quad (53)$$

If all resistances are zero, the above equation divides into two parts, viz:

$$Z_4(Z_5 + (\mu + 1)Z_1) = 0, \quad (54)$$

in which all terms are purely imaginary (except μ) and the product therefore real, and

$$Z_3(Z_1 + Z_4 + Z_5) = 0, \quad (55)$$

in which Z_3 is a pure resistance, Z_1 , Z_4 and Z_5 are pure reactances. The product is therefore imaginary. For (53) to be zero, both 54 and 55 must separately be equal to zero. Z_4 and Z_3 cannot have zero values in a freely oscillatory circuit and therefore the parenthetical terms must be zero. This means in 54 that either Z_5 or Z_1 must be negative—that is a capacitive reactance—while the other is inductance. Comparing, or substituting 54 in 55 show that $Z_4 = \mu Z_1$ and is of the same sign. That means, if a circuit is constructed with the three elements Z_1 , Z_4 , and Z_5 as shown in Fig. 13, that if Z_1 is an inductive reactance, Z_4 must also be an inductive reactance while Z_5 must be a capacitive reactance or, if Z_1

is a capacitive reactance, Z_4 must also be a capacitive reactance and Z_5 must be an inductive reactance. Phase relations are then such that sustained oscillations will occur.

16. EQUATIONS FOR COLPITTS' CIRCUIT (FIG. 14).

The same equations can be applied to Colpitts' circuit and similar solutions result. The circuit is similar in form to the Hartley circuit

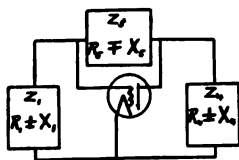


Fig. 13.

Schematic diagram of an oscillator. •

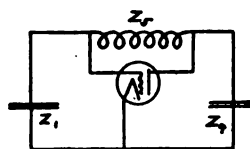


Fig. 14.

Colpitts oscillator circuit.

but has inductances instead of capacities and vice-versa. This circuit, however, has no mutual between Z_1 and Z_4 . Rewriting equation 18 with W equal to zero,

$$Z_3(Z_1 + Z_4 + Z_5) + (\mu + 1)Z_1Z_4 + Z_4Z_5 = 0. \quad (56)$$

Putting in the values for Z_1 , Z_4 , and Z_5 to fit Fig. 14,

$$\left. \begin{aligned} Z_1 &= R_1 + \frac{1}{aC_1}, \\ Z_4 &= R_4 + \frac{1}{aC_4}, \\ Z_5 &= R_5 + aL_5, \end{aligned} \right\} \quad (57)$$

we have

$$\begin{aligned} Z_3 \left(R_1 + R_4 + R_5 + \frac{1}{aC_1} + \frac{1}{aC_4} + aL_5 \right) + (\mu + 1) \left[\frac{R_1}{aC_4} + \frac{R_4}{aC_1} + R_1R_4 + \frac{1}{a^2C_1C_4} \right] \\ + R_4R_5 + \frac{L_5}{C_4} + R_4L_5a + \frac{R_5}{aC_4} = 0. \end{aligned} \quad (58)$$

Multiplying through by a^2

$$\begin{aligned} Z_3 \left[(R_1 + R_4 + R_5)a^2 + a^3L_5 + \frac{a}{C_1} + \frac{a}{C_4} \right] + (\mu + 1) \left[a^2R_1R_4 + \frac{1}{C_1C_4} \right. \\ \left. + \frac{aR_1}{C_4} + \frac{aR_4}{C_1} \right] + a^2 \left(R_4R_5 + \frac{L_5}{C_4} \right) + a^3R_4L_5 + \frac{aR_5}{C_4} = 0. \end{aligned} \quad (59)$$

Collecting in descending powers of a ,

$$a^3(R_4 + Z_3)L_5 + a^2 \left[Z_3(R_1 + R_4 + R_5) + R_4R_5 + \frac{L_5}{C_4} + (\mu + 1)R_1R_4 \right] \\ + a \left[\left(\frac{1}{C_1} + \frac{1}{C_4} \right) Z_3 + (\mu + 1) \left(\frac{R_1}{C_4} + \frac{R_4}{C_1} \right) + \frac{R_5}{C_4} \right] + (\mu + 1) \frac{1}{C_1C_4} = 0, \quad (60)$$

which has the same general form as equation 21

$$a^3 + a^2x + ay + z = 0, \quad (61)$$

where

$$\left. \begin{aligned} x &= \frac{Z_3(R_1 + R_4 + R_5) + R_4R_5 + \frac{L_5}{C_4} + (\mu + 1)R_1R_4}{(R_4 + Z_3)L_5}, \\ y &= \frac{Z_3 \left(\frac{1}{C_1} + \frac{1}{C_4} \right) + (\mu + 1) \left(\frac{R_1}{C_4} + \frac{R_4}{C_1} \right) + \frac{R_5}{C_4}}{(R_4 + Z_3)L_5}, \\ z &= \frac{(\mu + 1) \frac{1}{C_1C_4}}{(R_4 + Z_3)L_5}. \end{aligned} \right\} \quad (62)$$

The solution of a in this equation is somewhat different from that of the Hartley circuit. Referring back to equations 24, 25, and 26, it will be remembered that we neglected d_1 in comparison with d_2 , and d_1^2 in comparison with ω^2 . In this circuit, the values of d_1 and d_2 are such that we cannot neglect d_1 in comparison with d_2 but we can neglect d_1^2 and $2d_1d_2$ in comparison with ω^2 . This gives us

$$\left. \begin{aligned} \omega^2 &= y, \\ d_2 &= -\frac{z}{\omega^2}, \\ d_1 &= \frac{1}{2}(-x - d_2) = \frac{1}{2} \left(\frac{z}{\omega^2} - x \right). \end{aligned} \right\} \quad (63)$$

Inspection of d_2 shows that it will always be a negative quantity which is to be expected since it is concerned with the direct current transients in the circuit. If we write the approximate value of ω^2 (which holds when $R_1 = R_4 = R_5 = 0$) we have

$$d_2 = -\frac{(\mu + 1)}{(R_4 + Z_3)(C_1 + C_4)}. \quad (64)$$

If μ is zero, this reduces to

$$d_2 = -\frac{1}{(R_4 + Z_3)(C_1 + C_4)}. \quad (65)$$

which is the decrement in a circuit containing a capacity of $C_1 + C_4$

and a resistance of $R_4 + Z_3$. The resistance R_4 is not in series with both capacities C_1 and C_4 but in the assumption of $\omega^2 = 1/L_5 C_0$ it was assumed that R_4 was zero which accounts for the discrepancy. On making $R_4 = 0$ throughout as it should be to make the assumption hold, the equation assumes the exact and correct form for these conditions.

The effective capacity of C_1 and C_4 in series is called C_0 in the following equations and equals $(C_1 C_4)/(C_1 + C_4)$.

The value of ω^2 from equations 62 and 63 is

$$\omega^2 = \frac{1}{L_5 C_0 \left(1 + \frac{R_4}{Z_3}\right)} + \frac{(\mu + 1) \left(\frac{R_4}{C_1} + \frac{R_1}{C_4}\right) + \frac{R_5}{C_4}}{(R_4 + Z_3)L_5}. \quad (66)$$

This equation shows that the frequency is determined by the oscillation circuit L_5 , C_1 and C_4 in series and that if R_1 , R_4 , and R_5 are zero, the frequency is exactly the resonant frequency. Varying the resistances causes the frequency to vary around the resonant value. Raising resistance R_1 raises the frequency. Raising resistance R_5 will also raise the frequency but not in the same proportion as R_1 will. Increasing R_4 may either raise or lower the frequency depending upon the other circuit constants. This circuit operates at a frequency above the resonant value. The usual circuit constants when operating on an antenna are such as will cause the frequency to raise as R_1 is increased but drop if R_4 is increased.

The oscillation damping factor is

$$d_1 = \frac{1}{2} \left[\frac{(\mu + 1) \frac{1}{C_1 C_4}}{(R_4 + Z_3)L_5 \omega^2} - \frac{Z_3(R_1 + R_4 + R_5) + R_4 R_5 + \frac{L_5}{C_4} + (\mu + 1)R_1 R_4}{(R_4 + Z_3)L_5} \right] \quad (67)$$

which can be either positive or negative. For free oscillations, the equation must give a positive value for d_1 or

$$\frac{\mu + 1}{C_1 C_4 \omega^2} \geq Z_3(R_1 + R_4 + R_5) + R_4 R_5 + \frac{L_5}{C_4} + (\mu + 1)R_1 R_4. \quad (68)$$

Rearranging,

$$(\mu + 1) \left(\frac{1}{C_1 C_4 \omega^2} - R_1 R_4 \right) \geq Z_3(R_1 + R_4 + R_5) + R_4 R_5 + \frac{L_5}{C_4} \quad (69)$$

or

$$\mu + 1 \geq \frac{\frac{L_5}{C_4} + Z_3(R_1 + R_4 + R_5) + R_4 R_5}{\frac{1}{C_1 C_4 \omega^2} - R_1 R_4}. \quad (70)$$

This equation shows that increasing the resistances raises the value of μ necessary for free oscillations. If we put in the approximate value of ω^2

$$\mu + 1 \cong \frac{\frac{L_5}{C_4} + Z_3(R_1 + R_4 + R_5) + R_4R_5}{\frac{L_5}{C_1 + C_4} - R_1R_4} \quad (71)$$

and it becomes evident that the increasing of the resistances can be compensated for by increasing the ratio of L_5 to $(C_1 + C_4)$. That is, when R_1 , R_4 , or R_5 are increased, C_1 and C_4 must be both decreased. Decreasing C_1 alone will compensate to a certain extent but when it gets small compared to C_4 it has little influence. Increasing L_5 will help as $L_5/(C_1 + C_4)$ is the predominating denominator term while L_5/C_4 is of the order of magnitude of the remainder of the numerator.

If the resistances in the circuit are all zero, we have the condition

$$\mu \cong \frac{C_1}{C_4} \quad (72)$$

as the requirement for oscillation. This holds even though the circuit dissipates no energy. It corresponds with the requirement of a zero resistance Hartley circuit as given equation (40).

This type of circuit operates at a frequency slightly above the resonant frequency. The reduction in frequency due to the term $1 + (R_4/Z_3)$ is less than the increase due to the added term.

If we neglect the small term in equation 71 we can say

$$\mu + 1 \cong \frac{\frac{L_5}{C_4} + Z_3(R_1 + R_4 + R_5)}{\frac{L_5}{C_1 + C_4}}, \quad (73)$$

$$\mu \cong Z_3(R_1 + R_4 + R_5) \frac{C_1C_4}{L_5} \left(\frac{C_1C_4}{C_1C_4} \right) + \frac{C_1}{C_4}, \quad (74)$$

and calling C_0 the effective capacity of C_1 and C_4 in series.

$$\mu \cong Z_3(R_1 + R_4 + R_5) \frac{C_1C_4}{L_5C_0} + \frac{C_1}{C_4}, \quad (75)$$

which is exactly similar to equation 47 in form but has the reciprocal of a capacity wherever equation 47 has an inductance and vice versa. Curves 4, 5, and 6 for the Hartley circuit are therefore applicable to this equation for the Colpitts' circuit if we substitute $1/C_1$ for L_1 , $1/C_4$ for L_4 , $1/C_0$ for L_0 and $1/C_5$ for L_5 .

RESEARCH DEPARTMENT OF

THE AMERICAN TELEPHONE AND TELEGRAPH CO.

AND WESTERN ELECTRIC CO., INC.,

July, 1919.

AMPLIFICATION OF ELECTRIC CURRENTS IN THE
BUNSEN FLAME.

BY C. W. HEAPS.

SYNOPSIS.

Modulation of Electric Currents in a Bunsen Flame; the Use of a Third Electrode.

—When an electric current is sent through a Bunsen flame, entering and leaving the flame by platinum terminals, it has been found by the author that amplification effects similar to those in vacuum tubes may be secured by the use of a third electrode or grid placed near the lime-coated cathode in the flame. The potential of this grid is varied with respect to the cathode and direct current characteristic curves are given showing the current as a function of the grid potential. These curves are similar in appearance to those obtained with the audion, and may be utilized in the customary way for calculating the *amplification constants*. The factors influencing these amplification constants are discussed and it is found that under what are probably the most favorable conditions of the apparatus used, the voltage amplification is but little greater than unity. The power amplification is about 108 and the current amplification about 101. *The theory* of the action is outlined in a general way, the effectiveness of the grid being ascribed to its retarding influence on the electron emission of the cathode. •A consequent change in the cathode fall of potential alters the current through the flame. *The utility* of the device is not comparable with that of the vacuum tube amplifiers, largely because of the difficulty of securing permanent flame conditions. The energy output of the simple device used was also necessarily small because of the high flame resistance. It could be used, however, for the detection of electric waves.

THE phenomena associated with the passage of an electric current through a Bunsen flame have been extensively studied by H. A. Wilson and others. It is found that if two clean platinum wires are used as electrodes in the flame, and if they are about equally heated, then the potential gradient in the flame between the electrodes is by no means uniform. In general there is a small fall of potential in passing from the positive electrode into the gas, a uniform and very small potential gradient in the region midway between the electrodes, and a very large drop of potential in the immediate neighborhood of the negative electrode.¹ Practically all the fall of potential in the flame is found near the negative terminal. This condition of affairs is to be expected in an ionized gas if the negative ions are much more mobile than the positive ions. The cathode fall of potential is caused by a deficiency of negative ions in the gas around the cathode, and the net result of this deficiency

¹ H. A. Wilson: *Electrical Properties of Flames and Incandescent Solids*, p. 61.

is that the greater part of the resistance to the passage of the current is located very near the cathode.

Now when salt or lime is put on the hot cathode the drop of potential there is very much diminished and the potential gradient in the flame becomes much more uniform. Apparently the heated salt or lime by emitting electrons does away with the deficiency of negative ions in the gas near the cathode, with the result that there is a great decrease in the resistance of the flame to the passage of an electric current. The current through the flame may be increased as much as a hundred fold by this application of lime. It appears evident, therefore, that if one could regulate in a simple fashion the electron emission of the hot lime, then very large changes in the magnitude of the current through the flame could be effected. This regulation of electron emission from a hot wire is accomplished very effectively in the three electrode vacuum tubes of the audion type by a grid, or third electrode, the potential of which is adjusted by an outside battery. In this paper are described the results of using a third electrode for changing the magnitude of the electric current through a Bunsen flame.

Preliminary experiments were made with the electrodes all in the form of straight wires placed parallel with each other and in a vertical plane. The top wire was made the cathode and sealing wax was burned on it. The residue of burned sealing wax contains calcium and barium oxides, is tightly adherent to the wire, and serves as an efficient source of electrons when heated. The straight wire serving as grid was placed below the cathode, having first been cleaned by boiling in hydrochloric acid. This grid was set very close to the cathode, its effectiveness being much greater when so placed. However, it was found that when the distance between grid and cathode was less than about half a millimeter particles from the sealing wax deposit collected on the grid when the wires were placed in the flame. As the grid functioned properly only when clean it could not, therefore, be placed too close to the cathode. The third wire, the anode, was a short distance below the grid.

The anode and cathode were now connected in series with a galvanometer, a resistance, and a battery of dry cells. The grid was also connected through a battery and galvanometer to the cathode. The terminals were then placed in the vertical flame from a Meker burner. This burner was supplied with a mixture of gasoline vapor and air, the air being furnished by a compressor outside the room. A flame produced in this way is steadier and more homogeneous than when air from the room is taken in at the base of the burner by the usual method, because the entrance of dust and impurities into the flame is to some extent

prevented. The whole arrangement of apparatus is shown in Fig. 1. The galvanometer G_1 had a resistance of 240 ohms and gave a deflection of one division for a current of two microamperes. The galvanometer G deflected one division for a current of 3.3×10^{-7} amperes and had a resistance of 235 ohms. The resistance R could be adjusted by steps of 10^8 ohms from 0 to 2 megohms. The component parts of the electrical system were insulated from the earth by ebonite blocks.

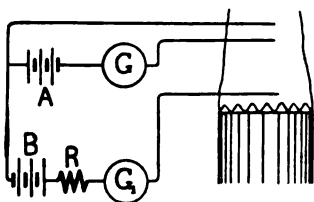


Fig. 1.

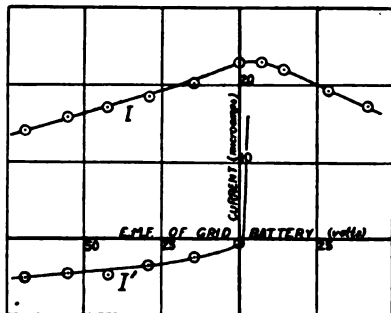


Fig. 2.

Fig. 2 shows one of a set of curves obtained with this apparatus. The grid was 1.5 mm. below the cathode and the anode 3 mm. below the grid. The battery B had a constant electromotive force of 56.5 volts and the resistance R a constant value of 2 megohms. The current flowing to the anode is plotted as ordinates in curve I and the current to the grid as ordinates in curve I' . Values of the grid current were multiplied by 10 before being plotted and the grid current is considered as negative when it flows from grid to cathode through the galvanometer G . Similar curves obtained when $R = 0$ show a greater effect of the grid potential on the main current to the anode than is indicated by Fig. 2.

From inspection of the curves it appears that the applying of a negative potential to the grid has the effect of decreasing the main current and increasing the grid current. Such an effect can be secured only when the grid is free from lime, as in this case there is a large resistance to the passage of a current into the grid. Furthermore, with a clean grid the current produced in the grid circuit does not increase proportionally with the E.M.F. of battery A , but at a slower rate if this E.M.F. is fairly large. This fact is a well known phenomenon in the case of flame conduction.¹ However, as soon as the grid becomes sufficiently positive with respect to the cathode the grid current changes in direction and increases very rapidly with the grid potential. At this point the current

¹ Electrical Properties of Flames, p. 59.

flowing to the anode begins to diminish,—a result to be expected as a consequence of the deviation of current from the cathode through the grid into the flame; for when the grid is positive with respect to the cathode it functions as an anode, hence manifesting a comparatively small resistance to the passage of a current.

It is apparent from the curve that a small change of current input causes a larger change in the current output, hence the device may be said to function as a current relay, or amplifier. If the effectiveness of the relay action is due to the retarding action of the grid potential on the electrons emitted from the cathode one would expect that another form might be given this third electrode so as to increase its effectiveness. The more completely the cathode is surrounded by the grid the greater should be the influence of the latter. However, the following points are to be considered. First, no part of the grid may be above the cathode, for in this position particles from the sealing wax deposit are carried upward and lodged on the grid, thus impairing its effectiveness as explained above. Second, the immediate proximity of a mass of metal exercises a cooling effect on the cathode, in which event the current flowing to it is considerably diminished. In view of these facts the following arrangement was adopted. The tip of the cathode was bent upward, the bend being made about half a centimeter from the end, and the sealing wax deposit confined to the tip of this upturned wire. A spiral coil of platinum wire, containing three turns, about 0.7 cm. long and 0.65 cm. in diameter was placed concentrically around the upturned tip of the cathode. The anode was placed 1 cm. below this combination. The electric circuits outside the flame were set up as in Fig. 1.

Some characteristic curves obtained with this arrangement are shown in Fig. 3. Here, as before, the grid current, multiplied by 10 before being plotted, is given by curves *I'* and *II'*. The curves giving the current to the anode are correspondingly marked *I* and *II*. Each point on the curves *II* and *II'* represents the mean of two separate observations—the first set of readings being taken as the grid battery was being increased

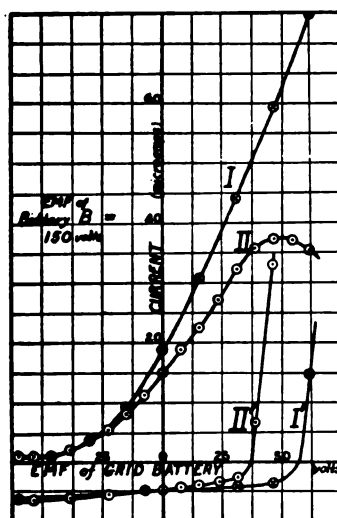


Fig. 3.

to its maximum E.M.F., the second set as it was being decreased. This method of observation was adopted in order to correct for slow steady changes which might occur in the nature of the flame or the lime deposit on the cathode. Such changes are difficult to prevent but in this case were found to be sufficiently small so that only one set of readings was taken for curves *I* and *I'*. For these curves the external resistance was that of the galvanometer, *i.e.*, 240 ohms; for *II* and *II'* a resistance of 2 megohms was added. Curve *I* bends downward in a fashion similar to *II* when the E.M.F. of the grid battery is slightly greater than 60 volts. A comparison of Fig. 3 with Fig. 2 shows very clearly that the spiral type of grid is more effective in modifying the main current than the single straight wire. Indeed the general form of the curves of Fig. 3 is strikingly similar to the well known characteristic curves of vacuum tube amplifiers. An important difference, however, lies in the failure of the main current curves to approach so closely the axis of grid E.M.F. as is the case with the audion. It is impossible in a flame amplifier for the main current to be reduced below a certain minimum by altering the grid potential because the grid current is always an essential part of the main current, and furthermore, the grid current is always appreciable.

In calculating the amplifying factors of this arrangement the same methods may be used as are practiced with vacuum tube amplifiers. The current is a function both of the E.M.F. of the grid battery and of the main battery *B*, so that the function expressing the relationship between these three variables will be the equation of a surface, called the characteristic surface. When a current flows between two clean electrodes in a flame it has been shown¹ by H. A. Wilson that the potential difference between the terminals is given by $E = aI + bI^2$ where *I* is the current and *a* and *b* are constants. If the action of the grid battery, of E.M.F. *V*, is to introduce an E.M.F. in the main circuit, we might expect the functional relation between the three variables to be of the type

$$E + kV = aI + bI^2,$$

where *E* is the E.M.F. of battery *B*. Obviously this equation does not fit the curves of Fig. 3 over their entire length unless *k* is a variable. However, if we confine our operations to a region on the characteristic surface which is sensibly plane the equation may be written in the form

$$E + kV = aI.$$

If *E* is kept constant and *V* varied we have $k/\delta V = a\delta I$. The constant *a* may be considered as equal to $R + R_0$, where *R*₀ is the resistance of

¹ Electrical Properties of Flames, p. 62.

the flame between anode and cathode and R is the external resistance. The amplifying factor k is thus given by

$$k = (R + R_0) \frac{\delta I}{\delta V}$$

and its value may be obtained by reference to the curves of Fig. 3. The change, δI , in the main current is associated with a change δi in the grid current—hence the current amplification may be defined as $\xi = \delta I / \delta i$. Obviously the change δV in the grid battery will produce a change $R\delta I$ in the potential drop across the external resistance. The ratio $\mu = R\delta I / \delta V$ may be called the voltage amplification. The power developed in R by this potential change is $R\delta I^2$. The power supplied by the grid is $\delta V \delta i$. The power amplification is thus $\eta = R\delta I^2 / \delta V \delta i$. The above quantities may be expressed in the form:

$$\xi = \frac{\delta I}{\delta i}, \quad \mu = \frac{kR}{R + R_0}, \quad \eta = \frac{R\xi k}{R + R_0}.$$

In calculating the numerical values of these expressions it will be convenient to let $\delta V = 25$ volts, the initial value of V being zero. Corresponding values, then, of the other variables are for curves I and I' : $\delta I = 20.7 \times 10^{-6}$, $\delta i = -0.56 \times 10^{-7}$, $R = 240$. For curves II and II' we have $\delta I = 13.4 \times 10^{-6}$, $\delta i = -1.32 \times 10^{-7}$ and $R = 2 \times 10^6$. The value of R_0 is about 8×10^6 ohms for both curves. Substituting these numbers in the equations we get

	$R = 240.$	$R = 2 \times 10^6.$
k	6.62	5.36
ξ	370	101.5
μ	1.98×10^{-4}	1.07
η	7.32×10^{-3}	108.6

There is thus no voltage or power amplification unless the external resistance is large. Even then the value of μ is small as compared with μ for a vacuum tube amplifier. However, in a flame a large value is not to be expected for the following reason. The action of the grid is supposed to consist in retarding or accelerating the electron emission of the cathode; hence, in order to be effective the grid must be instrumental in setting up an electric field of the proper value at the immediate surface of the cathode. Now throughout the range of the curves of Fig. 3 where amplification is apparent a small current is flowing from the flame into the grid, and since the grid is free from lime there will be, therefore, a big fall of potential at the grid. A much larger current flows from the

flame into the cathode as a result of the much smaller potential gradient at this terminal. Suppose, now, that the grid is given a greater negative potential by increasing the E.M.F. of battery *A* by an amount δV . Since the fall of potential at the grid is caused by the driving of negative ions out of the region around the grid one effect of δV would be to extend the region in which there is a scarcity of negative ions. For a sufficiently large δV this region might be extended to include the cathode, with the result that the electron emission of the latter is retarded. The current to the cathode would thus be diminished. Evidence in favor of this view is found in the fact that the action of the grid is enfeebled by taking it farther away from the cathode. The potential change, δV , does not, therefore, produce a proportional potential gradient change at the cathode; we should expect this δV to be distributed unevenly over a more extended region, a greater amount being located near the grid than at a distance from it. The net result of such a condition would be that δV must be fairly large to produce much of an effect. A large δV means a small voltage amplification. However, the change, δV , does not produce a big change of grid current so the power supplied may be small, and power amplification is thus possible.

In order to test out further the above theory of grid action some experiments were made with a lime-coated grid. Suppose that in some way the magnitude of the current taken in through the grid were a direct factor in diminishing the main current and that the potential

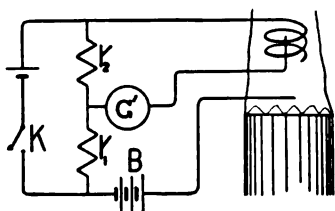


Fig. 4.

drop at the grid were not an important factor. Then a lime-coated grid should function even more effectively than a clean one because in this case much smaller values of δV are necessary to produce a given change in the grid current. However, experiments instituted to detect such an action failed to give any evidence in support of such a view.

A given grid current change, δi , in the case of a lime coated grid is associated with a very much smaller change of main current than is the case when the grid is clean.

An ingenious method has been devised by Miller¹ for determining k experimentally for the audion. His method as adapted for use with the flame is illustrated by Fig. 4. Here the resistances r_1 and r_2 are small compared with the flame resistance so that when the key *K* is closed E.M.F.'s are introduced into the grid circuit and the anode circuit which

¹ Proc. Inst. Radio Engineers, 6, p. 141, 1918.

are proportional, respectively, to r_2 and r_1 . If these two resistances are adjusted so that the galvanometer G' is unaffected by the closing of K their ratio gives the value of k . Experimentally it was found convenient to make $r_2 = 100$ and then vary r_1 . The values of k obtained were as follows:

E.M.F. of B.	Current Through G' .	k .
27.8 volts	4.7 microamps.	2.4
54.7 "	12.5 "	4.0
95.0 "	16.5 "	6.0

The arrangement of electrodes and grid had been disturbed before these measurements were made so that these values of k cannot be expected to compare with those obtained from the curves of Fig. 3. Also, as the E.M.F. of battery B was increased the accuracy of measurement by the above method was considerably decreased. It is to be noted, however, that an increase of E.M.F. in the main circuit produces an increase of k , though some later experiments indicated clearly that there is little to be gained in amplification by having battery B greater than 150 volts. The internal resistance R_0 is increased and the slope of the characteristic curve decreased when an E.M.F. of 580 volts is used in the main circuit. These results are probably produced by the setting up of larger potential gradients at the terminals. In a previous paper¹ experiments are described showing that the gradient increases more rapidly at the anode than at the lime-coated cathode when the E.M.F. is increased, and that with the appearance of the large gradient at the anode the current does not increase so rapidly. The flame resistance would thus be increased; and the fact that the gradient at the cathode increases explains why the grid would require a larger potential change to produce a given current change.

It was found that when the flame was made hotter by the admission of more air the current through G' was increased. The value of k , however, was made smaller. Such an effect is to be expected if the theory of the action of the grid is as outlined above. For when the cathode is raised to a higher temperature we may expect more electrons to be emitted, thus diminishing still further the cathode fall of potential and hence increasing the current. But we also suppose the average velocity of the electrons emitted to be increased by the rise of temperature. Hence, to retard the emission by a given amount the grid requires a greater negative potential than was necessary before the increased heating.

¹ PHYS. REV., 7, 1916, p. 663.

The direct current characteristics of the flame amplifier may be determined as above in a comparatively easy manner, but owing to the smallness of the currents involved there is some difficulty in making quantitative measurements using alternating grid potentials. A sensitive telephone may be used as indicator but care is necessary if capacity effects are to be avoided. The question also arises regarding the presence of a time lag in the action of the grid. Such a lag is entirely inappreciable in vacuum tube amplifiers, but there is reason for searching for such an effect in a flame amplifier. When a battery, telephone, and flame are connected in series and the current made by closing a key the click in the telephone is less sharp and distinct than is the case if for the flame a pure resistance of equal magnitude is substituted. Apparently the current does not build up through a flame in the same fashion as through a pure resistance when the E.M.F. is applied. Indeed, one would not expect this to be the case, for a flame has been shown to behave like a capacity for high frequency electrical oscillations.¹

These considerations seemed to make advisable a study of the time relations involved in the making and breaking of electric currents in a flame. A string galvanometer was accordingly inserted at *G* (Fig. 1) and a number of photographs made using a photographic film mounted on a revolving drum, showing the motion of the galvanometer string as the grid potential was changed. Comparison photographs were also obtained when the galvanometer was inserted in a complete metallic, inductionless circuit containing a battery, and the current made by closing a key. These comparison photographs indicated that the time lag of the galvanometer string was 0.019 seconds. Inasmuch as there was no measurable difference between the comparison photographs and the photographs showing the change of flame current it is to be concluded that if there is any time lag in the building up or breaking off of a current through a flame this lag must be less than 0.019 seconds.

The chief disadvantages at present apparent in the use of a flame amplifier for practical purposes are the smallness of the current output and the difficulty of securing steady and exactly reproducible conditions. In some instances, however, it may be used with convenience, *e.g.*, in the detection of electric waves for class demonstrations, etc. The use of a third electrode, or grid, is of some interest theoretically and experimentally in that it affords a method of studying the mechanism of flame conduction.

THE RICE INSTITUTE,
HOUSTON, TEXAS.

¹ Electrical Properties of Flames, p. 100.

THE PHYSICAL REVIEW.

THE TONES FROM BELLS.

BY ARTHUR TABER JONES.

SYNOPSIS.

Normal Modes of Vibration.—The positions of the *nodal lines* are examined for partial tones up to the ninth. Certain partials are heard only when the bell is tapped on the sound bow, others only when it is tapped above the sound bow.

Elasticity.—A value is obtained for the *Young's modulus* of bell metal.

Striking Notes.—The *principal striking note* is the note which is best heard when tunes are played on bells, and which gives its name to a bell. It cannot be picked up from the bell by a resonator, it cannot be elicited from the bell by resonance, and it does not beat with a tuning fork of nearly its own pitch. No partial tone of the bell has the same pitch as this striking note, but the fifth partial is an octave above it. A striking note is not a difference tone, and it does not arise from compressional waves running through the material of the bell. The pitch of the striking note seems to be determined by the fifth partial, the octave in which it lies being, however, generally misjudged. A possible reason for this general failure in correct estimation of the octave is found in the rates at which the different partials reach their maximum intensities.

A *secondary striking note*, an octave below the fourth partial of the bell, can be heard when the bell is tapped above the sound bow.

I. INTRODUCTION.

BELLS have been a subject of interest to many investigators,¹ but from the physical point of view very little work has been done. Papers by Lord Rayleigh,² Canon A. B. Simpson,³ and Mr. P. J. Blessing⁴ contain practically all that is known.⁵

¹ For bibliography of more than 250 titles see H. B. Walters, *Church Bells of England*, London, Frowde, 1912.

² *Phil. Mag.*, (5), 29, p. 1, 1890, or *Theory of Sound*, § 235a.

³ Simpson's two popularly written articles on "Why Bells Sound Out of Tune" and "How to Cure Them" were published in the *Pall Mall Magazine*, Oct., 1895, and Sept., 1896. They have also been published in the form of a booklet, which I believe is out of print. London, Skeffington, 1897.

⁴ *Physikal. Zeitschr.*, 12, p. 597, 1911.

⁵ From the last number of *Science Abstracts* (Jan., 1920) I learn of two publications on bells by J. Biehle. The copy of the *Physikalische Zeitschrift* in which these are reviewed (Vol. 20, pp. 429-431, Sept. 15, 1919) has not yet reached Smith College. I have, however, succeeded in seeing the review, and from it I judge that the material in my paper is sufficiently different from Biehle's to make it worth publishing. From the review of Biehle's work, from my study of the bells of the Dorothea Carlile chime, and from Simpson's statements about English and continental bells I think it likely that the bells of the Dorothea Carlile chime are more like English bells than like the 450 bells which Biehle has examined.

Rayleigh used a set of Helmholtz resonators and examined the partial tones given out by each of a number of bells. Each resonator was tuned by covering its mouth to a greater or less extent with a finger, and then the various partial tones of the bells were picked out by ear. In seven out of the eight church bells which Rayleigh examined the fifth partial was the only tone, irrespective of octave, which was close to the note that the bell was said to give, and in the case of at least one of the bells this fifth partial was an octave higher than the note which the founders gave as the pitch of the bell.

Simpson's description of his method of study is given in a single sentence: "Any note of a bell can be elicited separately by touching the bell with the stem of a vibrating fork which is of the same pitch as the note in question." Simpson examined a considerable number of bells and came to regard as the most important partials three tones which are usually in the neighborhood of three successive octaves. The lowest of these three is the *hum note*, which is the lowest tone given by the bell. The next is the so-called *fundamental*, and for the highest of the three Simpson introduced the term *nominal*. Simpson adduced reasons for believing that when English tuners tuned a peal of bells they tuned the nominals and paid little attention to the other tones, whereas continental tuners paid most attention to the fundamentals. He also remarks, "There is further this curious fact: That while a tuner [English] always gave the nominal as the note of the bell, he invariably gave the pitch an octave lower than it really was." Simpson pointed out that for the finest musical quality all the partial tones of a bell should harmonize. This they seldom do. The interval from the hum note for instance to the fundamental he found was usually less than an octave, and the interval from the fundamental to the nominal somewhat over an octave. Simpson suggested a method of tuning bells which appears to depend on the fact that different partials have nodal circles at different distances up the bell. Thus by thinning the bell at certain distances from the bottom certain partials may be more affected than others. The results thus obtained are said to be very fine.¹

Blessing distinguished the *principal note* or *striking note* of a bell from its *secondary notes*. The striking note is the note which is usually most noticed when tunes are played on bells. The secondary notes are pro-

¹ Simpson's method of tuning was at once taken up and developed by John Taylor and Co. of Loughborough, Eng. As to the results see T. L. Papillon, *Encycl. Brit.*, ed. 11, article Bell; Lord Grimthorpe, *Clocks, Watches, and Bells*, ed. 8, p. 393, London, Lockwood, 1903; W. W. Starmer, *Carillons*, p. 12, London, Novello, 1915; W. W. Starmer, *Musical Times*, 60, p. 522, Oct. 1, 1919. There are in North America three chimes by Messrs. Taylor—one of 10 bells at the Iowa State College, one of 13 bells at St. John's Church, Peterborough, Ontario, and one of 12 bells at the University of California—cast respectively in 1899, 1911, and 1915.

duced by division of the bell into segments separated by nodal lines, and are therefore the various partial tones of the bell. These partial tones can be picked out by properly tuned resonators and can be elicited from the bell by resonance. The striking note, on the contrary, sound no louder with a resonator than without it and cannot be elicited from the bell by resonance. The pitch of the striking note is usually not far from that of the second partial, but the manner in which the striking note is produced is a mystery.

These remarkable statements as to the striking note of a bell, and the curious fact observed by Rayleigh and Simpson that the pitch of the bell appears to be an octave below the fifth partial of the bell certainly suggest some interesting problems. The installing of the Dorothea Carlile chime of twelve bells¹ at Smith College provided opportunity for a study of the striking note and also for comparison of the partials of a number of bells, all cast in the same year and by the same founder.

2. METHOD OF STUDY.

Pitch Determinations.—The partial tones were found by Rayleigh's method with the use of Helmholtz resonators. The striking notes were picked up without resonators. The pitches, both of partial tones and of striking notes, were obtained by ear by comparison with a sonometer which carried a wire one meter long. Two positions of the sonometer bridge were found such that a note from the sonometer was in one case just perceptibly sharper and in the other just perceptibly flatter than the tone in question, and the mean of these two settings of the bridge was used. In order to avoid errors due to large amplitude the sonometer wire was plucked gently and the tuning fork was tapped lightly. As standard a c_4 [= 512 vd] tuning fork by König was used, and the sonometer wire was frequently compared with it.

The accuracy of the pitch determinations seemed at the time of making an observation to be often as good as 5 cents,² but observations made at different times usually differed by much more than this. The average deviation of half a dozen to a dozen observations made at different times was usually not far from 15 cents to 20 cents.³

Calibration of Sonometer.—For the shorter lengths of the sonometer wire the frequency of the note from the wire was not accurately proportional to the reciprocal of the length. This appeared to be due

¹ Cast in 1919 by the Meneely Bell Company, Troy, N. Y. Total weight 11,838 lbs. [= 5370 kg.] Weights of largest and smallest bells respectively 3006 lbs. [= 1364 kg.] and 268 lbs. [= 122 kg.].

² In designating intervals I use Mr. Ellis's very convenient interval the *cent*, 100 of which make an equally tempered half step.

³ 1 per cent. change in frequency corresponds to a change in pitch of 17.2 cents.

partly to the natural stiffness of the wire and partly to the increased tension when the bridge approached the end of the wire. In order to calibrate the sonometer two methods were used. For the range from c_3 to c_6 settings of the bridge were made at pitches which corresponded to a dozen selected notes on a reed organ which had a stop tuned to the untempered scale. Care was used to blow the organ at a pressure at which the tuning was good. For the range above c_6 it was at first thought that the first, second, third, etc., overtones of any part of the wire might be taken as the octave, twelfth, double octave, etc., of the note given by that part of the wire, and therefore that corresponding to any chosen position A of the bridge the positions for the octave, twelfth, double octave, etc., would be those which gave the same notes as the corresponding overtones obtained with the bridge at A . This, however, assumed that the natural stiffness of the wire was negligible and did not prove satisfactory. The method was therefore modified. No attempt was made to get the overtones of the wire, but the octave was estimated by ear, and only the octave was used. Thus corresponding to any chosen position A of the bridge two other positions were found, one at which the pitch sounded a trifle flatter and one at which it sounded a trifle sharper than the octave of the note given at A . The mean of these two was taken as the setting for the octave of the note given at A . In the range where this method overlapped the calibration from the reed organ the agreement of the two methods was very satisfactory. The corrections obtained have been applied to all readings, although for pitches below f_4 they are not more than 10 cents. In the neighborhood of c_6 they amount to about 18 cents, and in the neighborhood of g_6 to nearly 25 cents.

Temperature Correction.—The pitches of the bells appear to be very little affected by changes in temperature. Throughout the later part of the work the temperature was read at frequent intervals on a thermometer hung near the bells. The temperatures ranged from 10° C. to 30° C. The pitch of the hour bell—low f of the chime—in the range from -15° C. to 0° C., and the pitches of both the transverse and the longitudinal vibrations of a straight bar of bell metal in the range from -10° C. to $+20^\circ$ C. were also determined. The temperature coefficients obtained in the different cases were small and were not consistent.

It is not difficult to show that for the transverse and longitudinal vibrations of a straight bar, and for the extensional and flexural vibrations of a thin ring,

$$n = n_0 \left(1 + \frac{\alpha + \gamma}{2} \cdot t \right), \quad (1)$$

where n and n_0 stand for the frequencies when the bar is respectively at temperatures t and 0° , and α and γ stand for the temperature coefficients respectively of linear expansion and of Young's modulus. For many metals $\alpha + \gamma$ is negative, and the pitches of such bars and rings therefore fall with rise of temperature. Since equation (1) holds for all four of these cases it seems likely that it will also hold for bells. Taking α for bell metal as 0.000018 per degree C.,¹ and γ as -0.0003 per degree C.,² equation (1) leads to a temperature coefficient of about -0.24 cent per degree C.

In view of the lack of consistency in the experimental values for the temperature coefficient of pitch of the bells, and of the small value which such a correction would have, it was finally decided to make no attempt to reduce readings to a common temperature.

Pitch as a Function of Amplitude.—Throughout most of the work it was thought that the pitches of many of the partial tones—especially of the lower partials of the larger bells—were somewhat lower when the bells were first struck than when the sound had nearly died out. This effect was not confirmed by the frequency with which properly tuned forks would beat with the tones in question, and was probably a case of the subjective lowering of pitch which has been discussed by Dr. C. V. Burton.³ The pitches given below are averages from values which were obtained when tapping the bells gently.

3. THE PARTIAL TONES OF THE BELLS.

Nodal Lines.—Rayleigh determined the number of nodal meridians for the first five partials of two bells. His method was to find a number of successive meridians at which the beats of the tone in question vanished. The number of nodal meridians is then half the total number of the meridians at which the beats vanish. In the case of the sixth partial of the largest bell of the Dorothea Carlile chime this method failed. Instead of two normal modes of vibration which gave a single set of beats for this partial there seemed to be several normal modes of nearly the same frequency, so that there were beats of several different frequencies. The frequencies of the most prominent beats varied in an

¹ From the Landolt and Börnstein tables. This is about the value given for brass and for a bronze.

² From values found by Kiewiet [Winkelmann, Hbduch. d. Phys., ed. 2, Vol. 1, p. 567] for the temperature coefficients of various copper-tin alloys. Kiewiet's values have evidently been multiplied by some factor. On comparing his values with those for various substances as given in the Landolt and Börnstein tables I think it likely that this factor is 10^4 , and have so assumed. The coefficient given above is for the proportion of Cu 78 per cent. to Sn 22 per cent, which is the proportion used by the Meneely Bell Company.

³ E. H. Barton, A Text-book on Sound, p. 579.

erratic manner with the position at which the bell was tapped and the position at which the resonator was held, both along a circle of latitude and along a meridian. These variations may be partly due to the long inscription on this bell.

After some time spent in trying to unravel these different beats another method of determining the number of nodal meridians was hit upon which proved entirely satisfactory and was in most cases much more expeditious. The resonator was connected to the observer's head by a piece of rubber tubing and the binaurals of a stethoscope. A single position was found where the beats vanished, the bell was tapped at this one position, and the resonator was moved quickly some distance around the bell. At the nodal meridians the sound grew faint and between them swelled out. This method proved to be also of service in cases where no beats could be detected and Rayleigh's method would have involved fastening to the bell a local load.

As regards nodal circles, Rayleigh observed that certain partials were

TABLE I.

Nodal Lines.

In this table the positions of nodal circles are indicated by fractions. The unit chosen is the distance measured along the outer surface from the bottom of the bell to the point where the vertical "shoulder" joins the more or less horizontal "crown." Numbers in parentheses give the average deviations of the measurements on the different bells. The middle of the "sound bow" is at 0.164 (0.007).

Partial.	Bells on which Observed.	No. of Nodal Meridians.	Positions of Nodal Circles.	Remarks.
1 = hum note	All	4	None	
2 = fundamental	All	4	0.33 (0.015)	
3	All	6	0.47 (0.012)	
4	All	6	None found	Not certainly detected when bell was tapped below 0.21 (0.014).
5 = nominal	All	8	0.48 (0.017)	Very clear when bell is tapped on sound bow. Faint for other positions of tapping. On five smallest bells not detected when tapping above sound bow.
6	6 largest	8	0.20 (0.009) 0.53 (0.033)	
7	6 largest	10	None found	Detected only when tapping below 0.37 (0.035).
8	2 largest	10	None found	Detected only when tapping above 0.46 (0.025).
9	1 largest	12	None found	Detected only when tapping below 0.37.

very faint, if heard at all, when the bell was tapped at certain latitudes. My results check his very well and extend the observations beyond the first five partials. They are given in Table I. It will be seen that vibration of the material in the sound bow—on which the clapper

TABLE II.

Pitches of the Tones.

In the upper half of this table pitches are given on the basis of $c_3 = 256$ vd. Calculations on the basis of $a_3 = 435$ vd. would indicate that all but one of the bells are more or less flat. A note with no sign following it means that the value obtained did not differ from that equally tempered note by more than 10 cents; a note followed by + or - means that the value obtained did not differ from that note by more than 35 cents; and two notes mean that the value obtained lay between them, but did not lie as close to either as 35 cents.

In the lower half of the table the pitches are in cents above the principal striking note. The three bottom lines give the average, the average deviation, and the range of values for the lower half of the table.

Bell.	Principal Striking Note.	Partial Tones.								
		1.	2.	3.	4.	5.	6.	7.	8.	9.
<i>eb</i>	<i>eb</i> ₃	<i>f</i> ₃ +	<i>eb</i> ₃	<i>gb</i> ₃ <i>g</i> ₃	<i>c</i> ₄ <i>d</i> ₄	<i>eb</i> ₄ +	<i>a</i> ₄	<i>bb</i> ₄ +	<i>eb</i> ₅ +	<i>eb</i> ₅ <i>e</i> ₅
<i>f</i>	<i>f</i> ₃	<i>ab</i> ₃	<i>eb</i> ₃	<i>ab</i> ₃	<i>d</i> ₄ +	<i>f</i> ₄	<i>bb</i> ₄	<i>c</i> ₅ +	<i>eb</i> ₅ +	
<i>g</i>	<i>g</i> ₃	<i>a</i> ₃ <i>bb</i> ₃	<i>gb</i> ₃ -	<i>bb</i> ₃ <i>b</i> ₃	<i>e</i> ₄ <i>f</i> ₄	<i>g</i> ₄ +	<i>d</i> ₅ +	<i>d</i> ₅ +		
<i>ab</i>	<i>ab</i> ₃	<i>b</i> ₃ -	<i>gb</i> ₃	<i>b</i> ₃	<i>f</i> ₄ <i>g</i> ₄	<i>ab</i> ₄ +	<i>d</i> ₅	<i>eb</i> ₅ <i>e</i> ₅		
<i>a</i>	<i>a</i> ₃ +	<i>c</i> ₃	<i>g</i> ₃	<i>c</i> ₄	<i>g</i> ₄ -	<i>a</i> ₄ <i>bb</i> ₄	<i>eb</i> ₅ -	<i>e</i> ₅ +		
<i>bb</i>	<i>bb</i> ₃ -	<i>c</i> ₃ <i>d</i> ₃	<i>a</i> ₃	<i>d</i> ₄ +	<i>g</i> ₄	<i>bb</i> ₄	<i>e</i> ₅ -	<i>f</i> ₅		
<i>c</i>	<i>c</i> ₃	<i>d</i> ₃ <i>eb</i> ₃	<i>c</i> ₄ +	<i>eb</i> ₄ +	<i>a</i> ₄ <i>bb</i> ₄	<i>c</i> ₅ +				
<i>d</i>	<i>d</i> ₃	<i>e</i> ₃ -	<i>b</i> ₃ +	<i>e</i> ₄ -	<i>bb</i> ₄ -	<i>d</i> ₅				
<i>d</i>	<i>d</i> ₃ -	<i>f</i> ₃	<i>d</i> ₄ +	<i>f</i> ₄	<i>c</i> ₅ -	<i>d</i> ₅ -				
<i>eb</i>	<i>eb</i> ₄ +	<i>gb</i> ₃	<i>d</i> ₄	<i>gb</i> ₄	<i>d</i> ₅ -	<i>eb</i> ₅ +				
<i>f</i>	<i>f</i> ₄ +	<i>ab</i> ₃	<i>eb</i> ₄ +	<i>ab</i> ₄	<i>eb</i> ₅ -	<i>f</i> ₅ +				
<i>g</i>	<i>g</i> ₄ -	<i>a</i> ₃ <i>bb</i> ₃	<i>f</i> ₄ <i>g</i> ₄	<i>bb</i> ₄ +	<i>e</i> ₅	<i>gb</i> ₅ <i>g</i> ₅				
<i>eb</i>		-969	+ 2	+349	+ 948	+1235	+1813	+1934	+2414	+2466
<i>f</i>		-891	-203	+298	+ 930	+1200	+1693	+1923	+2213	
<i>g</i>		-936	-118	+341	+ 950	+1220	+1830	+1906		
<i>ab</i>		-939	-206	+292	+ 929	+1224	+1692	+1946		
<i>a</i>		-922	-220	+289	+ 964	+1239	+1768	+1910		
<i>bb</i>		-909	- 71	+358	+ 930	+1239	+1808	+1933		
<i>c</i>		-956	+ 9	+310	+ 943	+1210				
<i>d</i>		-930	-182	+265	+ 872	+1197				
<i>d</i>		-875	- 65	+334	+1000	+1210				
<i>eb</i>		-915	-224	+283	+ 955	+1202				
<i>f</i>		-904	-188	+293	+ 961	+1204				
<i>g</i>		-926	-131	+333	+ 922	+1172				
Average		-923	-133	+312	+ 942	+1213	+1768	+1925	+2314	+2466
Av. Dev.		20	71	26	21	16	50	12	100	
Range		94	233	93	128	67	138	40	201	

strikes—has little to do with the production of partials 4, 6, and 8, so that these partials are relatively faint when the bell is struck in the

usual manner by a clapper, whereas in the production of partials 5, 7, and 9 the vibration of the material above the sound-bow appears to be unimportant, and these partials—especially 5, which is the lowest of them—will be more strongly brought out by a blow of the clapper.

The Pitches of the Partial Tones.—The pitches of the partial tones are given in Table II. It will be seen that it is only in a rather rough way that the successive partials form the same intervals for the different bells of the Dorothea Carlile chime. It will also be seen that in every case a partial of even order lies closer to the partial next above it than to the one next below, and an odd partial lies closer to the partial next below it than to the one next above. The averages for all bells are given in Table III.

TABLE III.
Average Interval from One Partial to the Next.

Partials.....	1, 2	2, 3	3, 4	4, 5	5, 6	6, 7	7, 8	8, 9
Intervals in cents.....	790	445	630	271	541	158	385	52

This difference in the intervals according as we pass up from an odd or an even partial may to some extent be understood by referring to Table I., where it is seen that the transition from an even partial to the next above it involves an increase in the number of nodal meridians, whereas the transition from an odd partial to the next above does not. For instance, on passing from the fourth partial to the fifth the width of each vibrating segment is reduced by about one fourth, whereas on passing from the fifth partial to the sixth the height of a vibrating segment is reduced by something like a half. If nothing else changed we should therefore expect a larger interval between the fifth and sixth partials than between the fourth and fifth. Table III. shows that this expectation is justified.

4. THE STRIKING NOTES.

Principal and Secondary Striking Notes.—From Table II. it will be seen that there is in general no partial which approximates at all closely to the pitch of the striking note. Thus Blessing's statement that a resonator does not respond to a striking note appears to be correct. Now when a bell is tapped on the waist or shoulder, *i.e.*, above the sound bow, it is known¹ that the bell sounds flatter than when tapped on the sound bow. And when tapping on the waist or shoulder I seemed to hear a note, flatter than the striking note, to which a resonator would not

¹ Helmholtz, *Sensations of Tone*, 4th Eng. ed., p. 72.

respond. I am therefore using the name *principal striking note* for the note obtained from the sound bow and not reinforced by a resonator, and the name *secondary striking note* for the note obtained from the waist or shoulder and not reinforced by a resonator. The secondary striking note is not as readily heard as the principal striking note. It is usually some 200 cents to 300 cents flatter than the principal striking note and somewhat flatter than the second partial of the bell.

Resonance.—On two of the bells of the Dorothea Carlile chime the principal striking note is very close to the second partial. At some half dozen different times I thought that I picked up with a resonator the principal striking note of some one of the other bells, but later attempts with the same bells seem to show that I was mistaken. A resonator does not, in general, respond to a striking note.

The results were similar when I attempted to get the bells to respond to a tuning fork which could be adjusted by moveable loads to give pitches throughout a range of about an octave. The bell was tapped and the fork adjusted by ear until its pitch was about that of the partial in question. The fork was then struck and its stem pressed against the bell. Various partial tones within the range of the fork responded clearly, but the striking notes did not thus respond.

Beats.—There was no difficulty in tuning a fork until its pitch was about that of a given partial and then hearing distinct beats when both fork and bell were tapped. But no beats could be detected when the fork had approximately the pitch of a striking note.

Difference Tones.—The first explanation of the striking notes which suggests itself is perhaps that they may be combination tones. Blessing says that Rudolph König suggested this possibility, but that no combination of the partial tones of a bell would give the proper frequency. Blessing, however, gives no data in support of his statement, and it seemed worth while to examine the question.

If a striking note is a combination tone of any sort it is most likely that it is a first order difference tone. From the observed frequencies of the various partials the difference tones given in Table IV. have been calculated. This table has reference to the principal striking notes. A calculation for the secondary striking notes shows similar results.

On comparing with Table II. it will be seen that Table IV. includes the difference tones arising from the fifth and seventh partials for all the bells on which the seventh partial was observed. But it will also be seen that in only half of these cases does the difference tone which arises from the fifth and seventh partials lie within a quarter of a step from the principal striking note. Moreover if the striking note were a combina-

TABLE IV.

Difference Tones.

This table includes all the first order difference tones which lie within half a step from the principal striking note. Those which lie within quarter of a step are starred.

Partials.	Bell.	Cents from Prin. Strik. Note Up to Dif. Tone.
2,5	Low <i>eb</i>	+58
	<i>c</i>	+13*
	<i>d</i>	+85
	High <i>g</i>	+75
4,6	Low <i>f</i>	-91
	<i>ab</i>	-79
	<i>a</i>	+59
5,7	Low <i>eb</i>	+28*
	Low <i>f</i>	+71
	Low <i>g</i>	-22*
	<i>ab</i>	+79
	<i>a</i>	-55
	<i>bb</i>	+15*
7,8	Low <i>eb</i>	-40*

tion tone we should expect it to be very faint, if heard at all, when the bell is tapped gently. As a matter of fact, the principal striking note comes out clearly when the bell is tapped gently. It seems then to be clear that a striking note is not a difference tone.

Compressional Waves.—It seemed possible that the striking notes might be due to compressional waves which spread through the material of a bell and returned periodically to the point where the bell had been struck. If this were the case the frequency of the principal striking note would be roughly the same as that of the longitudinal vibration of a straight bar which had a length equal to half the circumference of the sound bow.

To enable me to examine this matter the Meneely Bell Company kindly cast for me a rod of bell metal. After the ends had been trued off this rod had a length of 93.0 cm. The frequency of longitudinal vibration of the lowest mode was about 1810 vd.¹ To have the same frequency as that of the low *eb* bell (304 vd) a rod of bell metal would therefore have to be 553 cm. long. The diameter of the mouth of the

¹ The density of this rod is about 8.86 g./c.c. Its Young's modulus is therefore about $10.0 \cdot 10^{11}$ dynes/cm². This value for the Young's modulus [bell metal = 78 per cent. Cu and 22 per cent. Sn] fits excellently on a curve coördinating the values obtained by Voigt [Wied. An., 48, p. 674, 1893] for copper, tin, and an alloy of 88 per cent. Cu and 12 per cent. Sn. Voigt's values were obtained by forcing a short bar of the given material to vibrate with a frequency of from one to two vibrations per second.

low *eb* bell is 134 cm., so that the rod required would have a length much greater than half the circumference of the sound bow—in fact considerably in excess of the entire circumference of the mouth. Similar statements hold for the other bells.

Moreover, if the principal striking note were due to a compressional wave running, say, around through the sound bow, the diameters of the sound bows of the various bells should be inversely proportional to the frequencies of the principal striking notes. A calculation of the relative frequencies of the bells on this basis leads to values which are considerably in error—in the cases of three of the bells by something like 100 cents.

From this failure of the diameters of the bells to be proportional to the periods of the principal striking notes, and from the fact that a compressional wave would run through the material of a bell too quickly to give the observed pitch, it is clear that the principal striking note is not produced by a compressional wave.

Misjudged Octave.—From Table II. it will be seen that the pitch of the principal striking note is not far from an octave below the fifth partial of the bell. This checks with Rayleigh's and Simpson's observations that in most cases the only partial which is close to the pitch of a bell is the fifth, and that this fifth partial is an octave higher than the pitch of the bell is supposed to be.

It is well known that an error of an octave in judging the pitch of a note is easily made, and Simpson evidently believed that there is in general no note which has the pitch of the principal striking note, but that we hear most clearly the fifth partial of the bell and think it is an octave lower than it is. If that is the case it would explain why a striking note cannot be picked up from a bell by a resonator nor elicited from the bell by resonance, and why it will not beat with a tuning fork. It would probably also explain a statement made by Blessing that if the sound bow of a bell is gradually turned thinner and thinner the striking note grows fainter and fainter and finally disappears. Blessing says nothing as to how this process affects the partial tones of the bell. But since the fifth partial appears to be produced almost entirely by vibration of material in the sound bow it seems likely that reducing the thickness of the sound bow would weaken the fifth partial.

What evidence is there as to the octave in which the principal striking note lies? There is the judgment of the founders and tuners referred to by Rayleigh and Simpson. As to myself, I have at times felt very sure that the note I heard was really of the pitch which I have called that of the principal striking note and was not an octave higher. At other times

I did not feel so sure. When the chime was being played I have compared the notes which I heard with those on a reed organ. When doing this it seemed to me there could be no question but that the pitches really are those which they are ordinarily supposed to be, *i.e.*, each one an octave below the fifth partial. A statement from the Meneely Bell Company also corroborates this. With reference to the low *f* bell of the Dorothea Carlile chime they write me, "This is supposed to be about F natural above middle C, although bells as a rule, from their nature, seem to sound lower." The evidence seems to show that if the pitch of the principal striking note is determined by the fifth partial the octave is very generally misjudged—even by bell founders.

Is there any probable reason for such an error in judgment? In attempting to answer this question we may exclude from consideration the fourth, sixth, and eighth partials, all of which are faint, if heard at all, when the bell is struck on the sound bow. From Table II. it will be seen that the seventh partial is almost exactly a musical fifth (700 cents) above the fifth partial. Now in the harmonic series of tones to which the notes from strings, pipes, etc., approximate, we are accustomed to hear a fundamental accompanied by its octave and twelfth, so that even if the fifth and seventh partials do not produce a combination tone an octave below the fifth partial, they may nevertheless help to suggest a fundamental note of that pitch.

Another, and probably much more important, reason for misjudging the octave lies in the rates at which the different partials reach their full intensities. The fifth partial seems to reach its maximum intensity almost as soon as the bell is struck, the second and third, especially on the larger bells, not quite so soon. Thus it is possible that when a bell is struck the fifth partial at once attracts attention, and the second and third add a considerable volume of sound so soon afterward as to make the pitch seem an octave lower than that of the fifth partial. On the smaller bells the second and third partials seem to be more prompt in their response, and this may have something to do with the difficulty which bell founders are said to have usually experienced in casting small church bells of good musical quality.

My present hypothesis as to the principal striking note of a bell is, then, that it is a note of which the pitch, except for octave, is determined by the fifth partial, and that the octave in which we think we hear it is determined by the more sluggishly responding second and third partials. It is desirable that a considerable number of bells should be investigated, and that photographic records should be obtained showing the rates at which the various partials, especially the second, third, and fifth,

grow to their maximum intensity when the bells are struck in the usual manner by a clapper.

In conclusion I wish to express my thanks to the Meneely Bell Company for their kindness in giving me various data and in casting for me the rod of bell metal.

SMITH COLLEGE,
March 30, 1920.

THE VALENCY OF PHOTO-ELECTRONS AND THE PHOTO-ELECTRIC PROPERTIES OF SOME INSULATORS.

BY M. J. KELLY.

SYNOPSIS.

Photo-emission from insulators; the valency of photo-electrons.—Charged droplets of different insulators are suspended in the field of a parallel plate condenser by regulating the strength and direction of the electric field so that the electric force on the droplet is equal and opposite the gravitational force. Ultra-violet light of various intensities and frequencies is allowed to fall on the droplets and the photo-emission is observed. The photo-emission of several insulators is studied in this manner. By making the light intensities sufficiently low, the author finds that *only one electron escapes at each emission*. This is true for all insulators studied and is verified in the case of each insulator by the observation of several hundred emissions. This result is in line with similar work on ionization of gases by X-rays, γ -rays, β -particles, and α -particles by Professor Millikan and his students. By the use of different absorption screens the *long wave-length limit of photo-emission* from sulphur, shellac, oil and paraffine is located. These are located with greater accuracy than given by any other methods. The effect of water vapor and various surface impurities on photo-emission is discussed.

I. INTRODUCTION.

IN 1911 Millikan and Fletcher¹ furnished the first conclusive evidence that the mechanism of ionization by X-rays, gamma rays and β particles consisted in the detachment from the neutral molecule of one single elementary charge. Millikan, Gottschalk and Kelly² by using a similar method found the same to be true for ionization by α particles of several different gases and vapors. Mr. J. B. Dereux,³ at Professor Millikan's suggestion, using the same general method studied the photo-emission from mercury drops. This work gave evidence which indicated that only one electron was expelled from the mercury atom in the process of photo-emission.

It would indeed be interesting if high speed particles of atomic size, high speed electrons and electromagnetic radiations of as widely differing frequencies as that of gamma rays and ultra-violet light all showed the uniform property of liberating one and only one electron from the atom in its ionization.

¹ Millikan and Fletcher, Phil. Mag. (6), 21, p. 753, 1911.

² Millikan, Gottschalk and Kelly, Phys. Rev.

³ J. B. Dereux, Phys. Rev., 11, p. 276, 1918.

As mentioned above Mr. J. B. Dereux suspended single mercury drops, charged statically in the process of vaporization in the electric field of a parallel plate condenser and allowed ultra-violet light to fall directly on the drops. At the instant of a change in speed, indicating a change in the charge on the drop by its photo-emission, the light was shut off and its speed measured. It was possible to calculate from the change in speed the number of electrons liberated. The majority of the changes observed were those corresponding to emission of single electrons. Since a number of changes corresponded to emissions of more than one electron it was thought worth while to extend the investigation to other substances.

If the changes observed which correspond to emission of more than one electron were due to simultaneous emissions from different atoms on the surface, the chance of this could be decreased by using less active surfaces and decreasing the intensity of the radiation. In choosing surfaces whose emission currents would be small the insulators would naturally suggest themselves. They were selected for the further reason that this is the only method of study of photo-emission which is capable of furnishing very direct and very reliable information with regard to the photo-electric properties of insulators.

II. HISTORICAL SURVEY OF PHOTO-ELECTRIC WORK ON INSULATORS.

In the study of photo-emissions of insulators the general method of procedure has been to place a thin sheet of the insulator over the surface of one of the plates of a condenser, to apply an accelerating field, allow the radiation to fall on the surface and measure the current. In good insulators the surface charges up positively as the photo-electrons escape, until the potential gradient at the surface is sufficiently high to neutralize the accelerating potential and prevent the escape of further electrons. This gives a fatigue effect which makes consistent results difficult to obtain. Sheets of good insulators will remain for a long time in this polarized state and fresh samples of the material must be used. Using this general method Goldmann and Kalandyk¹ investigated the photo-electric effect in sulphur. Photo-electric currents were obtained when a source of ultra-violet light was used and the polarization effect mentioned above was encountered and studied. They found that the photo currents completely disappeared when a plate of glass was interposed between the source and the sulphur, which would indicate the long wave limit somewhere below λ 3200. R. Reiger² investigated a number of insulating

¹ Ann. d. Phys., XXXVI., p. 589, 1911.

² R. Reiger, Ann. d. Physik., XVII., p. 935, 1905.

materials, and found a number including, ebonite, mica, sealing wax and glass that showed small emission currents when exposed to the light of an electric arc. W. Wilson¹ examined shellac and found no emission with ultra-violet light.

III. EXPERIMENTAL.

The apparatus as described by Millikan² was used in the present work, with such changes as were found necessary in the problem. The apparatus consisted essentially of a parallel plate condenser with plates horizontal, the plates were 22 cm. in diameter and were separated 1.586 cm. An ebonite strip surrounded the condenser, with windows set in at proper intervals; one for the illumination of the drop, one for observation and one of quartz for admission of the ultra-violet light. Several small holes were bored in the center of the upper plate for the admission of drops and an electromagnetically controlled shutter was provided to close these holes when the drops were under observation. The condenser plates were placed in an air tight iron tank about 30 cm. in diameter and 50 cm. high. This cylinder was placed in a tank, about 50 cm. in diameter leaving a space between the walls of tank and cylinder which was filled with oil to hold the temperature constant. Tubes at the level of the condenser provided with vacuum tight, transparent windows ran from the outer tank into the inner tank at the proper places for admission of light for illuminating the drop, for observation, and for admitting the ultra-violet light. The windows in the ebonite strip were directly in front of these tubes.

The space between the condenser plates was illuminated by the light from a right-angled carbon arc. The light was focused on the drop by a cylindrical lens. A telescope having a high magnifying power was used for observing the drops. In the focal plane of the eye piece was a scale, the smallest division of which corresponded to 2 mm. Observations were made through a window 150 degrees from the one through which the light entered. A stop watch was used in timing.

The potential of the plates was furnished by 3,500 small lead cells giving a potential of about 7,000 volts. By means of a special switch the condenser could be charged, short circuited, or reversed by changing the position of a handle.

For changing the charge on the drops X-rays were used. The X-ray tube was placed just outside the apparatus near one of the windows. The source of radiation, usually a mercury vapor lamp was placed on a line of centers with the center of the condenser, and opposite a quartz window.

¹ W. Wilson, *Ann. d. Physik.*, XXIII., p. 127, 1907.

² R. A. Millikan, *Phys. Rev.*, 32, p. 349, 1911.

The drops of the various insulators were formed by getting the insulator in the liquid state and discharging it through an atomizer placed in the upper part of the iron cylinder and manipulated by air pressure from without. After atomizing there was always a rain of drops through the holes of the upper plate and one of proper gravity speed was selected; the shutter over the holes closed and the drop balanced by regulating its charge with X-rays. An electromagnetically controlled shutter over the quartz window was then lifted and the radiation allowed to enter until a change in speed showed an emission to have occurred, the shutter was then closed and the speed determined. From this speed and its speed under gravity the number of unit charges on the drop could be obtained and the difference between the number before and after emission gives the number of electrons emitted.

The number of charges on the droplet was determined from the relation

$$n = C \left(1 \pm \frac{t_1}{t_2} \right) \frac{1}{V},^1$$

where n = number of unit charges.

C = a constant for a given drop.

t_1 = number seconds it takes drop to traverse a certain distance d under gravity.

t_2 = number seconds it takes drop to traverse the same distance d under the field V .

V = potential across plates.

In determining the spectral range of photo-sensitiveness of the various insulators the method followed was to balance a drop in the field and allow radiations of a certain range to fall on it and observe whether or not emissions occurred. A large number of drops were used for each range so that there was no uncertainty in any case.

IV. SULPHUR.

Drops of sulphur were obtained by heating chemically pure sulphur to a proper temperature in a specially made atomizer. The atomizer was placed in the large chamber about 20 cm. above the upper plate of the condenser and heated by a coil which kept the entire atomizer at a temperature of 150° C. At this temperature the sulphur is in the form of a pale yellow mobile liquid and atomizes quite readily. The temperature regulation had to be close in order to obtain satisfactory performance for at about 160° C., the sulphur changes to a dark viscous liquid which is too viscous to permit being atomized. The sulphur drops were always charged statically and a satisfactory drop was easily obtained.

¹ Millikan and Fletcher, *Phil. Mag.* (6), 21, 753, 1911.

At the beginning of the work no special precautions were taken with the gas content of the chamber; it consisted of the air of the room freed fairly well of water vapor. The sulphur drops evaporated very rapidly; in fact so fast that all calculation of the number of charges on the drop or the changes in charge were made impossible by the enormous change in the size of the drop. Table I. gives the gravity speeds, taken at 5-minute intervals, of a drop held under observation for 30 minutes. The charge on the drop was varied by X-ray ionization so that as it evaporated it remained just balanced in the field. At 5-minute intervals the electric field was removed and readings were taken of its speed under gravity.

TABLE I.

Sulphur Drop No. 8.

Time to pass over 50 divisions:

1.	8.2 seconds.		
2.	10.4	"	at the end of 5 minutes.
3.	13.6	"	10 "
4.	17.7	"	15 "
5.	28.0	"	30 "

While evaporating the drops drifted badly and were soon out of the illuminated space. It was necessary to get rid of the evaporation before observations could be made. After a number of unsuccessful attempts the trouble was eliminated by boiling sulphur in the sealed chamber until the walls were coated with flowers of sulphur, and moisture-free air then allowed to stand in the chamber for days before the observations were taken. While the drops still evaporated, the rate was slow enough so that no uncertainty was introduced into the calculations. Table II. gives data on a drop observed for one hour in the same manner as the drop in Table I.

The readings given in Table II. for each time interval are the average of a number taken at each of the intervals. I was considerably surprised to find a variation of as much as 10 per cent. in successive timings, one

TABLE II.

Drop No. 27.

Time to pass over 50 divisions:

1.	7.6 seconds.		
2.	7.6	"	at end of 15 minutes.
3.	7.4	"	30 "
4.	7.0	"	60 "

taken immediately after the other as from experience in timing drops in another investigation I had acquired sufficient skill to check readings to .1 seconds. Upon examining a number of the drops under a micro-

scope they proved not to be perfect spheres, but spheres with minute crystal faces on the surface. The amount of this deformation varied considerably from drop to drop being apparently absent in some. This seemed to offer grounds for a satisfactory explanation of the variations, for a change in the orientation of the drop between two different excursions would change the resistance offered to its fall. In selecting drops for observation several timings of their gravity speed were taken one immediately after the other and if the variations were large the drop was discarded and another obtained.

In studying the valency of photo-electrons the following procedure was employed; after a drop with a proper gravity speed had been selected it was given a sufficient positive charge by X-ray ionization to just balance it in the field with the voltage used. The shutter covering the quartz window was opened and the radiations from a quartz mercury lamp allowed to fall on the drop until an increase in speed indicated an emission. The shutter was immediately closed and the speed determined. The shutter was again opened and the process repeated. This was continued until 4 or 5 emissions had occurred, then the drop was brought back to its initial number of charges with X-rays and the process repeated. Some of the smaller drops were started with an excess of 3 or 4 electrons and emissions allowed to occur until the drop had lost 3 or 4 electrons beyond the neutral point. This was not possible with the larger drops due to the electric force with the small number of unit charges and with voltages available being insufficient to balance the force of gravity. Some drops were kept under observation for two hours. 57 separate emissions were observed on one drop and over 500 emissions were observed under good experimental conditions and calculated.

When the observations were first begun the drops captured about as many electrons as they emitted. This was at first attributed to residual radio-activity as this same apparatus had been used by Millikan, Gottschalk and Kelly (*loc. cit.*) in studying ionization by α particles and had had radium open in the chamber. But on running a blank, that is observing the drop for an hour with the shutter closed, the drop was found to capture but two electrons; so it was concluded that the plates were emitting. In order to prevent this the plates were covered with lamp black, but the photo-sensitiveness was increased several hundred-fold, in fact, as soon as the shutter was opened the drop would capture electrons in such rapid succession that the individual captures were indistinguishable. This emissivity was most probably due to the carbon in the lamp black as it was found that its long wave limit was $\lambda 2550$ (approx.) and this is about the limit Hughes¹ found for carbon.

Hughes, Photo-Electricity, p. 102.

Shellac was then tried, as W. Wilson (loc. cit.) had pronounced it not photo-sensitive to the light of a quartz mercury-vapor lamp. However, it emitted electrons freely, so a thin coating of paraffine was next tried and no emission was found.

Table III. gives the results of observation on 10 droplets of sulphur.

TABLE III.

Drop No.	No. Minutes Under Obs.	No. Emissions.	No. Singles.	No. Mult.
21	90	35	35	
22	100	40	40	
24	100	44	44	
25	120	57	56	1
26	30	12	12	
28	60	38	37	1
29	110	45	45	
30	120	40	40	
31	60	20	20	
32	100	25	25	

As is shown all emissions with the exception of two were of a single electron and both of these were emissions of two electrons. As is evident from the "times under observation" the emissions were very frequent, in fact more so than the table indicates as this time includes time consumed in regulating charge on the drop, timing it and adjusting the illumination.

In order to decrease the rate of emissions a plate of fused quartz about 5 mm. thick was placed in the path of the beam. From evidence to be discussed later this plate appeared to cut off all radiation below $\lambda 2250$. This decreased the emission rate to six to eight an hour. About 200 emissions were observed at this reduced rate and 40 hours were consumed in observation. In every instance the emissions were unmistakably emissions of single electrons. Table IV. gives the data on a representative drop.

The activity of various drops differed considerably. The rate of emission from some drops was so slow that after observing them for a time to be sure they possessed activity they were discarded. This difference in activity must be attributed to surface conditions. All drops were most active initially and their activity gradually decreased with time; the rate of emission was decreased roughly one half at the end of two hours.

The surface polarization effect which was observed in sheets of sulphur was somewhat evident with the drops. If electrons were emitted until the drop became positively charged then as the positive charge increased

TABLE IV.

Drop No. 32.

λ_1	λ_2	K. V.	$\left(1 \pm \frac{\lambda_1}{\lambda_2}\right) \frac{1}{K\lambda}$	No. Units on Drop.	No. Electrons Emitted.
4.1	80.0	24	.0438	4 Pos.	
	12.6	24	.0552	5	1
	7.0	24	.0660	6	1
	4.9	24	.0765	7	1
4.1	19.5*	36	.0219	2 Pos.	
	22.4	36	.0330	3	1
	7.1	36	.0436	4	1
	12.2	24	.0557	5	1
	6.8	24	.0672	6	1
	5.0	24	.0756	7	1
4.0	5.0	24	.0750	7 Neg.	
	6.9	24	.0658	6	1
	12.8	24	.0550	5	1
	80.8	24	.0436	4	1
	19.0*	24	.0328	3	1
4.0	21.6	36	.0326	3 Pos.	
	7.0	36	.0434	4	1
	12.0	24	.0552	5	1
	6.8	24	.0660	6	1
	4.8	24	.0763	7	1
3.9	6.7	24	.0659	6 Neg.	
	12.0	24	.0551	5	1
	78.0	24	.0435	4	1
	19.8*	24	.0334	3	1
3.8	20.0*	36	.0221	2 Pos.	
	22.0	36	.0325	3	1
	6.9	36	.0430	4	1
	12.0	24	.0547	5	1
	6.6	24	.0656	6	1
	4.9	24	.0738	7	1

All λ_2 marked with * the drop was going downward due to the force of gravity being greater than the opposite directed force of the electric field. In these cases the minus sign is used in the formula $\left(1 \pm \frac{\lambda_1}{\lambda_2}\right)$.

by further emissions, the rate of emission was greatly decreased. A drop that had lost 25 electrons beyond its neutral value was held under observation for one hour and only three emissions observed. The excess positive charge was neutralized by X-ray ionization and the drop was found to emit at the rate that was to be expected after it had been under

observation for one hour. Data was taken on a number of drops with varying excess of positive charge hoping to find some quantitative relation of stopping potentials but the variation from drop to drop, probably due to differences in surface contamination masked any such relation, if it exists.

In order to find the long-wave limit of emission for the sulphur a Hilger quartz spectroscope was set up outside the quartz window and by means of extra quartz lenses the radiation from any desired frequency was focused at the center of the condenser in the path of the drops. No emissions were obtainable with any frequency. To test the adjustment of the light system mercury drops were suspended between the plates of the condenser and photo-emissions were obtained when the instrument was set for λ 2536. As no emission could be obtained from the mercury for the adjustments of the spectroscope at wave-lengths shorter than λ 2536 due to their low intensity, the only conclusion that could be drawn from this work was that the long-wave limit for the sulphur was some wave-length shorter than λ 2536. It was next attempted to set some closer limits for the long-wave limit by using absorption screens.

After studying a number of filters Cobalt chloride in absolute methyl alcohol was found most suitable. An absorption cell with quartz windows about 8 mm. wide was used. Solutions of Cobalt chloride in this cell, varying in strength from 2 *N* to .01 *N* were studied photographically with a quartz spectrograph and a mercury vapor lamp. The greatest precautions to prevent stray light and fogging were taken and the exposure times were made as long as fogging permitted. These photographs showed the solutions to have a narrow absorption band covering the range λ 4600 to λ 5100 and complete absorption of radiations of wave-lengths shorter than λ 2650 for the 2 *N* solution, and of all radiations shorter than λ 2400 for the .01 *N* solution. The methyl alcohol cut off all radiations of wave-length shorter than λ 2350. With the quartz mercury lamp that was used, photographing directly and using every precaution lines were obtained down to λ 2150. These photographs were repeatedly taken and the sharpness and completeness of the absorption ranges well substantiated.

No emissions from the sulphur were obtained when the radiations passed through the cell filled with any of the cobalt chloride solutions or with the methyl alcohol only. Many drops were held under observation for more than an hour each with the methyl alcohol cell before the quartz window. After a drop was secured it was exposed to the direct radiations and several emissions observed then the cell would be inter-

posed and the emissions would immediately stop; at the end of 30 minutes the cell would be removed and emissions from the drop would begin immediately in every instance. This definitely placed the long-wave limit at some wave-length shorter than λ 2350.

An absorption screen was looked for that would transmit some frequencies to which the sulphur was photo-sensitive; some transparent commercial *fused* quartz was found which when interposed in the path of the radiation cut down the emissivity from 40 emissions to the hour to 5 an hour. These plates were about 5 mm. thick and only one was used. If a plate of crystal quartz of the same thickness was placed in the path of the radiation there was no reduction in the rate of emission. This was verified in a number of drops. On examining the transmission of this plate it was found to cut off (at least photographically) all wavelengths shorter than λ 2250.

Some time after completing this work I noticed that Hughes in his book, *Photo-Electricity*, had remarked that a column of water 1 cm. long cut off all radiations shorter than λ 2200. This was confirmed photographically and as a check on the earlier work, the emission of sulphur through this filter was examined and its rate of emission was found to be decreased in about the same ratio as with the fused quartz plate filter.

The long-wave limit of photo-sensitiveness of sulphur can be placed with a great amount of certainty between λ 2400 and λ 2200 and it is probably within the narrower limits of λ 2350 and λ 2250.

V. SHELLAC.

Drops of shellac were obtained by atomizing a filtered solution of shellac flakes dissolved in ethyl alcohol. The alcohol evaporates rapidly and the density of the sphere reaches a constant value in a very few minutes, as is shown by the constancy of the gravity speeds. The drops showed perfect sphericity when examined under a microscope and showed none of the variations in speed on succeeding excursions under gravity as did sulphur.

All the sulphur was washed out of the large cylinder and removed from the condenser plates. The entire interior of the iron cylinder was coated with shellac and this allowed to stand several days after which the air was pumped out of the tank several times and the air that was admitted each time was filtered and dried. A thin coating of paraffine was used on the plates as with the sulphur. The shellac drops were much more satisfactory to work with as there was no evaporation, and little, if any, fatigue effect of photo-emission.

TABLE V.

Drop No. 49.

P.D. = 6,000 volts.

Time under obs. = 90 minutes.

η_1	η_2	$\left(1 \pm \frac{\eta_1}{\eta_2}\right) \frac{1}{V} \times 10^3$	No Units on Drop.	No Electrons Emitted.	Time Interval Before Emission, Secs.
23.0	31.8*	.0465	1 Pos.		
	51.3*	.0920	2	1	20
	128.	.1367	3	1	90
	Inf.	.1666	4	1	10
	62.0	.2286	5	1	120
	35.6	.2745	6	1	40
	25.0	.3200	7	1	0
	18.2	.3775	8	1	80
	15.6	.4125	9	1	30
23.2	35.4	.2757	6 Neg.		
	61.0	.2296	5	1	15
	Inf.	.1666	4	1	40
	130.0*	.1370	3	1	65
	52.0*	.0926	2	1	5
	32.2*	.0466	1	1	10
	23.0*	.0000	0	1	12
	31.8*	.0448	1 Pos.	1	28
23.2	Inf.	.1666	4 Neg.		
	133.0*	.1375	3	1	68
	50.0*	.0895	2	1	70
	32.0*	.0462	1	1	10
	23.0*	.0000	0	1	25
23.4	32.2*	.0461	1 Pos.		
	52.0*	.0916	2	1	70
	134.0*	.1375	3	1	45
	Inf.	.1666	4	1	150
	62.5	.2290	5	1	10
	35.6	.2761	6	1	35
	25.4	.3200	7	1	40
23.4	32.5*	.0466	1 Pos.		
	52.4*	.0923	2	1	140
	133.0*	.1373	3	1	30
	Inf.	.1666	4	1	25
	62.0	.2295	5	1	100
	35.4	.2766	6	1	75
	25.2	.3211	7	1	5
	18.2	.3808	8	1	2

All η_2 marked with * the drop was going downward due to the force of gravity being greater than the oppositely directed force of the electric field. In these cases the minus sign is used in the formula $\left(1 \pm \frac{\eta_1}{\eta_2}\right)$.

With the complete radiation from the mercury vapor lamp the photo-current from the shellac drops was less than that obtained initially from the best sulphur drops. However there was much less variation in photo-emission from drop to drop than from the sulphur.

The time interval from the opening of the shutter until an emission had occurred was observed for about 200 emissions. This was taken when the excess positive or negative charge was never more than 5 units. The average time interval was 50 seconds, the maximum time was 4 minutes and the minimum was too small to be measured by a stop watch.

In the study of the valency of the emission data were taken on 40 drops. More than 800 emissions were observed and the number of electrons liberated at each emission determined. The same procedure was followed as in the case of the sulphur.

Table No. V. gives the data on a typical drop. The time interval given under "time of exposure" is the interval of time between the opening of the shutter and the occurrence of the emission.

Table No. VI. contains a summary of data on 10 drops.

TABLE VI.

Drop. No.	Time Under Obs.	No. Emissions.	No. Singles.	No. Mult.
17	120	47	47	0
19	80	29	29	
20	100	28	28	
21	60	23	23	
27	120	44	44	
31	120	40	40	
37	60	19	19	
38	50	17	17	
40	70	26	26	
49	90	30	30	

As seen from the table every emission observed corresponded to the liberation of a single electron, this was also true of the observations on the remaining drops. The conclusion thus seems fully warranted that the process of photo-emission liberates one and only one electron from the molecule in sulphur and shellac. This is rather interesting especially in the case of shellac in which there are complex molecules made up of a large number of atoms.

As it is generally recognized that there is an absence of free electrons in non-conductors and insulators, the actual observation and identification of electrons emitted photo-electrically from them gives added weight to the accumulating evidence that photo-electrons come from the atom structure and are not free electrons.

The long-wave limit of the shellac was found to be quite definitely somewhat shorter than that of sulphur, although I believe not a great deal shorter. No emissions were obtained from a large number of drops when the radiations were passed through the methyl alcohol solution. A fused quartz plate was then interposed in the path of the beam and 10 drops were observed for intervals of time exceeding an hour and no emissions occurred, but immediately after the plate was removed emissions would begin quite normally. The filter of water one cm. in thickness was interposed in the path of the beam for each of the 10 drops and emission was entirely stopped, but immediately after removing the filter emission began in every case. The conclusion then seems fully warranted that the long-wave limit for shellac is shorter than λ 2200.

Allen in his book Photo-electricity, p. 83, in discussing W. Wilson's work says, "Although shellac itself is not photo-electrically active, it allows the photo-electric current to pass through it when a thin layer is laid upon a metal plate." As is evident from what has been given we found that shellac was photo-sensitive. The photo-sensitiveness of shellac in thin sheets was examined by covering the metal plates of the condenser with a thin coating. The photo-emission from the coated plates was stopped by the same filters that stopped the emission from the shellac spheres; although this was not found to be the case with the cleaned metal plates, which indicates that this emission is from the shellac and not from the plates.

VI. PARAFFINE AND OIL.

Commercial paraffine was melted and a thin coating of it placed on the condenser plates in a portion of the work with shellac and sulphur because there was no emission from it when illuminated by the mercury vapor radiation direct.

The oil used throughout the work on the evaluation of "e" by Professor Millikan was examined for its photo-emission by suspending the drops in the path of the radiation and found not to be photo-sensitive to the radiations from the mercury light direct.

As only a small piece of fluorite just large enough for a window in the ebonite strip was available, a pair of zinc electrodes were placed just inside the inner cylinder and in the narrow space between the edge of the condenser plates and the wall, directly in front of the small fluorite window. By using a tuned circuit very much energy was put into the spark. When the spark was running it caused currents of air in the cylinder which made the drops drift badly.

Great precautions were taken in sealing the condenser so that its

interior would not be affected by outer currents. By doing this, drops of either oil or paraffine could be kept under observation for more than 30 minutes while the spark was running. The drops of paraffine were obtained by melting the paraffine in the atomizer used for the sulphur. The oil drops were obtained by atomizing in the usual method.

With the radiation from the zinc spark passing through a thin fluorite window and about 12 cm. of air photo-emissions were obtained from both the oil and the paraffine drops. Several emissions were obtained from each of 20 drops of both materials and no emissions were obtained from any drops of either material when radiations from the mercury lamp fell on the drops. The conclusion that both oil and paraffine are photo-sensitive and that their long-wave limit is shorter than $\lambda 2150$ (this is the shortest wave-length photographed through the mercury lamp used) seems fully warranted.

VII. SUMMARY.

1. A method has been given for detecting very weak photo-emission currents from insulators. Photo-emission currents were observed from sulphur, shellac, oil and paraffine.

2. The long-wave limit for sulphur has been placed between $\lambda 2400$ and $\lambda 2200$.

3. The long-wave limit for shellac was found to be some wave-length shorter than $\lambda 2200$.

4. The long-wave limits for oil and paraffine were found to be below $\lambda 2150$.

5. The photo-emission from molecules of sulphur and shellac consist in the ejection of a single electron from the molecule at each emission.

This work was suggested by Professor Millikan and carried out under his direction. I desire to express my appreciation for his interest and helpful suggestions, also to Mr. V. H. Gottschalk who was associated with me in the preliminary work.

THE DETECTING EFFICIENCY OF THE ELECTRON TUBE AMPLIFIER.

BY E. O. HULBURT AND G. BREIT.

SYNOPSIS.

Definition of Detecting Efficiency.—The detecting efficiency of an amplifier is defined as $\lim_{A \rightarrow 0} \frac{b_0}{A^2}$ where b_0 is the average change in the plate current of the last tube and A is the amplitude of the impressed grid voltage.

Relative Importance of Detecting Efficiency and Input Impedance.—A discussion of the necessity of taking into account both the detecting efficiency and the input impedance is given.

Measurements of Detecting Efficiency for a Transformer-Coupled Radio-Frequency Amplifier.—Measurements of the detecting efficiency of a transformer-coupled radio-frequency amplifier were made by means of a condenser potential divider and a sensitive quadrant electrometer.

Measurements of Amplification.—Measurements of the amplification due to each tube of the above amplifier were made. It was found that the *sound intensity* in the telephones was increased in the ratio of $9 \times 10^3 : 1$ owing to the use of the first two tubes.

I. INTRODUCTORY.

THE multi-stage electron tube amplifier such as is frequently used in radio practice and laboratory experiment is essentially an instrument for giving a relatively strong response, in the form of an electric current or a sound, to a relatively small alternating voltage impressed on its input terminals. It is of basic importance to be able to describe the behavior of an amplifier, or in other words, to be able to predict how a given amplifier will act under given conditions. The behavior of an amplifier depends upon the manner in which it reacts upon the external input circuit and upon the actions which take place inside itself. The general problem, then, of investigating the behavior of an amplifier requires the consideration of two interrelated problems, the first being the determination of the reaction between the amplifier and the external input circuit, the second being the determination of the action of the amplifier itself. The first we term the *problem of input impedance*, the second the *problem of detecting efficiency*.

To emphasize the importance of distinguishing between the two phases of the general problem let us consider a specific example. Suppose it were required to compare two different amplifiers. To do this it would seem sufficient to listen by means of them to a steady signal, connecting

each one in turn to the same circuit. Such a procedure, however, would not in general give a fair test. It may happen that the first of these amplifiers, say *A*, is equivalent to a capacity in series with a large resistance and gives a large change in the output plate current for a small input E.M.F.; while the second, say *B*, is equivalent to a capacity in series with a small resistance but gives a much smaller change than *A* in the output plate current for the same input voltage. If then the test were performed on a receiving circuit having a low resistance and capacity *B* would not influence the antenna current appreciably while *A* would decrease it on account of a large input resistance. The decrease might be so large that the rectification in the plate circuit might be less with *A* than with *B*. Thus the test would be in favor of *B*. If the same test were performed on a high resistance circuit the amplifiers being connected across a large capacity, then neither amplifier would affect the currents in the receiving circuit appreciably. Hence, since *A* gives a larger rectification for the same input voltage than *B*, the test will be in favor of *A*. It is thus seen that an intelligent choice between the two amplifiers cannot be made unless both phases of the problem have been solved.

2. THE INPUT IMPEDANCE PROBLEM.

Let us suppose that the grid and filament of the first tube of an amplifier are connected to a circuit in which high frequency current is flowing. This circuit is known as the input circuit. In general it is not legitimate to assume that the current in the input circuit is the same as it is if the amplifier were absent even though there may be no direct inductive coupling between the input circuit and the various internal circuits of the amplifier. The problem of unravelling the factors which enter into this effect is termed the input impedance problem.

This question has been discussed by H. W. Nichols¹ and by J. M. Miller.² They showed that the effect of a single tube, when the grid current is zero, is equivalent to that of an electric circuit having a definite resistance and reactance which was calculated. In a paper³ by one of us the single electron tube has been dealt with for the cases of positive and negative grid voltage, and the exact formulas have been worked out.

Thus, the problem of the input impedance is considered to have received a complete theoretical solution for the case of a single tube. The more complicated case of the multistep amplifier has as yet not been treated theoretically, although it would appear that the problem offers no fundamental difficulty.

¹ *Phys. Rev.*, 13, 404, 1919.

² Bureau of Standards Scientific Paper, No. 351.

³ "The Calculation of Detecting and Amplifying Properties of an Electron Tube from its Static Characteristics," G. Breit. (As yet unpublished.)

3. THE DETECTING EFFICIENCY OF THE AMPLIFIER.

The main purpose of the present work was the consideration of the second phase of the general amplifier problem, namely, the detecting efficiency of the amplifier. The detecting efficiency of the amplifier means, in general words, the efficiency of the amplifier to make weak signals intelligible. A precise definition of detecting efficiency is given later on. From general considerations it can be seen that the detecting efficiency depends upon the relation between the input grid voltage change and the resulting change in the output plate current. Therefore this relation must be determined either by theory or experiment before much can be said about the detecting efficiency of the amplifier. To determine this relation from theoretical considerations is, however, a somewhat difficult matter. Let us point out some of the difficulties briefly. Since the relation between the input grid voltage change and the output plate current change depends upon the amplification and rectification taking place in each tube, it is necessary to have an accurate knowledge of the characteristics of the electron tubes. These characteristics must be known with a precision sufficient to enable one to compute the first and second derivatives of the plate current and grid current with respect to plate voltage and grid voltage, respectively. It is further necessary to know the resistance and reactance of all the electrical circuits connected to the tubes for all frequencies. If the tube and circuit constants have been ascertained, then the output plate current can be computed for a given input grid voltage. Since both the measurements of the constants and the computations in the case of the amplifier are somewhat involved it appeared better from an experimental standpoint to measure directly the output plate current as a function of input grid voltage.

4. GENERAL EXPERIMENTAL PROCEDURE.

The type of amplifier chosen for investigation was the three tube high

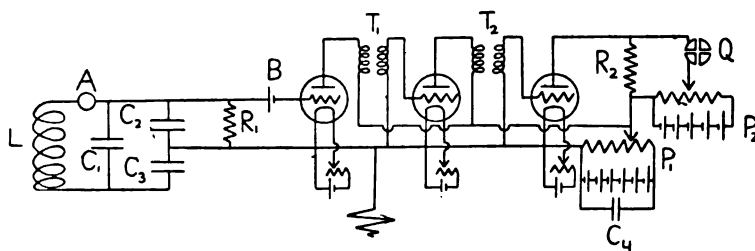


Fig. 1.

frequency transformer-coupled amplifier. This amplifier, shown schematically in Fig. 1, may be used to receive modulated radio frequency

signals. Apparatus was arranged to investigate the effect on the rectified component of the plate current of the last tube of impressing in some branch of the input grid circuit a high frequency E.M.F. of the form $A \cos \omega t$, where A is a constant. It is shown in the following paragraph that the results obtained from such a procedure are also true for the case where A is not a constant but varies in a certain manner.

Suppose that the frequency of modulation is so low that the reactances of the plate and grid circuits are negligible for that frequency. Let there be an external E.M.F. impressed in some branch of the circuit between F_1 and G_1 of the form

$$\varphi(\omega't) \cdot \cos(\omega t - \epsilon),$$

where ω/π is the radio frequency and ω'/π is the frequency of modulation. φ is a periodic function of period 2π , which is supposed to be always positive for real values of the argument. If $\varphi(x)$ is zero for some real value of x the modulation will be said to be *complete*. If there is no real value of x which makes $\varphi(x)$ zero the modulation is said to be *incomplete*. Since the reactances of all the circuits are negligible at the frequency $\omega'/2\pi$, it is legitimate to assume that at any instant of time, t_0 , the plate current of the last tube is the same as it would be if an E.M.F. $\varphi(\omega't_0) \cos(\omega t - \epsilon)$ had been impressed on the amplifier for an infinite time. If such were the case, the plate current would in general be different for different values of $\varphi(\omega't_0)$. Thus it is sufficient to investigate the effect on the plate circuit of the last tube of impressing in some branch of the input circuit an E.M.F. of the form $A \cos \omega t$ where A is a constant.

5. EXPERIMENTAL DETAILS.

The apparatus consisted of a condenser potential divider, the amplifier, and a quadrant electrometer connected in the plate circuit of the last tube of the amplifier. These will be described in the order named. The arrangement is shown schematically in Fig. 1.

In order to impress on the grid of the amplifier a high frequency voltage of known amplitude of the same order of magnitude as that obtained in the reception of radio signals a condenser potential divider was devised. This consisted of three variable condensers of capacities C_1 , C_2 , and C_3 connected to a coil L in which high frequency current was induced by a suitable electron tube generating set. A thermo-galvanometer A measured the total current through the combination of condensers. The amplifier was connected across C_2 , so that the voltage across C_2 was E_g , the effective grid voltage of the input tube of the amplifier. The high resistance leak R_1 (about 2 megohms) was con-

nected across C_2 to ensure a definite value of E_g during the experiment. The effect of R_1 upon the impedance of C_2 was negligible, because C_2 was large (about $0.05 \mu F$) and the frequency used was of the order of 3×10^6 . If the effective value of the current through A is I and the frequency of the current is $\omega/2\pi$, then it may be shown that

$$E_g = \frac{C_3 I}{\omega(C_1 C_2 + C_1 C_3 + C_2 C_3)}.$$

By adjusting C_3 to a small value and C_2 to a large value the coefficient of I in the above equation may be made so small that a readable value of I is obtained when E_g is comparatively small. The absence of stray capacity effects was tested by using different condenser settings and different values of current. In order to eliminate direct action between the generating set and the amplifier, twisted leads about five meters in length were used between L and C_1 , and between C_2 and the amplifier. This difficulty increases as the number of stages in the amplifier is increased, but by increasing the distance between the various parts of the apparatus it appears to be possible to reduce any direct action sufficiently for practical purposes.

The amplifier was a three-tube high-frequency transformer-coupled amplifier. The tubes were Western Electric Company tubes, Type VT1; they were used with the filament current always 1.10 amperes and the plate voltage always 22.0 volts. Separate storage cells supplied each filament; the plate voltage supply was common to all the tubes. By means of a standard cell B , Fig. 1, the input voltage E_g was kept always at a standard value. The plate battery was shunted by a $2 \mu F$ condenser C_4 . The transformers T_1 and T_2 were resonance transformers, both tuned to the same radio frequency. They were made of No. 36 silk-covered copper wire wound on paraffined wooden spools 3 cms. in diameter, 200 turns on the primary and 250 turns on the secondary winding.

The change in the value of the rectified component of the output plate current of the amplifier was measured by a Dolezalek quadrant electrometer Q , Fig. 1, connected across a high resistance R_2 placed in the plate circuit. The connections are shown in Fig. 1. The sensibility of the electrometer was 2500 mm. deflection per volt difference of potential between the quadrants. R_2 was 60,000 ohms. Therefore the electrometer deflection in millimeters could be reduced to the plate current change in amperes by dividing by 15×10^7 . P_1 and P_2 are potential dividers, P_1 serving to keep the plate voltage at a standard value, and P_2 to compensate for the potential drop in the resistance R_2 , so that the electrometer rested approximately at zero. When the input grid voltage

was changed, a deflection of the electrometer resulted which was proportional to the change in the rectified component of the output plate current.

It was important that the filament voltage of the last tube and the voltages of P_1 and P_2 be constant. Storage cells were found to be sufficiently steady for the purpose. When a slow drift of the electrometer occurred, the error was eliminated by averaging deflections.

6. EXPERIMENTAL RESULTS.

The electrometer deflections, which were proportional to the change in the value of the rectified component of the output plate current, were recorded for a series of values of the amplitude of the radio frequency voltage impressed on the input grid for various frequencies. It was seen that the electrometer deflection was nearly proportional to the square of the amplitude of the grid voltage. This meant that the rectifying action of the last electron tube was represented approximately by the expansion of the plate-current grid-voltage relation, by Taylor's theorem, in which derivatives of higher order than the second were neglected. In Fig. 2 are shown the curves for the electrometer deflection plotted against the square of the grid voltage for the five different frequencies corresponding to the wave-lengths 800, 825, 850, 875, and 900 meters.

7. THE DETECTING EFFICIENCY.

If A and b_0 denote the amplitude of the change in the input grid potential and in the rectified component of the output plate current, respectively, the detecting efficiency for a given frequency is defined conveniently by the relation

$$\text{detecting efficiency} = \lim_{A \rightarrow 0} \frac{b_0}{A^2}.$$

The detecting efficiency for a specified wave-length is obtained from the slope at the origin of the curve of Fig. 2 for that wave-length. The slopes at the origin have been computed for each curve of Fig. 2, and are shown in Fig. 3 plotted as ordinates against wave-lengths as abscissas. These slopes are reduced to the detecting efficiency by dividing by 15×10^7 which is the factor of proportionality between electrometer deflection in millimeters and the change in the rectified plate current in amperes. It is seen from Fig. 3 that the detecting efficiency of the amplifier is greatest at wave-length 850 meters. It is to be noted that the results shown in Fig. 3 give a complete solution of the question of the numerical value of the detecting efficiency of the amplifier. They do not, however, give any information as to the various factors upon which the detecting

efficiency depends. Nor do they yield information concerning the input impedance of the amplifier.

In order to check experimentally the assumptions of section 4 the following test was carried out. With a given current in A , Fig. 1, the

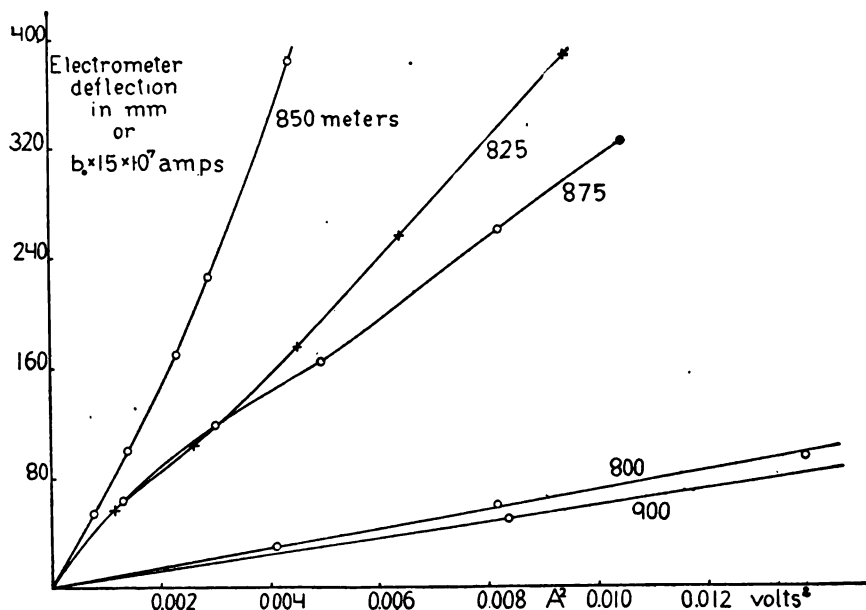


Fig. 2.

electrometer deflection was noted. A 500-cycle interrupter was introduced to chop the exciting current. When the coupling between the generating set and L was increased until the current had its previous

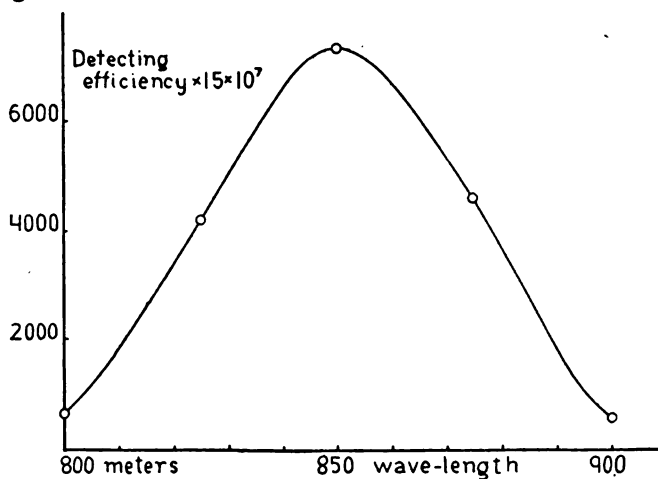


Fig. 3.

value, it was found that the electrometer deflection was the same as before. This test was repeated for various frequencies of the interrupter with the same result. This showed that modulating the exciting wave in the manner described above did not change the detecting efficiency of the amplifier.

8. AMPLIFICATION OF EACH STAGE.

The degree to which the detecting efficiency depended on each stage of the amplifier was determined by measuring the detecting efficiency of the amplifier for wave-length 850 meters with the first tube disconnected and then with the second tube also disconnected. It was found that the detecting efficiency of the amplifier with all three tubes was proportional to 9710, with two tubes 1020, and with one tube 104. The amplification due to the first tube was, therefore $\frac{9710}{1020} = 9.7$ and that due to the second tube was $\frac{1020}{104} = 9.8$.

This meant that the square of the radio frequency voltage from filament to grid of the second tube was 9.7 times as great as the square of the input grid voltage provided that the voltage was sufficiently small. Any rectifying action which occurred in the successive stages did not influence the result, because the slope of the curve was taken at the origin, and rectification is known to depend on terms in ΔE , of higher order than the first. For amplitudes which can no longer be considered as infinitesimal the rectification may manifest itself by shifting the operating point on the characteristic of each tube and thus changing the amplification constant and internal resistance of the tube. This may be the explanation of the curious bends in the curves of Fig. 2.

9. SOUND INTENSITY AMPLIFICATION.

If a modulated radio frequency signal is received by means of the amplifier the kinetic energy of the vibrations of the telephone diaphragm, and also the energy of the sound waves produced thereby, is proportional to the square of the amplitude of the change in the rectified component of the plate current. The amplitude itself is, however, proportional for the same input radio frequency voltage to the square of the detecting efficiency as defined above. Consequently, for weak signals the sound intensity is proportional to the square of the detecting efficiency. In the case of the amplifier cited above the sound intensity was increased in the ratio of $9 \times 10^3 : 1$ owing to the use of the first two tubes.

JOHNS HOPKINS UNIVERSITY,

March, 1920.

THE MINIMUM ARCING VOLTAGE IN HELIUM.

BY K. T. COMPTON, E. G. LILLY, P. S. OLMSTEAD.

SYNOPSIS.

Helium Arc; Minimum Arcing Voltage.—The arc was stimulated by an intense thermionic current passed between electrodes in very pure helium. 20 volts was found to be a well-defined minimum voltage at which the arc will strike or break, although it may be maintained on voltages as low as 8 volts after having struck.

Helium Spectrum; Voltage for Excitation of Lines and Bands.—Apparently the ordinary helium and parhelium lines and the bands are excited whenever the arc strikes. The line 4686 of the *enhanced system* was never observed below 55 volts, and was stronger above 80 volts. The lines of the *sharp subordinate series of pairs* are peculiar in that their intensity decreases, relatively to that of the rest of the spectrum, as the voltage is increased.

The results are in accord with *Bohr's theory of radiation and atomic structure*.

INTRODUCTION.

RECENT investigations of the production of radiation and ionization by electron impacts in helium¹ have shown that the minimum radiating potential of this gas is close to 20.2 volts and its minimum ionizing potential is 25.5 volts. One of the writers has shown² that these values apply to radiation and ionization set up by a single electron impact against a normal unexcited atom, whereas, if the electron current and gas density are relatively large, ionization may occur at any voltage above 20.2 volts. This ionization at abnormally low speed is presumably due to impacts against atoms which are in a relatively unstable condition due either to preceding impacts or to the absorption of radiation coming from neighboring atoms which have been struck. The latter of these causes is much more important than the former. Neither radiation nor ionization has ever been observed below 20 volts.

These considerations suggest that 20.2 volts should be the minimum voltage at which an arc can strike in helium and that the necessary conditions for obtaining an arc at this voltage are an intense bombarding electron current and relatively high gas pressure, so that the amount of ionization due to the cumulative effect of impacts and radiation may be sufficient to cause an arc.

Rau³ and Richardson and Bazzoni⁴ have published results of experi-

¹ F. Horton and A. C. Davies, Roy. Soc. Proc. A., 95, p. 408, 1919; Franck and Knipping, Phys. Zeit., 20, p. 481, 1919.

² K. T. Compton, Phil. Mag. (in print)

³ Sitz. Ber. d. Phys. Med. Ges. zu Würzburg, p. 20, 1914.

⁴ Nature, 98, p. 5, 1916.

ments on arcing potentials and the arc spectrum in helium. The lowest arcing voltage found by Rau was 24.5 volts. This was in the presence of mercury vapor, whose ionization increased the current density and thus caused the helium arc to strike at a lower voltage than in pure helium. Apparently 29.5 volts is the lowest voltage at which Rau obtained the arc in pure helium. Richardson and Bazzoni, working with helium in the presence of mercury vapor, obtained the helium arc spectrum at 22.5 volts, the mercury arc spectrum appearing first at a slightly lower voltage.

The following experiments were made to determine the voltage at which the arc strikes under various conditions of gas pressure and electron current density in very pure helium. It was found that the arc could be made to strike at voltages as low as 20 volts, but never lower. Under favorable conditions, however, the arc could be maintained at much lower voltages, the lowest voltage observed being 8 volts, with a gas pressure of 5 mm. and a current of about an ampere through the gas. Observations of the spectrum were also made under various conditions.

APPARATUS.

The arc was obtained between an incandescent tungsten wire cathode and a nickel disk anode, enclosed in a glass bulb. Wires of various diameters from 0.06 mm. to 0.25 mm. were used, and their lengths varied from 1 cm. to 2 cm. The distance between the electrodes varied between 1 cm. and 3 cm. in different bulbs. A large bulb of cocoanut charcoal was sealed directly to one end of the experimental tube, while to the other end was attached the glass tubing connection to the helium reservoir and pump. This connecting tube was bent into two U tubes near the bulb. The one nearest the bulb was a trap to prevent the entrance into the bulb of water or mercury vapors and the further one contained charcoal. The bulb and the charcoal tubes were baked out in an electric furnace maintained at a temperature between 300° and 350° C. for several days while the apparatus was evacuated by a diffusion pump. Liquid air was then applied to the trap and later to the two charcoal tubes before the helium gas was admitted. A mercury hand pump with a magnetically operated valve permitted the gas pressure to be adjusted to any value between 0 and 24 mm. A Hilger wave-length spectrometer was used for the examination of the spectrum. In every case observations were made at increasing filament temperatures until the wire burned out. The voltage across the arc and the current through it were regulated by series and parallel resistances connecting it to a 110 volt storage battery.

RESULTS.

From among the large number of sets of observations made only a few examples can be given in the accompanying figures. Possibly the results can be most concisely presented by discussing them under the four following cases, which are somewhat arbitrary and not always mutually exclusive.

Case I.—No arc: The current increases as ionization and radiation set in above 20 volts, but there is no discontinuous change or visible

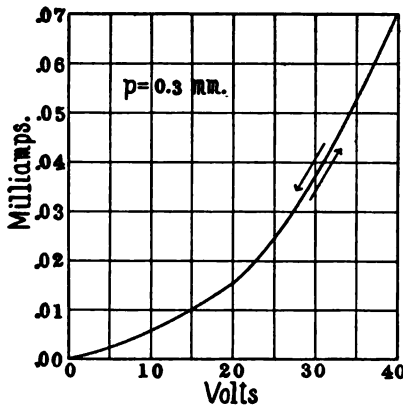


Fig. 1.

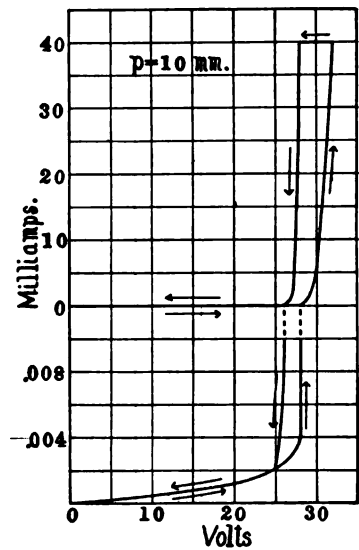


Fig. 2.

radiation. Variations of current with voltage are reversible. This case is observed at any gas pressure if the thermionic current is small and with any thermionic current if the gas pressure is small. Example: Fig. 1.

Case II.—The arc strikes at a voltage above about 29 volts and breaks at a voltage above 20 volts. There is no characteristic value for either the striking or breaking voltage, since they may be varied by varying the thermionic current or pressure. In general any change which increases one, also increases the other. The arc current always increases as the applied voltage is increased, but there is a region of irreversible changes between the striking and breaking voltages. Example: Fig. 2.

Case III.—The arc strikes at a voltage below 29 volts and breaks at 20 volts. Here it is found that, by increasing the thermionic current by raising the filament to a higher temperature, the striking voltage may be reduced as low as 20 volts, but the breaking voltage stays constant

at 20 volts. Between the striking and breaking voltages the current-voltage changes are irreversible. Example: Fig. 3.

Case IV.—The arc strikes at 20 volts and breaks at 20 volts, although it may be maintained at lower voltages if the current of the arc is increased by decreasing the series resistance. But no further increase in the thermionic current will cause the arc to strike or break at voltages below 20. This seems, therefore, to be a definite minimum value for the arcing voltage—and, therefore, for the excitation of the spectrum. If, after the arc has struck, the series resistance is decreased, the current increases and the potential drop across the arc decreases. In one case the arc was thus maintained on 8 volts and in another case on 10 volts. There seems to be no lower limit for the maintenance of the arc, once it has struck, for this seems to be limited only by the size of the currents which the electrodes will

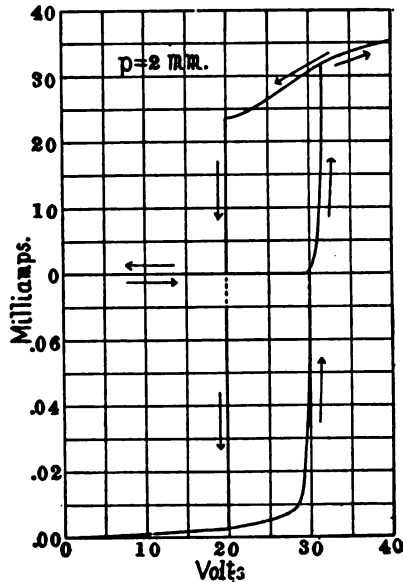


Fig. 3.

carry. In every such case, as the series resistance is again increased, the voltage again rises to 20, where the arc breaks. Example: Fig. 4.

There are several interesting peculiarities of the arc which have been noticed during the experiments. Between Case III. and Case IV. there is an intermediate condition which may be considered as marking the division between them. Here the arc strikes at 20 volts, and remains at 20 volts however the current through the arc be increased by decreasing the series resistance. In this condition the resistance of the arc varies exactly inversely with the current through the arc. In one case the current between the electrodes was varied at will from 50 microamperes to 50,000 microamperes without producing any variation in the potential drop of 20 volts.

The current and voltage of the arc, with large currents, often exhibited discontinuous changes as the geometrical distribution of the arc in the tube suddenly changed. As far as we could tell, these shifts gave no indication of critical voltages in addition to that at 20 volts, since they seemed to be more or less accidental and occurred at different voltages at different pressures and with different positions and shapes of the electrodes.

It should be stated that the observed critical voltage which we have called 20 volts, in reality was sometimes as high as 20.5 volts and sometimes as low as 19 volts. These variations, however, are accounted for by the average initial velocities of emission of electrons from the filament at the temperatures used.

In securing an arc under the conditions of Case IV., the optimum gas pressure was between 4 mm. and 10 mm. Higher gas pressures would be advantageous, because of the greater probability of ionization by

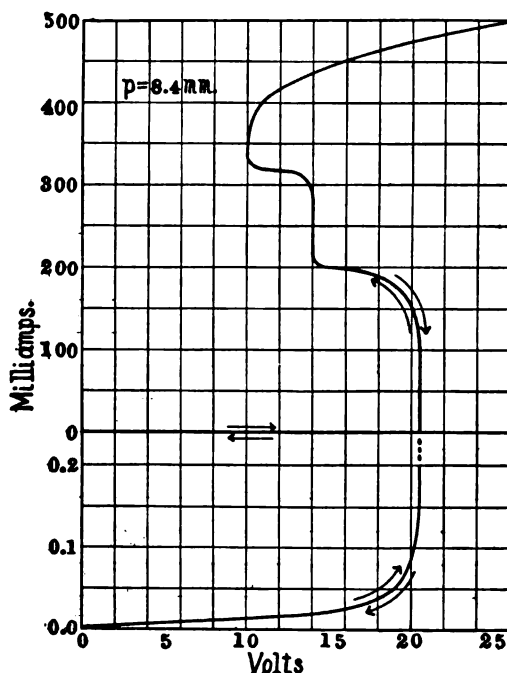


Fig. 4.

cumulative impacts, if it were not for the fact that, at such high pressures, the electrons make so many collisions in their zig-zag course that the aggregate amount of energy which they lose as a result of the momentum imparted to the atoms becomes considerable.¹ Thus, at the higher pressures, voltage greater than 20 volts must be applied in order that the electrons may acquire a velocity corresponding to 20 volts.

SPECTROSCOPIC OBSERVATIONS.

As far as we could tell, the entire helium spectrum, with the exception of the enhanced lines, appeared whenever the arc struck, and therefore

¹ Benade and Compton, *PHYS. REV.*, 11, p. 184, 1918.

appeared at potential differences as low as 20 volts. This was apparently true of the band as well as of the line spectrum, although the relative faintness of the bands made it impossible to see them at voltages quite as low as 20. With the most intense arc, the lowest voltage at which the bands were observed was 26 volts, but the indications were that we missed them below this voltage because of lack of intensity rather than because they were not excited. The most prominent line of the enhanced spectrum is the first member 4686 of the series $4N[(1/3^2) - (1/m^2)]$. Rau was able to excite this line only with potential drops of 80 volts or more. At gas pressures of less than about 3 mm. and with thermionic currents not too intense, we have found that this line suddenly appears as the voltage is raised above 80 volts, but, with more intense currents and pressures between 3 mm. and 5 mm., the line may be seen at voltages as low as 55 volts. In these cases, however, the intensity of the line considerably increases if the voltage is raised above 80 volts. At still higher gas pressures, such as 10 mm., we were unable to excite this line at any voltage used.

We verified Rau's and Richardson and Bazzoni's observations that the line 4713, which is the second member of the sharp subordinate series of the system of pairs, decreased in intensity with increasing voltage, although the lines of the other series increased strongly. We observed this also in the case of the first member 7066 of the same series.

Although the band spectrum was excited at apparently the same voltage as the rest of the arc spectrum, it appeared that it was relatively most intense at high gas pressures, from 10 mm. up. At these pressures we observed the bands noted by Fowler¹ very easily, and, in addition, some fainter bands. One of these consists of two or more sections extending from about 5748 to 6044. Another seems to consist of six pairs, equally spaced, between 5719.5 and 5657. There is another set of fine lines beginning at about 4575 and extending to 4378, with evidence of the convergence of a series at the latter wave-length. We are somewhat hesitant about mentioning these, since our spectroscopic equipment was not sufficiently elaborate to permit of refined observations, and yet we are unable to attribute them to an impurity, since the gas seemed to be very pure. There was no trace of the spectra of mercury, other inert gases (with the possible exception of neon) or of hydrogen (except in the case of one bulb on which a quartz window was waxed and which could not be given adequate "baking out"). Several of the faint lines of these bands corresponded, within the limits of accuracy of our observations, with certain neon lines, but there was no trace of the three or four strongest neon lines in this region, or elsewhere in the spectrum, at pres-

¹ Roy. Soc. Proc. A, 91, p. 208, 1915.

tures at which the bands were prominent, and there was no uniformity between the neon spectrum in general and the observed spectrum.

DISCUSSION.

The results of this investigation confirm the view that ionization may occur at any voltage above the resonance potential as a result of the impact of an electron against an atom which is already partially ionized through the absorption of a quantum of resonance radiation from neighboring atoms. In other words, the presence of resonance radiation changes the condition of the atom to one of easier ionization. In the intense arcs in which the discharge was maintained far below 20 volts, it is evident that practically all the atoms were in an abnormal, easily ionizable, condition. We may picture the state of affairs as one in which an electron, returning toward its normal most stable position step by step, and radiating energy at each step, is again partially or entirely ejected towards less stable positions by absorption of radiant energy or by impact before it has an opportunity to return to the most stable position. Thus the bulb contains a large density of radiant energy of various spectral frequencies and helium atoms in all stages of ionization, or degrees of stability. The more intense the arc, the larger is the proportion of atoms whose electrons are in the outer orbits, or least stable configurations, so that the arc may be then operated on reduced voltage. The first step, however, essential to reaching this condition and the excitation of visible radiation, is the displacement of the electrons from their most stable to their next most stable configuration—a displacement requiring an amount of energy which is acquired by an electron in falling through 20 volts.

The investigations of low voltage arcs in mercury and other metallic vapors¹ are to be interpreted in a similar way. In every case the arc may be made to strike at a voltage equal to the resonance potential, and may be maintained at a lower voltage. Here, however, the voltages are low, so that the initial velocity distribution of the electrons from the filament introduces uncertainties of a relatively large magnitude as compared with those in the present case. By analogy with these metallic vapors, we should expect to find a very strong emission series in helium, with the first member (also an absorption line) at about 605 Å. and convergence at about 484 Å., corresponding to 20.2 volts and 25.5 volts.

The theories of atomic structure, which are rather uncertain with regard to the constitution and radiation of systems with more than one electron, have been quite successful in accounting for the spectra of

¹ McLennan, *Phys. Soc. Lond. Proc.*, 31, p. 1, 1918; Hebb., *Phys. Rev.*, 9, p. 372, 1917; 11, p. 170, 1918; 12, p. 482, 1918 and others.

atoms with a nucleus and a single electron. On Bohr's theory, the spectral series from helium atoms which have lost one electron should be given by the formulæ

$$4N\left(\frac{1}{n^2} - \frac{1}{m^2}\right),$$

whence the negative energies of the various configurations of an electron are given by $4N/n^2$, where n may have various integral values from 1 to ∞ . Thus the energy required to change the electron from its most stable to its next most stable condition is $4N(1/1^2 - 1/2^2)$, or $3N$; while the energy required to completely remove it is $4N$. The minimum energy required to change the electron from the most stable condition to that condition in which it may emit the line 4686 is $4N(1/1^2 - 1/4^2)$, or $15N/4$. Thus the energy, in equivalent volts, required to remove the second electron from an atom already ionized, is 54.3 volts, while the energy required to cause such an atom to emit the 4686 line corresponds to 50.8 volts. Similarly, the minimum energy required to cause an atom, originally neutral and normal, to emit this radiation corresponds to $50.8 + 25.5 = 76.3$ volts; that required to cause an atom which has absorbed a quantum of the resonance radiation to emit this line corresponds to $76.3 - 20.2 = 56.1$ volts; that required to doubly ionize an atom at a single impact is $25.5 + 54.3 = 79.8$ volts if the atom is originally normal and is $79.8 - 20.2 = 59.6$ volts if the atom has absorbed resonance radiation.

There are thus various ways in which the 4686 radiation may be excited. Of these, the excitation by a single impact at 79.8 volts or more should predominate at low pressures and currents, while at higher pressures, with most of the atoms in the abnormal state, the excitation should be caused by 56.1 volts or possibly even by 50.8 volt impacts. These possibilities accord with our observations, in which the 4686 line was observed at voltages as low as 55 volts in a very intense arc, but with evidence of another method of excitation at 80 volts which was relatively more important at low pressures and arc intensities. The failure to observe the line at pressures above 10 mm. is evidently due to the very small chance of the electrons falling through a sufficiently large potential difference without losing their energy at an ionizing or radiating impact.

The fact that the lines of the so-called Parhelium series appear at the same voltages as those of the Helium series renders untenable Stark's conclusion that they are due to atoms which have lost more electrons than those atoms which give rise to the helium series lines.

ELECTRIFICATION BY IMPACT.

BY HAROLD F. RICHARDS.

SYNOPSIS.

Electrification by Impact; Measurement of the Charge Produced by Collision between a Metal and a Dielectric.—After briefly discussing the unsuccessful attempts which have been made to formulate a satisfactory theory to explain electrification by friction, the author suggests that impact of dielectric upon metal, without sliding friction, may cause an electrical effect whose laws will shed light upon the frictional phenomenon. An apparatus is described for measuring the electric charge produced when a disc or sphere of dielectric material collides with a metal disc. The charges obtained in this manner ranged from 0.16 to 9.83 e.s.u., and produced potentials of 2.41 to 183.8 volts upon the metallic systems employed. These charges are of the same order of magnitude as those obtained by friction. The experiment was performed with *various metals and dielectrics*, and in every instance the metal received a positive charge. In no case was there any evidence of the erratic variation which others have found to be characteristic of electrification by friction. Curves are given which show the *variation of charge with velocity of impact* and with the *mass of the impinging system*. The charge produced by a single collision increases with each of these factors, but the velocity of impact was found to exert a greater influence than the mass of the moving body, in determining the amount of the charge. In certain cases velocities were attained at which the electrification due to a single impact reached a maximum value.

Relation of Charge to Capacity.—The quantity of charge produced by a given collision was shown to be independent of the capacity of the metallic system.

Effect of Repeated Impacts.—When many impacts were performed in rapid succession, the amount of charge increased to a maximum. This maximum was shown to be conditioned rather by the quantity of charge present upon the dielectric than by the potential of the metal anvil.

Discussion of Results.—The author concludes that there is no direct dependence of the electrical energy upon the mechanical energy lost in impact, and that electrification by impact is similar in nature to the contact effect between metals. The results are considered to support Helmholtz's theory regarding the nature of electrification by friction.

I. INTRODUCTION.

THE very meager quantitative results and theoretical study of electrification by friction have prevented the formulation of a satisfactory theory to explain the phenomena involved. Perhaps the most important work in this field is that of Owen,¹ who finds many indications that the effect is similar to the contact difference of potential between metals, thus agreeing with Helmholtz.² The latter had previously suggested that the function performed by the frictional work is

¹ Phil. Mag., XVII., p. 457 (1909).

² Wissenschaftliche Abhandlungen, Erster Band, p. 860.

merely to bring the molecules of the two substances into closer contact. Owen concludes that, with a sufficient amount of frictional work, the charge reaches a constant maximum, and that this maximum is attained with a smaller amount of work the greater the pressure between the rubbing surfaces. He notes further that less work is required to produce a given charge if contact is improved by a series of rubs. Jones,¹ however, definitely rejects these views, as is shown by the following quotation from his paper: "Frictional electricity appears, therefore, to be an effect of a different order from that of contact electricity, and it is worth while considering whether the facts cannot be accounted for on some other hypothesis." He assumes that the rate of production of charge is proportional to the rate at which work is performed, and explains the fact that the charge reaches a maximum value by assuming that back-leakage occurs in an amount proportional to the total charge present. It is difficult to see how this theory can account for the fact that one substance remains positive with respect to another, a phenomenon which is readily explained by the theory of Helmholtz. Thus, in spite of attempts to formulate a consistent theory, no great success has been attained, because the data of electrification by friction remain essentially erratic. This is abundantly shown by the experiments of Owen and Jones, and also by the later work of French² and McClelland and Power,³ who find that not only the amount, but even the sign, of the charge varies with the conditions to which the rubbing surfaces are exposed before an experiment.

The present work was therefore undertaken to find what electrical effect would be produced by intimate contact, *without friction*, between metals and dielectrics. Both Owen and Jones have found that mere contact of such substances does not produce a detectable amount of electrification, but it is probable that the contact in their experiments was not such as would bring the surfaces sufficiently close together to furnish an adequate test. It was my belief that the more intimate contact produced by collision might cause an effect whose laws would shed light upon the nature of electrification by friction. It seems evident that the more definite nature of a single impact at a known velocity will give to the data of such an effect a greater consistency than that which characterizes the results of electrification by friction.

¹ Phil. Mag., XXIX., p. 272 (1915).

² Phys. Rev., IX., No. 2, p. 151 (1917).

³ Roy. Irish Acad. Proc. 34, Sect. A, p. 40 (1918).

2. APPARATUS.

The apparatus used in the first experiments is indicated in Fig. 1. An ebonite disc *E* fits firmly into a brass socket attached to the lower end of a brass tube *A* sliding in a cylinder *D*. *A* is 2.5 cm. outside diameter, 17.5 cm. long, while *D* is 3.1 cm. inside diameter and 35 cm. long. The sliding tube is provided with studs to form a three-point contact at top and bottom, and these were carefully machined so as to permit free fall with minimum lateral freedom. Uniformity in bore of the stationary cylinder, the low-friction close-fitting contact of the bearing studs, and the relatively large ratio of the length of the falling

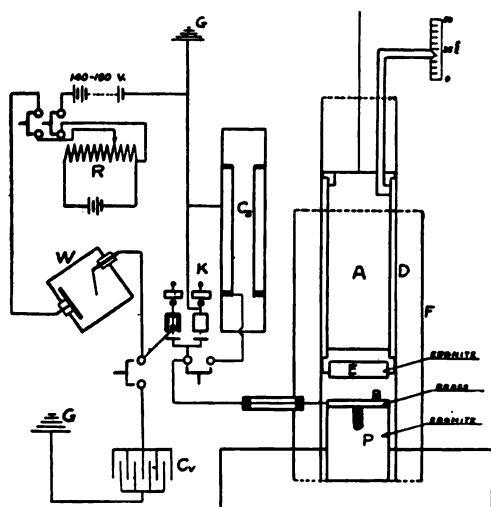


Fig. 1.

tube to its diameter, all combined to ensure uniform orientation of the ebonite surface at the moment of impact with a brass disc *B*. The latter was fastened securely to an ebonite pedestal *P* by means of a threaded projection of its own material. The pedestal was firmly held in a heavy iron base. All the horizontal surfaces were rendered parallel by careful turning in a lathe. An index rigidly attached to the plunging cylinder indicated the height of its fall. The air space between the cylinders provided windage, but windows were also constructed at the base of *D* to prevent compression of air. *F* is an earthed metal cylinder. The ebonite and brass discs were each 2.5 cm. in diameter, and of a thickness of 0.95 and 0.32 cm., respectively.

The charge produced upon the metal disc was measured by means of a Wilson tilted electroscope *W*, which was chosen on account of its low

capacity (2.3 cm.), and consequent high sensitivity to quantity of charge. Deflections were observed with a microscope containing a micrometer eyepiece. The plate of the Wilson electroscope was maintained at a constant potential of 140 to 180 volts by a battery of accumulators, and the sign of this potential was always kept the same as that of the charge to be measured, so that the instrument was used in its most stable condition. The scale of the microscope contained 40 divisions, and the position of the goldleaf could readily be estimated to 0.1 division. Calibrations made with multiples of Weston standard cells furnished curves showing the voltages corresponding to given deflections. Various sensitivities were used, ranging from 3 to 10 divisions per volt, and for these values the deflections were nearly, but not quite, proportional to the potential of the goldleaf. Accurate adjustment to the desired sensitivity was obtained by the use of a potentiometer arrangement, indicated at *R*. Under the conditions described, the Wilson electroscope furnished a most convenient and satisfactory means of measuring the charges produced by impact. The metal disc and measuring apparatus could be earthed or insulated either together or separately, by means of a double key *K* operated by a simple arrangement of two strings. Care to prevent leakage was taken and, in addition, all charged portions of the apparatus were housed in earthed metal boxes for electrostatic protection.

3. MEASUREMENT OF CAPACITY.

The capacities used in these experiments were so small that they were measured by comparison with a specially-constructed condenser of concentric cylinders possessing a capacity of the same order. The dimensions of the inner tube were 2.70 cm. outside diameter and 43.15 cm. length; the outer cylinder was 4.45 cm. inside diameter and 52.50 cm. long. As the length compared to the space between the cylinders was large, the ordinary formula for capacity can be used. The calculated capacity proved to be 43.23 cm. (electrostatic units). All values of capacity used in the experiments were determined by comparison with this condenser. The capacity of the system was altered whenever necessary by inserting in the circuit an adjustable parallel-plate condenser, whose range of capacity was from 33.9 to 346.1 cm. The sliding plates of this condenser were earthed, and the plates receiving the charge were mounted rigidly on sulphur pedestals.

4. EXPERIMENTAL RESULTS.

A. Variation of Charge with Velocity of Impact.

The surfaces of the brass and ebonite impact discs were made as flat and smooth as possible, and mounted so as to be parallel at the moment

of contact. The cylinder, carrying the ebonite disc, was loaded with lead shot to give a total weight to the falling system of 753.8 grams. The ebonite disc was raised to various heights by means of a string passing over a light pulley, and then released by a mechanical trip so as to fall freely upon the brass anvil. The index indicated that there was no rebound, and therefore only a single impact. The ebonite was then raised 20 cm. above the brass disc and the charge on the latter measured. Tests showed that the charged ebonite plunger was sufficiently distant to produce no appreciable inductive effect upon the measuring system. Between successive impacts the ebonite surface was discharged by means of radium. Velocities of impact were calculated from the observed distances of fall.

A single curve was obtained as follows: Three readings were taken at each selected velocity when the height of fall was successively increased, and then the run was repeated with decreasing velocities. The charge

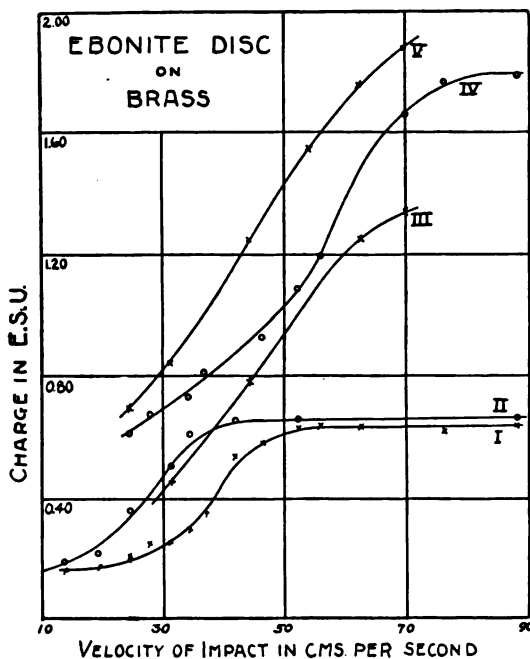


Fig. 2.

produced at a given velocity was then calculated from the six readings of the ascending and descending series, and this average was plotted. The charges obtained at a given velocity usually increased slightly after repeated impacts, probably due to further flattening of the surfaces, and

the effect of this small secular variation was eliminated by averaging together the ascending and descending runs. The usual variation from the mean in a series of observations at a single velocity was 3 per cent. No especial care was taken of the surfaces other than an occasional cleansing with alcohol. This treatment made only a small difference, usually increasing the charge by a small amount and rendering consecutive readings slightly more consistent. Upon one occasion cleaning the ebonite with a fine, flat file increased the charge from 0.54 to 0.99 e.s.u., but, in a velocity run immediately following, the charge reverted to its former value.

The curves exhibited in Fig. 2 show the charges obtained with a single impact at various velocities. These representative curves were selected from observations extending over a period of a month or more, during which many hundreds of collisions were produced between the same surfaces. It is seen at once that impact produces charges of the same order as those obtained by friction. These charges were of sufficient magnitude to raise the brass disc to fairly high potentials. Even the relatively low maximum charge shown in Curve II produced a potential of 9.75 volts upon the metallic system, whose capacity was 19.9 cm. (2.21×10^{-8} microfarads). The curves show a steady secular increase in the charge obtained at a given velocity, and also that, in the early work, when the surfaces were fresh, a maximum was attained at the velocities used, whereas no maximum was reached in the later experiments with the same surfaces. It may be supposed that this is due to increase in the contact-area, inasmuch as many collisions would render the ebonite surface more nearly plane.

It is worthy of note that the ebonite disc *always* produced a positive charge upon the brass anvil. There was no tendency towards the reversal of sign observed when charge is produced by friction. Even the slight residual charge obtained when the dielectric was lowered upon the metal with as nearly zero-velocity as possible was always positive, even when in amount it was not more than 0.02 e.s.u.

B. Relation of Charge to Capacity.

An exhaustive series of experiments was performed to find the relation between the capacity of the metallic system and the charge produced upon it by a given impact. The variable condenser was adjusted so that the charge due to a single collision gave a convenient deflection. Three readings were taken when the charge was formed at the capacity of the whole system, then three readings when the capacity of the measuring system was not added until after the impact had taken place. This

process was repeated a number of times in close succession, the charge being formed alternately upon the metal anvil and upon the whole system, but always measured at the same capacity. The velocity of impact remained constant throughout. Thus the only variable was the capacity at moment of production of charge. The mean charges found in four such experiments are given in Table I.

TABLE I.

Experiment No.	Velocity of Impact (cm./sec.).	Capacity at Impact (cm.).	Charge (e.s.u.).
1.....	67.1	{ 19.2 152.5	{ 1.42 1.43
2.....	62.6	{ 19.2 134.9	{ 1.98 1.97
3.....	70.0	{ 123.2 168.8	{ 2.22 2.23
4.....	62.6	{ ∞ 168.8	{ 1.04 1.02

In order to employ infinite capacity the metal disc was earthed at the moment of impact, but insulated before the ebonite disc was raised. It had previously been found that the potential of the system was not altered until the dielectric was lifted.

These results show that the charge is independent of the capacity. On account of the important conclusions which can be drawn from this fact, the observations for one experiment are given (Table II.). Readings were taken alternately at the two capacities.

TABLE II.

Capacity at Impact.	Volts.						Mean Charge.
19.2.....	2.77	2.82	2.85	2.88	2.83	mean 2.83	1.42
152.5.....	2.78	2.84	2.85	2.91	2.87	mean 2.85	1.43

C. Influence of Mass of the Plunging System.

Fig. 3 shows velocity curves obtained in the same manner as previously described. The ordinates are the potentials produced by a single impact at the capacity of 133.3 cm. The run represented by curve *B* immediately followed *A*, the mass of the impinging system being decreased as indicated. Runs *C* and *E*, and *G* and *F*, were performed similarly. For the smaller mass, the charge is seen to be more nearly proportional to the velocity, and there is not the tendency to approach a maximum shown by the curves for greater mass. The curves show that a mean re-

duction in weight of 48 per cent. produced a mean decrease in charge of only 16 per cent., for constant velocity of impact.

D. Effect of Repeated Impacts.

Experiments were performed with repeated impacts in order to deter-

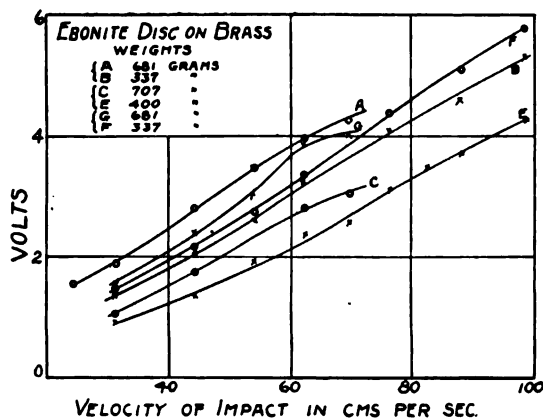


Fig. 3.

mine whether there is a maximum potential or a maximum charge obtainable by collision. Fig. 4, Curve I., shows the results when the

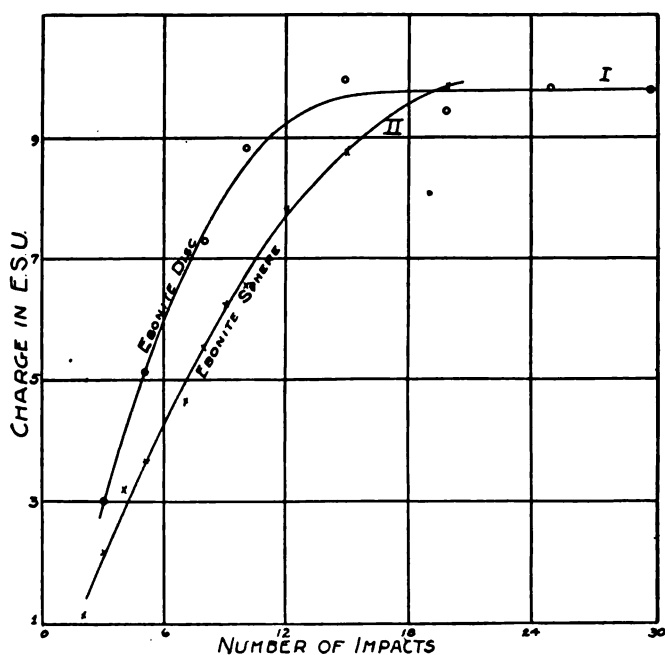


Fig. 4.

ebonite surface was not discharged between successive impacts. Fourteen collisions at a constant velocity of 62.6 cm./sec. could be produced in one minute. Insulation was excellent, so that leakage during the largest number of impacts was nearly negligible and could readily be allowed for by a small correction. The curve shows a maximum charge of 9.78 e.s.u., which produced a potential of 165.1 volts upon the metal disc. The charge was shared with a larger capacity for measurement. Then the ebonite disc, initially uncharged, was allowed to make five impacts upon the brass anvil, which was charged to various potentials by means of accumulators. The results are given in Table III.

TABLE III.

Velocity of Impact (cm./sec.).	Initial Potential of Brass Disc (Volts).	Charge Added by Five Impacts (e.s.u.).
62.6	280.2	3.31
62.6	350.2	3.42
62.6	386.0	3.09

There is seen to be no indication of a limiting potential, at the voltages used. It is thus evident that the limiting factor is not the potential of the metal anvil, but the amount of charge present upon the dielectric.

E. Effect of Static Pressure.

It has already been mentioned that a charge of approximately 0.02 e.s.u. was always produced when the ebonite disc was lowered upon the brass anvil with as nearly no velocity as possible. To determine whether greater pressure would increase this charge, weights ranging from 4.38 to 12.08 kilograms were placed upon the ebonite disc while it rested upon the brass. No increase in charge was observed.

5. THE HURLING EXPERIMENTS.

A. Apparatus.

In order to perform the experiments under widely different conditions, an apparatus was devised for projecting a sphere of dielectric upon an insulated metal disc. The construction is indicated in Fig. 5. A weight *P* falling upon the short end of a light, rigid oak lever *L* imparted a high velocity to a small brass tube *H* sliding in a heavy sleeve. The rise of the sliding tube was small, its motion being suddenly arrested by the abutments shown. The pole of the electromagnet *M* was provided with a collar *A* of non-magnetic material, in order to ensure uniform release of the projectile without lateral motion. The ball of dielectric rested in the smoothly beveled edge of the sliding tube. A calibration curve

was obtained showing the maximum free rise of the ball for different distances traversed by the falling weight. The action of this hurling device was constant to such a degree that the maximum variation from the mean in a series of flights was only 1.5 per cent. at the highest velocities used. The velocity of the ball at a given height h could readily be calculated from the formula $v = \sqrt{2g(H - h)}$, where H is the maximum flight for the drop used. The ball could be projected vertically, so as to be pocketed in the hurling cup upon rebound, although in practice the ball was received upon heavy lead foil spread a little to one side of

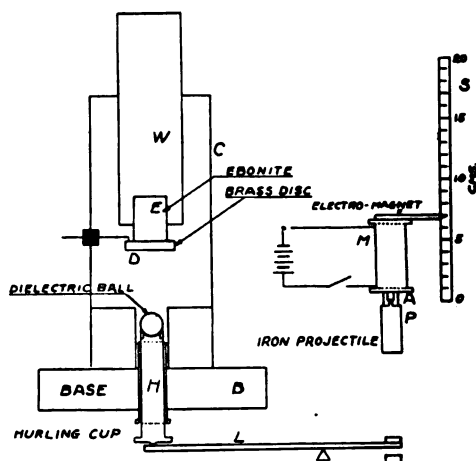


Fig. 5.

the sliding tube, and then rolled into the cup. The metal disc upon which the ball impinged was attached to an ebonite pedestal E inserted in a massive wood cylinder. C is a metal cylinder to afford electrostatic protection. The charge received by the disc was measured in the manner previously described. The capacity of the metal disc and its connecting wire was 16.0 cm.

B. Variation of Charge with Velocity.

Fig. 6 exhibits specimen curves showing the charge produced at various velocities by a single impact of ebonite upon brass. The dielectric sphere was 2.5 cm. in diameter and had a mass of 10.88 grams. The brass disc was 5.0 cm. in diameter by 0.32 cm. thick, and possessed a fairly high polish. The flight of the ball through the air before collision occurred was 11.6 cm. Discharge of the ball was effected before every impact by passing it rapidly through a flame, as it was found that the results were not affected by substitution of this agent for radium. Each

point on the curve is the mean of five or six observations at the velocity indicated. The maximum variation of a single reading from the mean was 8 per cent. As in the experiments with the plunging ebonite disc, the initial curves show that the charge reaches a maximum for certain velocities, whereas later work indicated that a much higher velocity would be required to produce a maximum. A possible explanation for this is that after a large number of impacts the ebonite surface was

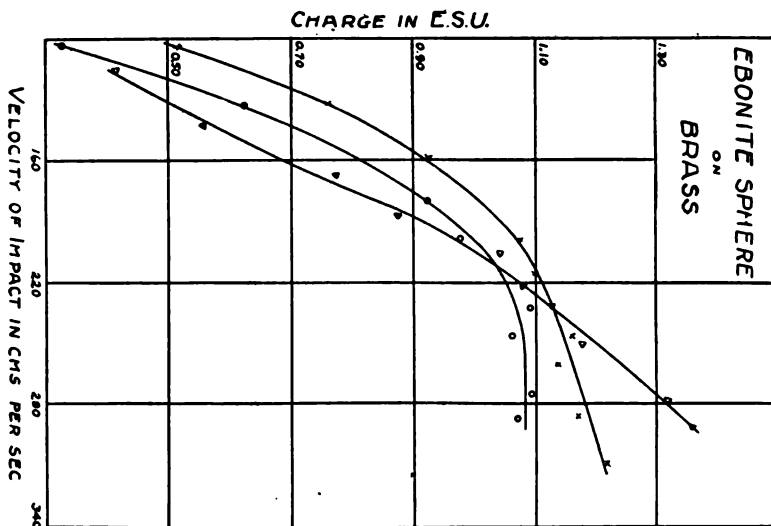


Fig. 6.

slightly bruised and flattened, although the brass showed no signs of wear. The surfaces of the dielectric and metal received no especial care other than an occasional cleansing with alcohol. No reversal of sign of the charge was ever observed; the charge on the brass was positive for all velocities and at every trial.

C. Repeated Impacts.

Fig. 4, Curve II., shows the charge produced by a given number of collisions at a velocity of 179.8 cm./sec. The ebonite ball was discharged between impacts. Seven collisions could be effected in one minute, and leakage during the time required for the largest number of impacts was nearly negligible, so that only a small correction was necessary. Twenty impacts produced a potential of 183.8 volts upon the brass disc. The curve shows that there is a tendency towards a maximum, although the dielectric was discharged between impacts. This result differs from the conclusion based upon the data given in Table III. for a similar experiment with the ebonite disc. The reasons for this will be discussed later.

D. Various Metals and Dielectrics.

Tests were made when a zinc disc was substituted for brass, and a glass ball for ebonite, the results of which are shown in Table IV. The charge is due to a single impact. The zinc disc was 5.0 cm. in diameter and 0.5 cm. thick. The glass sphere had a diameter of 1.9 cm. and a mass of 8.23 grams.

TABLE IV.

Materials.	Coefficient of Resiliency	Velocity of Impact (cm./sec.).	Charge (e.s.u.).
Ebonite on brass	0.74	292.0	1.37
Ebonite on zinc	0.70	166.8 to 252.8	0.87
Glass on brass	0.40	200.0 to 285.9	0.57
Glass on zinc	0.30	200.0 to 246.5	0.81

In every instance the metal received a positive charge. An ivory ball also produced a positive charge on brass. In all cases except that of ebonite on brass the metal was permanently indented by the impinging sphere, so that consistent velocity curves could not be obtained. For these cases the mean charge is given for a range of velocities at which the charge was approximately constant.

6. DISCUSSION OF RESULTS.

The results prove that intimate contact, without friction, between metals and dielectrics produces charges of the same order as those which were obtained by friction in the experiments previously mentioned. It is unlikely that any appreciable portion of the charge is due to friction. In both forms of the experiment, every precaution was taken to eliminate sliding contact of the surfaces. The lateral play of the ebonite disc was exceedingly small, and in the hurling experiments the sphere was projected vertically so as to avoid slipping of the surfaces. The hurling cup fitted the sphere evenly, so that the ball could hardly have possessed any considerable rotation at the instant of collision, and the time of contact was probably only a minute fraction of a second, since the speed of separation due to the large values of the velocity and resiliency was further augmented by gravity. In all published accounts of electrification by friction, repeated mention is made of the erratic character of the effect, which extends even to the sign of the charge. In the present experiments there was never the slightest tendency towards a change of sign, and the value of the charge remained remarkably constant during any one set of runs extending over several days, although it varied between fairly wide limits during the course of several months. In the

entire work there were not more than ten instances when the metal received a negative charge, and all of these occurred immediately after the specimen had been cleaned with alcohol, making it clear that, in these cases, the surfaces had not dried completely. The charge was always positive even when it was produced by a contact of almost no velocity. Any residual friction would in all probability have been extremely variable in amount, and this variation, superimposed upon the erratic effect of constant friction, would have seriously affected the consistency of results. Upon several occasions the ebonite disc was rotated by hand upon the brass anvil, which acquired a charge sometimes positive and sometimes negative. Sliding friction may therefore be completely dismissed in considering the results.

Apparently electrification by impact is a surface effect. Since the area of surfaces actually in contact increases with velocity of impact, greater charges can be expected for the higher velocities. This would explain the velocity curves obtained. The secular variation in the amount of charge may be accounted for by the fact that alteration in the shape of the surfaces occurs during lapse of time and repeated collisions.

It does not appear that there is any direct relation between the charge produced and the energy lost by collision. If Q and C represent, respectively, the charge and the capacity of the metallic system, and E the corresponding electrical energy, we may write $Q = \sqrt{2CE}$. If any constant fraction of the mechanical energy lost in impact at a given velocity were transformed into electrical energy, then E would be a constant for impact at that velocity, and the charge would vary directly as the square root of the capacity. Inasmuch as it has been shown that the charge is independent of the capacity, we may conclude that there is no direct dependence of the electrical energy upon the mechanical work performed in collision. As it seems probable that the frictional and impact effects are produced by the same mechanism, it is advisable to discuss the soundness of Jones's¹ conclusions. He states that the charge produced by a given amount of frictional work increases with the capacity of the metallic system. However, he does not mention the values of the capacities used, giving one dimension only for the cylinders employed to vary the capacity. This dimension he terms variously the length or the "thickness." Furthermore, his results are vitiated by a large secular variation, inasmuch as he found it necessary to perform complete runs at each of the different capacities in succession, whereas in the present experiments many readings could be taken alternately at the various capacities in a short time, so that not only would the

¹ Loc. cit.

secular change be small, but it could be eliminated by taking an average value. In this connection it may be noted that the recent work of McClelland and Power¹ contradicts Jones's conclusions that the maximum charge is independent of the temperature and velocity of the rubbing surfaces.

Further evidence in support of the view that there is no direct relation between the mechanical energy and the charge produced is furnished by the curves in Fig. 3. If, for a given velocity of impact, there were a direct proportionality between the electrical and mechanical energies, then in the expression given above, $Q = \sqrt{2CE}$, E would vary directly as the mass of the plunging system. Thus a reduction of 28 per cent. in the charge might be expected when the mass is reduced by 48 per cent. The actual decrease in the charge, however, was only 16 per cent. Additional evidence is afforded by the wide variation in mechanical energy lost per unit charge. For the ebonite ball, using the mean charge (Fig. 6) produced at a velocity of 295 cm./sec., the coefficient of resiliency being 0.74, we find 1.76×10^6 ergs lost per e.s.u. of charge. A similar calculation for the ebonite disc (Fig. 2) gives 1.50×10^6 ergs per e.s.u. Owen² finds that for ebonite rubbed with copper the same ratio has values ranging between 4.3×10^6 and 186.0×10^6 .

So far as conservation of energy is concerned, the total electrical energy acquired is compensated by the mechanical work done in separating the charged surfaces. This is apparent at once from the consideration that at the moment the charge is produced the layers of positive and negative electricity almost coincide and therefore the potential of the system is not appreciably raised. No deflection of the goldleaf occurs until the surfaces are separated. However, the failure of the charge to increase when additional pressure is added, unaccompanied by a sudden impact, indicates that a certain amount of work is required to effect the passage of electrons from the metal to the dielectric, although it is possible that static pressures much greater than those used in the present experiments would increase the electrification. In this connection it should be recalled that, in the photoelectric effect, work is done by the electrons in emerging from the surface film. But in the present case so small a portion of the energy lost in impact is utilized in this manner that it cannot be stated how much work is necessary to produce the charge. Probably in the case of two optically flat surfaces the velocity of impact necessary to produce a given charge would be much smaller than in the present experiments.

¹ Loc. cit.

² Loc. cit.

The results for repeated impacts with the ebonite disc (Fig. 4) show a limiting charge of 9.78 e.s.u. Some view such as that of Helmholtz, who regarded the frictional effect as similar to the contact difference of potential between metals, must be assumed to account for the direction of the effect. From this point of view a very high maximum potential of the metal might be expected, on account of the vastly greater concentration of free electrons in metals than in dielectrics. This is borne out by the experiments in which an ebonite disc impinged upon metal surfaces at various potentials (Table III.), the charge produced showing no tendency towards a maximum. We may conclude, therefore, that the maximum is conditioned rather by the amount of charge present upon the dielectric than by the potential of the metal anvil.

In the experiment of repeated impacts with the ebonite ball, the latter was discharged between collisions. The curve (Fig. 4) shows that in this case there seems to be a tendency for the positive charge on the metal to leak back to the dielectric. This is at variance with the results mentioned in the preceding paragraph. This tendency towards back-leakage, however, might be greater for the sphere than for the disc, on account of the greater intimacy which higher velocities and curvature of surface would cause. The dissipation of energy per unit area is probably greater in the case of the sphere than of the disc, and the heat developed might also facilitate back-leakage.

It is a pleasure to acknowledge here many valuable suggestions received from Professor L. T. More and Professor R. C. Gowdy during the course of the experiments.

UNIVERSITY OF CINCINNATI,

February 11, 1920.

A NEW DESIGN OF PRECISION X-RAY SPECTROMETER.

BY C. D. COOKSEY.

SYNOPSIS.

Principle of the Spectrometer.—The spectrometer is especially designed to make use of the "*Method of Displacement*," already published, for the determination of X-ray wave-lengths. This method is briefly described.

Description of the Spectrometer.—A detailed description of the spectrometer with a plan and side elevation are given.

Adjustments.—The processes involved in the adjustment of the spectrometer for precision measurements on X-ray wave-lengths are explained in detail.

Verification of the Fundamental Assumption on which the "Method of Displacement" is Based.—The method requires that a portion of the length of a beam of X-rays, after reflection, shall be of constant width. The possibility of this has been previously demonstrated on theoretical grounds. An exploration of the reflected beam for the α_1 line of the K series of silver has been made and the results are found to agree with theory.

PRINCIPLE.

A METHOD has been devised for measuring wave-lengths in the X-ray region of the spectrum, by Professor H. S. Uhler,¹ which promises to eliminate many of the errors involved in earlier methods. The method requires the use of two slits of equal width between the source of rays and the crystal and a means of displacing the photographic plate in a straight line parallel to the line of collimation of the spectrometer, the plate undergoing no rotation during the displacement. The glancing angle of reflection at the crystal is then determined solely by the distance apart of the spectral lines, taken on both sides of the direct image of the slits in the two positions of the plate, and the distance through which the plate has been displaced. These distances can both be measured on the same comparator, thus making the determination of glancing angles depend solely on two linear measurements in the same units.

This method of displacement for determining glancing angles was given a preliminary trial by Uhler and myself² in our work on the K series of gallium. The spectrometer we then used had not been constructed originally for this method, but the results obtained with the remodelled apparatus encouraged us to believe that with a properly designed instrument the method would be capable of great precision.

¹ Uhler, *Phys. Rev.*, N.S., Vol. XI., No. 1, p. 1, 1918.

² Uhler and Cooksey, *Phys. Rev.*, N.S., Vol. X., No. 6, p. 645, 1917.

DESCRIPTION.

The spectrometer is shown in plan and elevation in Figs. 1 and 2. The main part of the spectrometer consists of a heavy cast iron base *A*, in the form of a rectangle, about 78 cm. long by 29 cm. wide. At the center of one end of this base is a vertical bracket carrying a vertical slit, *S*₁, with adjustable gold jaws 2.4 mm. thick. The base *A* is provided on one side with a V rail and on the other with a plane surface running parallel to the long axis of the rectangle. Another rectangular base, *L*, having a foot with a V groove at each end of one side, and a foot with a plane bottom at the middle of the other side, can slide on the rails of base *A* toward or away from *S*₁. A Société Genevoise circular dividing engine, without its knife carriage, is bolted on the upper surface of *L* in such a manner that the axis of rotation of the revolving table is parallel to the long axis of *S*₁. The normal from the center of *S*₁ to the axis of rotation forms the collimating line of the spectrometer. A second slit, *S*₂, similar to *S*₁, is fastened to the base of the dividing engine

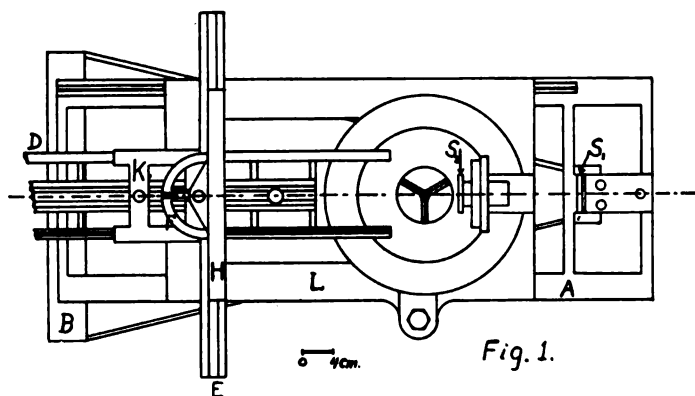


Fig. 1.

between its axis of rotation and *S*₁, and at the same height as *S*₁. *S*₂ is normally at a distance of 4.5 cm. from the axis of rotation, but this distance can be altered by a few centimeters. *S*₂ is provided with a slow motion adjustment by which it can be moved horizontally at right angles to the line of collimation. Another adjustment permits its rotation about this line or a parallel line as axis.

The face-plate of the dividing engine has fastened at its center a brass plate with three V grooves 120° apart, radiating from the axis of rotation. Two stands, not shown in the figures, are provided with three legs each to fit into these grooves. One of these stands has fastened to it the crystal holder. This holder has two horizontal linear adjustments, one parallel to, and the other at right angles to, the crystal face. A third

adjustment permits the crystal to be rotated about a horizontal axis parallel to its face. The other stand supports a slit, S_3 , used only in adjusting the spectrometer. This slit is of fixed width of about 0.02 mm., and the jaws of the slit, when in place, do not come above the collimating line. Thus this slit does not limit those rays coming through S_1 and S_2 which are directed above the collimating line when they reach the third slit. S_3 is provided with adjustments by which the center of the upper end of the opening between the jaws can be made to remain accurately in the axis of rotation while the divided circle is turned through 360° . This adjustment was made under a high-power microscope, provided with cross hairs, and the fact that it is possible to so adjust the slit that no motion of the point referred to can be detected when the divided circle is rotated shows that the axle and bearings of the dividing engine are very true.

On the dividing engine base, diametrically opposite S_2 is mounted a pair of rails, D , about 42 cm. long and 10 cm. between centers. The casting carrying these rails is pivoted on the engine base so that the rails can be made parallel to the line of collimation. One of these rails is

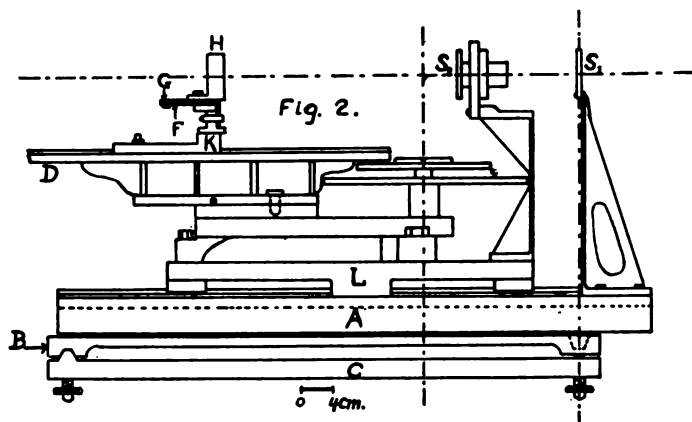


Fig. 2.

V-shaped in section, and the other square, with its upper surface plane. They were very carefully machined and ground smooth.

A carriage, K , fits on these rails and is furnished with a clamping screw to lock it tight to them at any point of their length. The extreme distance of this carriage from the axis of rotation of the spectrometer can be limited by an adjustable stop clamped to one of the rails, or by a rod of known length placed between the stop and the end of the carriage. It is this feature which permits the use of the "Method of Displacement"

already referred to. The end of the carriage nearer S_2 has fastened to it a horizontal aluminium rail, E , 42 cm. long, with its long axis at right angles to the direction of motion of the carriage. The transverse section of this rail is an inverted T. A horizontal brass semicircle, F , with a flange projecting vertically downward can be clamped on the T rail at any point. A flat brass plate, G , rests on top of the semicircle and is pivoted at its center. It can be rotated on the semicircle and clamped at any desired angle, F and G together forming a sort of protractor.

The plateholder, H , is a cast aluminium rectangular box, with plane surfaces inside, against which the plate can be pressed flat by means of strong springs on a wooden cover at the back. To allow for variations in the thickness of plates, the cover is lined with thick felt on the inside. A rectangular opening about 24 cm. long by 1.5 cm. high is cut in the front of the box and covered with black paper. The paper can easily be removed to permit a view through the plateholder or the measurement of the distance from the plate to the axis of rotation. The plateholder accommodates plates 25.4 cm. by 3 cm., the latter figure being determined by the dimensions of the device for holding the plates on the comparator used for measuring them. The central portion of the bottom of the casting extends behind the plane of the back of the rectangle in the form of a tongue, the bottom of which was machined flat. This tongue is provided with two dowel pins and a screw by which the plateholder can be screwed tight to the azimuth plate, G .

ADJUSTMENTS.

The object of this method of mounting the plateholder is to be able to photograph several spectral lines at a long distance from the direct beam without having to use excessively long plates and to be able to adjust the plane of the plate so that it may be either at right angles to the collimating line, or to the beam reflected from the crystal. A very convenient use of the aluminium T rail is in taking what I call "Range Plates." In using two very narrow slits it is necessary to know very closely what setting of the crystal gives the best reflection for a particular wave-length. To ascertain this a series of exposures of equal length are taken on the same plate, the plateholder being displaced a small distance along the T rail between each exposure, and the crystal setting altered a small amount. A comparison of these exposures shows, not only the best setting of the crystal, but, if enough are taken, the effective range of reflection of the crystal for the particular width of slits chosen.

The object of mounting the plateholder, crystal, and S_2 on a separate base from that on which S_1 is mounted is to be able to conveniently

vary the distance between S_1 and the axis of rotation of the spectrometer, which at the same time would, of course, vary the distance between S_1 and S_2 . The distance of S_1 from the axis can be varied from about 11 cm. to about 35 cm.

The base A of the spectrometer is pivoted on a triangular cast iron base, B , in such a manner that A can be given a small rotation about the long axis of S_1 as axis. The base B rests, in turn, on a similar base, C , provided with ways on which B can slide a short distance at right angles to the collimating line. The base C is supported on leveling screws. Finally the table of the spectrometer can be rotated back and forth through a small angle by means of a spring motor connected to the tangent screw of the dividing engine through a connecting rod and crank.

The object of this rather elaborate mounting is to facilitate the lining up of the spectrometer with the focal spot on the anti-cathode of the X-ray tube. In working with soft X-rays it is necessary to have a window in the tube which will not absorb too much of the energy. Thin aluminium is very good for this purpose, but, when very thin, the window must be quite small, thus giving an emergent pencil of rays of small angular width. Experience has taught me that it is quite difficult, without special means, to make the collimating axis of the spectrometer coincide with the axis of this pencil. This adjustment is simple with the mounting described above. The instrument is first brought roughly into line with the aid of a fluorescent screen. A plate is then placed immediately behind S_1 and two exposures taken on it; a long exposure with the slit narrow and a short one with the slit wide open or removed entirely. This gives the projected images of the slit and the aluminium window superposed on each other. The center of the image of the window is then marked with a fine scratch, and the plate replaced in the position in which it was when the exposures were taken. By focusing the cross hairs of a microscope on the center of the image of the window, and moving the spectrometer across base C till the image of the slit comes under the cross hairs, S_1 is brought into the axis of the pencil of X-rays. In order to bring the collimating line of the spectrometer in coincidence with the axis of the pencil, S_2 is placed in position on the spectrometer table, and a plate placed immediately behind it. An exposure is then taken with S_1 narrow, and, if the lineup is badly out, another with S_1 wide open, and a screen placed directly above S_2 . This gives two images, one above the other; the center of the upper one gives the position of the axis of the pencil, while the center of the lower one gives the position of the axis of rotation of the spectrometer with respect to the former. The plate is replaced on the spectrometer, and a microscope

focused on the center of the upper image. The spectrometer is then rotated on base B , about S_1 as an axis, till the image of S_2 comes under the cross hairs. The axis of the pencil of rays coming through the window on the tube will then pass through S_1 and the axis of rotation of the spectrometer. This being done, a similar method of superposed exposures will show the position of S_2 with respect to the collimating line, and any error can be corrected by means of the transverse adjustment referred to above. By means of a pair of parallels temporarily supported on the divided circle and extending between S_1 and S_2 , the plateholder can be placed first in front of one slit, and then in front of the other without any rotation about the normal to the plate. Images of both slits can be taken close together on the same plate and any lack of parallelism between the slits corrected by means of the rotational adjustment on S_2 referred to above.

The rails D are made parallel to the collimating line by making S_1 and S_2 very narrow, and taking one exposure on the upper half of the plate when it is directly over the bearing on which the rails are pivoted, and a second on the lower half of the plate when it is at the extreme end of the rails. The rails can then be rotated so that their extreme end moves through the distance between these images. The straightness of the rails was tested by covering the plate with a screen having a narrow horizontal slit, and taking exposures with S_1 and S_2 narrow and the plate at different positions on the rails, the horizontal slit being moved vertically by its own width between each exposure. At the time of writing, the rails have been found to be very true, though there seems to be a very slight transverse shift of the plate when moved between certain positions. This may be due to dirt, but if it persists in future tests it can easily be measured and allowed for in determining wave-lengths by the "Method of Displacement."

The azimuth angle between the normal to the plane of the plate and the collimating line is completely determined by the difference in distance between the same spectral line photographed on each side of the central beam in two different positions of the plate on the rails D , and the distance between these positions. This angle can be made zero by rotating the plate G on the circle F or can be applied as a correction in the calculation of wave-lengths.

If the plane of the crystal is not exactly parallel to the axis of rotation of the spectrometer, the lines taken on either side of the central beam will be inclined to each other by four times the angle between the plane of the crystal and the axis. If two plates are taken at the same distance from the axis of rotation, each with the same line on both sides of the

central beam, and are then placed with their gelatin sides together, and the top of one at the bottom of the other, in such a manner that the corresponding lines on one side of the center are superposed, then the corresponding lines on the other side will be inclined to each other by eight times the angle of tilt of the crystal. This makes a very delicate method of detecting any error in the adjustment of the crystal face.

VERIFICATION OF THE THEORY.

It has been shown theoretically by Uhler¹ that a certain portion of a beam of monochromatic X-rays passing through two slits of equal width and reflected from a crystal is of a rectangular cross-section in a plane normal to the mutually parallel long axes of the slits, the length of the rectangular portion being equal to the distance between the slits, and the width being equal to the width of the slits. I have explored this rectangular portion of the beam in the case of the α_1 line of the K series of silver. The distances from the axis of rotation to S_1 and S_2 were 20 cm. and 4.5 cm. respectively, thus making the rectangular portion of the reflected beam 15.5 cm. long. Exposures were taken on the same plate at both ends of the rectangle and at points one quarter, one half, and three quarters of its length. This was done with both slits about 0.02 mm. wide and again with them 0.1 mm. wide. During the exposures the crystal was rotated back and forth through an angle of $3'$ about the position which gave the best reflection.

The results of these photographs are in good agreement with Uhler's deduction that the portion of the reflected beam between the foci of the slits is rectangular in cross-section and of the same width as the slits. The exposures with the wider slits were purposely made short that they might bring out any lack of uniformity in the distribution of energy across the width of the beam at the different parts of its length. The results showed that the energy was fairly uniformly distributed and that, if the lines were thoroughly exposed, there should be no danger of an asymmetric blackening with respect to their axes.

Though this spectrometer has not yet been used for the direct determination of wave-lengths, every indication points to its giving a high degree of precision in such work. It should be pointed out here that there is nothing about the construction of the instrument to prevent its being used in the ordinary way for determining glancing angles by measuring the distance of the plate from the axis of rotation. Indeed, a determination of the glancing angle by both methods might give a measure of the depth of the mean reflecting plane of the crystal.

¹ *Loc. cit.*

In conclusion I wish to express my thanks to Professor Uhler for his many helpful suggestions and advice in connection with the design and adjusting of this instrument and especially for the plateholder and its adjustable mounting which were designed by him.

SLOANE PHYSICAL LABORATORY,

YALE UNIVERSITY,

March 15, 1920.

THE PERFORMANCE OF CONICAL HORNS.

By G. W. STEWART.

SYNOPSIS.

The experiments were performed to secure quantitative results with a conical horn used as a receiver. The approximate theory of A. G. Webster, when extended, and the experimental data herein presented lead to the following conclusions concerning such a horn acting as a receiver:

Intensity at Vertex as a Function of Frequency.—The experimental variations of intensity at the vertex plotted with changing frequency, intensity of the arriving waves constant, demonstrate that:

(a) The frequencies producing resonance are very closely integral multiples of the fundamental frequency.

(b) The resonance peaks broaden and the minima between the peaks increase with increasing frequency.

(c) Therefore the conical horn used as a receiver gives an amplification for any frequency nearly the same or greater than the fundamental.

(d) The greater the frequency, the less the differences between maxima and minima, and thus a horn long compared to the wave-length of the least frequency involved, gives an amplification without highly marked resonance characteristics.

(e) The function of frequency obtained experimentally varies with the horn angle chosen, but retains the same general character.

Optimum Horn Angle for Fundamental Resonance.—There is a horn angle for which the amplification will be a maximum, assuming fundamental resonance. The optimum angle is greater the higher the frequency, 256 and 512 d.v. being used.

Optimum Horn Angle; Overtones.—Using 512 d.v. as the fundamental and first and second overtone, similarly 256 d.v. as the fundamental and first overtone, the optimum horn angle decreases with the order of the harmonic, being greatest for the fundamental.

End Correction.—The end correction of the conical horn is approximately 0.7 of the radius.

Conical Attachments.—Cylindrical and conical attachments to a conical horn introduce their own resonance characteristics, which may be estimated as to frequency by treating the end attached as a free open end but applying a positive correction to the length if the angle of the attached tube is less, and a negative correction if greater.

Limitations of Conclusions.—The conclusions apply strictly to the limits of frequency used in the experiment. Nevertheless they make more clear the nature of the action for all frequencies.

THE published theory¹ of the action of conical horns and pipes is unsatisfactory in at least two respects, namely, there is a masking of certain physical aspects of the action through the assumption of spherical waves, and there exists too great an inaccuracy in these com-

¹ See Rayleigh's Theory of Sound, Vol. II., Ch. XIV., and Barton, Phil. Mag., Series 5, Vol. 15, 1908, pp. 69-81, and Webster, Proc. Nat'l. Acad. of Sciences, Vol. 5, p. 275, 1919.

puted quantitative relations which should enable one to anticipate the amplification produced by a conical horn when used as a receiver. The present contribution furnishes certain quantitative information concerning the action of conical horns as receivers, under conditions as specified, and also gives a brief explanatory discussion expressed in terms of the physical action involved.

I. APPARATUS AND METHODS.

Sources of Sound.—The sources of sound were two electrically driven tuning forks mounted upon wooden box resonators having openings of 10×5 cm. for the 256 fork and 7.3×3.7 cm. for the 512 fork.

General Method of Measurement.—A Rayleigh disc, connected by rubber tubing to the conical horn investigated, was used to obtain the relative intensities of the sound in the conical horn. The internal diameter of the rubber tubing was .45 cm. and the length about 150 cm. Readings were taken by the means of a telescope and scale. The deflections of the disc were proportional to the intensities in the disc and hence to the intensities at the vertices of the horns. A disc was constructed for each frequency, the upper one shown in Fig. 1 being for the 512 fork.

Location.—The source of sound, horn and disc were mounted on the edge of the roof of the physical laboratory 21 meters high, where the

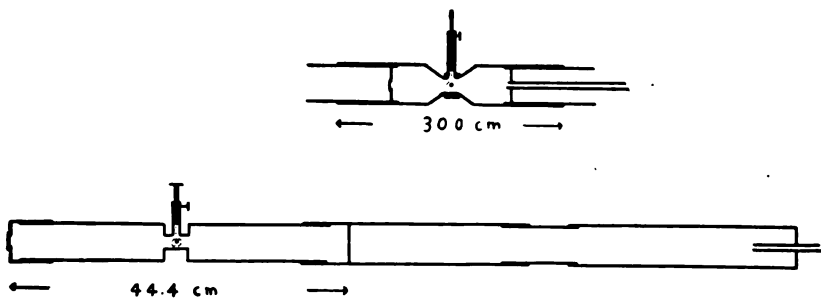


Fig. 1.

absence of other buildings, combined with the elevation of the site, made the reflection of sound nil, excepting that from the roof itself. This reflection was diminished in two ways: first, by covering the adjacent portions of the roof with thick hair felt whose absorption coefficient is about 50 per cent. and by mounting the fork and horn approximately 150 cm. above the roof and separating them from each other not further than this same distance. Thus the effect of roof reflection was reduced to a small per cent. However, the proximity of the horn and fork

introduced an error in that the wave falling upon the horn was not strictly plane, though in all cases the wave surface deviated from a plane only by a small fraction of a wave-length.

Horns.—The horns were made of galvanized iron, U. S. standard plate No. 24. It was found that, if thin metal were used, there was resonance in the conical wall affording sufficient dissipation to affect the intensity in the horn itself. This effect could be demonstrated by changing the rigidity of the clamping of the horn. With the horns actually used, however, the intensity measurement was not capable of being changed by clamping and the walls did not vibrate noticeably. Inasmuch as the absorption of the metal when not vibrating is very small, it is assumed that the results here presented are those found in conical horns having rigid walls and possessing no absorption.

Each horn was cut off at the vertex, leaving a diameter of 0.5 cm.; thick wall rubber tubing 0.45 cm. in internal diameter and approximately 150 cm. in length, was slipped over the open vertex and connected to the Rayleigh disc.

In many of the experiments the lengths of the horns were altered a few centimeters in order to produce the length required for resonance. This was accomplished by means of heavy drawing paper slipped over the large end. It was experimentally shown that when the length of the addition of drawing paper did not exceed a few centimeters the paper served just as well as metal itself. The reason is, of course, that the changes in pressure near the large end of the horn are small.

Detailed Method.—Deflections of the disc were read and assumed (and justifiably so) to be proportional to the intensities in the horns. But in order that measurements of relative intensities with varying experimental conditions might be secured, corrections were made for the lack of constancy in the intensity of the source of sound, and for the change of sensitiveness of the disc caused by changes of temperature. The data for the first correction were obtained by microscopic measurement of the amplitude of the fork at the time of each reading of the disc. The second correction was secured through an occasional reading of the disc when attached to a tube terminating in a position fixed relative to the source of sound, the amplitude of the fork being noted. The details of applying these corrections are simple and almost obvious. The only assumption involved is that the intensity is proportional to the square of the amplitude of the fork.

II. EXPERIMENTAL RESULTS.

The experimental results herein presented are not as accurate as can be obtained, but they will give fairly satisfactory quantitative information concerning the action of conical horns. The experimental conditions

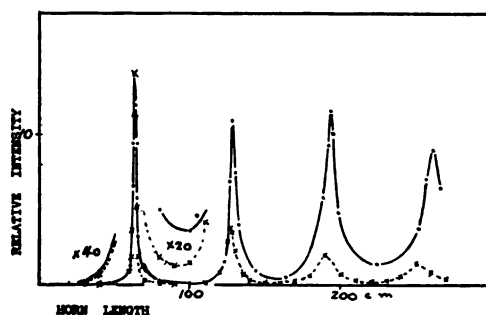


Fig. 2.

necessary to obtain more accurate results are difficult to secure. The experimental points shown in the curves are usually each the result of a single setting and observation. Nevertheless, they do not comprise the entire data but are merely typical. All results have been corrected for temperature, the reduction being made to 20° C.

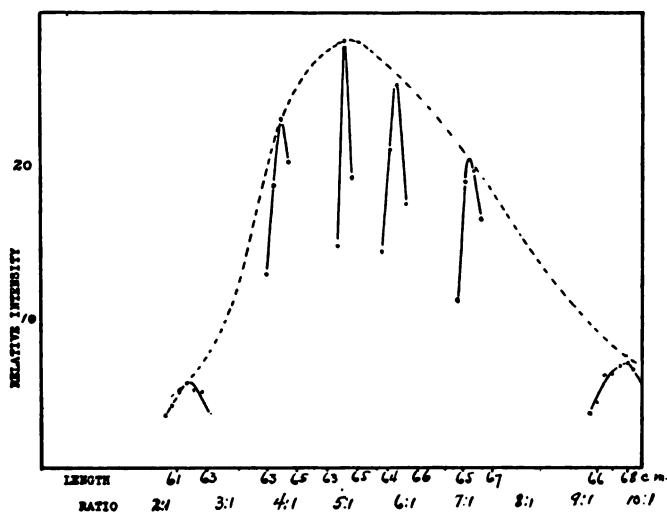


Fig. 3.

Variation of Intensity with Length, Frequency Constant.—The first question one might ask in regard to the action of a conical horn or tube acting as a receiver is in regard to the resonance characteristics and the

amplifying effect throughout the range of frequencies of interest. A series of trials was made with conical horns of constant angle but of varying length, using the same source of sound. The data were taken on several evenings and are shown, after corrections are made for the

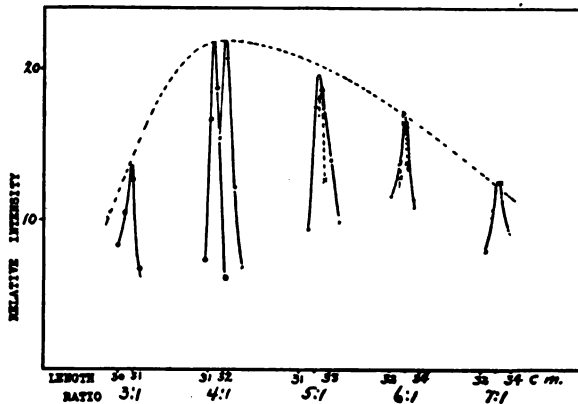


Fig. 4.

effect of temperature differences upon length, in the full line curve plotted in Fig. 2. Over two portions of the curve magnifications of 40 and 20 are also plotted. There should be added to the data in Fig. 2 an important relevant fact, namely, that the intensity without the horn as tested by an open tube at the position occupied by the open end of the

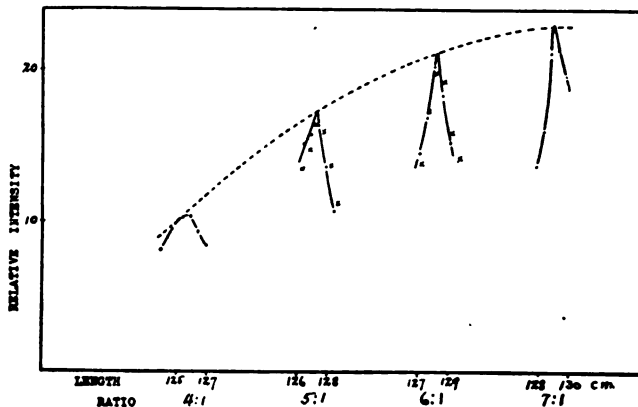


Fig. 5.

horn, was of the same order as the observation with the horn of shortest length.

Variation of Intensity with Angle in a Conical Horn, Assuming Maximum Resonance and Fundamental Frequency.—In Fig. 3 it is shown that

the maximum intensity at resonance occurs when the ratio of radial length to diameter of opening is approximately 5 : 1 for a frequency of 256 d.v.

Fig. 4 gives similar information in regard to the frequency of 512 d.v. Here the optimum ratio is approximately 4 : 1, that is, the radial length if four times the diameter of the open end. The observations indicated

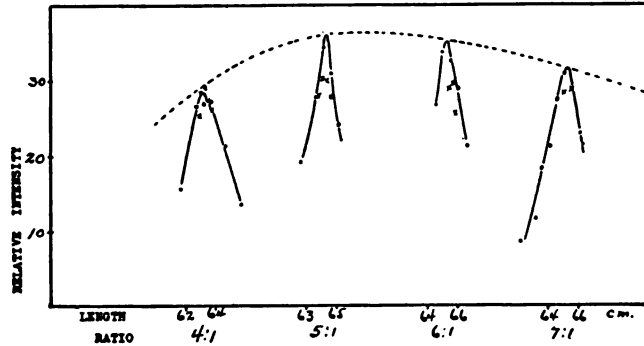


Fig. 6.

by circles and crosses were taken at other times. There appears a disagreement of peak location for the 4 : 1 probably due to an error in reading of 1 cm.

Resonance Peaks, Similar to those in Figs. 3 and 4, but with a Frequency of the First Overtone.—Fig. 5 for 256 d.v. and Fig. 6 for 512 d.v. are self-

explanatory. Again crosses and dots indicate separate series. The point worthy of note is that, frequency constant, the optimum angle decreases when the horn is lengthened so that the fundamental becomes the first overtone.

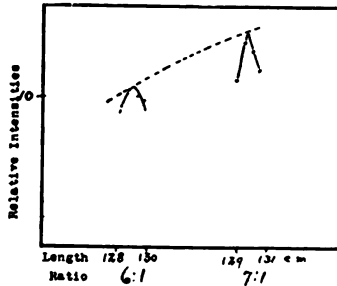


Fig. 7.

Resonance Peaks, with Frequency that of the Second Overtone.—With 512 d.v. only observations on two horns were taken and these are shown in Fig. 7.

Although these data are too meager to interpret accurately, yet there is sufficient evidence that the optimum angle continues to decrease with an increase in the order of the overtone.

Variation in Optimum Angle with Order of Overtone.—Fig. 8 shows how the optimum angle changes with the order of the overtone and it is here seen that the ratio of change for the 256 and 512 is approximately the same. The reliability of the data gives only a general idea of the change.

Effect upon Resonance Characteristics of Conical and Cylindrical Attachments.—It is to be anticipated that the resonance characteristics of the tube attached to a conical horn for listening purposes will be superimposed upon the characteristics of the cone itself. By simple tests with a tonometer and attached cones and cylindrical tubes the writer found that the resonance frequencies of the attachments can be estimated by the following rule: The attached tube, cylindrical or conical, will introduce resonance characteristics which may be computed as to

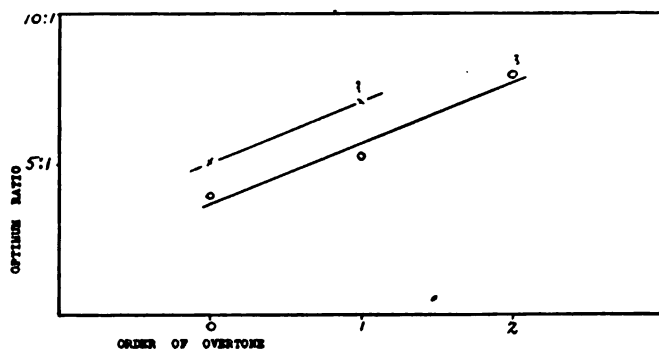


Fig. 8.

frequency by treating the end attached as a free open end but at the same time remembering that if the angle of the attached tube is less than that of the conical horn there is a positive end correction and if greater a negative end correction.

Directivity of a Conical Horn.—As is well known, when the source of sound is along the extended axis of the horn, the greatest intensity is produced at the vertex. This effect, because it depends upon the phase relations at the opening, becomes more marked the larger the opening of the horn. This comment is introduced merely because in practical application of the conical horn as a receiver, the exclusion of side noises is of great importance.

End Correction.—The experiments may be used to determine, the end correction of a conical horn. If we assume that,

$$L + KR = n \frac{\lambda}{2},$$

where L is the length of the horn, R the radius, K a correction factor, and n an integer, and plot L/n , and R/n as coördinates, K may be computed from the slope. If the results are in agreement with the above linear relation, it may reasonably be concluded that the resonance frequencies are integral multiples of the fundamental. The results of the experi-

ments have been so plotted and the assumption of the linear relation seems to be fully justified, whether the frequency is a fundamental or an overtone. The value of K appears to be 0.7, with a possible error of 10 per cent. Experiments can readily be designed to determine the exact relation between the resonance frequencies of the conical horn, and thus the end correction, with a much greater accuracy than here employed.

III. THEORY.

A brief reference to theory will aid in extending the application of the experimental results and in understanding the action of the conical horn. The only theory of which the writer is aware that offers a possibility of obtaining quantitative results for conical horns is that of Dr. A. G. Webster¹ of Clark University. In this theory he makes the following assumptions:

(a) That the tube or horn has a cross-section which is always small but is a function of the length, x .

(b) That the sound waves travelling in positive and negative directions of x are plane, *i.e.*, all variations in y and z directions are zero.

(c) That the conductivity for an open end is the same as a channel of a selected length.

(d) That the general equations of sound are applicable.

It will be necessary to extend the published work of Webster somewhat in order that we may secure the quantitative results. Let X be the volume of air periodically entering the horn under a maximum excess pressure p , and z be the impedance and equal to p/x (see Webster, *loc. cit.*). Let subscripts 1, 2, and 3 refer respectively to the values at the small end of the horn or in this case the vertex, the opening of the horn, and outside the opening at a point where the influence of the horn can be considered as vanishingly small. Assume the distance between p_2 and p_3 to be small compared with a wave-length and that Z_0 represents the impedance of the opening treated as a short tube or channel, the air escaping from a circular hole in an infinite plane. Then, by definition,

$$X_2 = \frac{p_2 - p_3}{Z_0} = \frac{Z_2 X_2 - p_3}{Z_0} = \frac{p_3}{Z_2 - Z_0}.$$

But according to Webster, equation (24)

$$X_2 = c p_1 + d X_1 = (c Z_1 + d) X_1,$$

where c and d are defined as follows:

$$c = \frac{-\sigma_2 x_1 \sin k(l + \epsilon_1 - \epsilon_2)}{\beta x_2 \sin k \epsilon_1 \sin k \epsilon_2}, \quad d = \frac{\sigma_2 x_1 \sin k(l - \epsilon_2)}{\sigma_1 x_2}.$$

¹ *Loc. cit.*

σ_1 and σ_2 represent the cross sections at the small and large ends, respectively, $\beta = \rho a^2 k$, where ρ is density, a velocity of sound, and k the frequency divided by the velocity, x_1 and x_2 represent the distances from the vertex of the small and large ends respectively, and

$$\begin{aligned} l &= x_2 - x_1, \\ \tan k\epsilon_1 &= kx_1, \\ \tan k\epsilon_2 &= kx_2; \\ \therefore X_1 &= \frac{p_1}{Z_1} = \frac{X_2}{(cZ_1 + d)} = \frac{p_2}{(cZ_1 + d)(Z_2 - Z_0)}. \end{aligned}$$

But since

$$Z_2 = \frac{aZ_1 + b}{cZ_1 + d} \quad (\text{Webster's equation (25)}),$$

with

$$a = \frac{x_1 \sin k(l + \epsilon_1)}{x_2 \sin k\epsilon_1}, \quad b = \frac{\beta x_1}{\sigma_1 x_2} \sin kl,$$

then

$$p_1 = \frac{z_1 p_2}{(cz_1 + d) \left(\frac{az_1 + b}{cz_1 + d} - Z_0 \right)},$$

and we have a relation between the exterior and vertex maximum excess pressures; and these pressures are proportional to the square roots of the respective intensities.

For the conical horn closed at the vertex,

$$z_1 = \infty,$$

and therefore

$$p_1 = \frac{p_2}{(a - cz_0)}.$$

Now Webster shows that, assuming the conductivity of the open end to be that of a short tube of conductivity c_0 , ending in an infinite plane,

$$z_0 = \rho a^2 k^2 \left(\frac{k}{2\pi} i - \frac{1}{c_0} \right).$$

Hence the complete expression that will enable us to determine quantitative relationship between the incident sound intensity and that found at the vertex is

$$p_1 = \frac{p_2}{\frac{x_1 \sin k(l + \epsilon_1)}{x_2 \sin k\epsilon_1} + \frac{\sigma_2 k x_1 \sin k(l + \epsilon_1 - \epsilon_2)}{x_2 \sin k\epsilon_1 \sin k\epsilon_2} \left(\frac{k}{2\pi} i - \frac{1}{c_0} \right)}$$

and if p is to be determined for the vertex, $x_1 = 0$,

$$p_1 = \frac{p_2}{\frac{\sin kx_2}{kx_2} + \frac{\sigma_2 \sin k(x_2 - \epsilon_2)}{x_2 \sin k\epsilon_2} \left(\frac{k}{2\pi} i - \frac{1}{c_0} \right)}. \quad (1)$$

It can be shown (Professor Webster has shown this, as well as the derivation just given by the author, but has not published either) that Webster's theory may be modified by omitting the assumption of small cross section, and by assuming spherical waves, p being a function of r , the radial length, and the time, with the following equation:

$$p_1 = \frac{p_3}{\frac{\sin kr}{kr} + \frac{\sigma \sin(kr - \epsilon)}{r \sin k\epsilon} \left(\frac{k}{2\pi} i - \frac{1}{c_0} \right)}, \quad (2)$$

where

$$\tan k\epsilon = kr.$$

For our case of conical horns (2) is the more accurate.

For the conductivity c_0 , it is assumed that the open end is equivalent to a channel of length 0.6 of the radius, Thus

$$c_0 = \frac{\pi R^2}{0.6 \times R} = \frac{\pi R}{0.6}.$$

With this value of c_0 and using (2), computations were made and relative values of $(p_1/p_3)^2$ or relative values of the intensity, were secured. In computing, one must of course use the modulus of the fraction in the above equation. The results are plotted in the dotted curve in Fig. 2.

An additional point of the theory should be noted. Equation (2) indicates that the computed values depend upon the product kr which is proportional to the product of radial length and frequency. In Fig. 2, the experimental results were plotted with changing radial length, frequency remaining constant. From the foregoing statement it is obvious that precisely the same curve would have been obtained with varying frequency, radial length constant, the values of the ordinates depending only upon the product of radial length and frequency.

IV. DISCUSSION OF RESULTS.

Extension of Results. Fig. 2.—Inasmuch as we are apparently justified, by the preceding theoretical considerations, in regarding Fig. 2 as showing the relationship between relative intensity and frequency, the following conclusions may be stated:

- (a) The frequencies producing resonance are very closely integral multiples of the fundamental frequency.
- (b) The resonance peaks broaden with increasing frequency.
- (c) The intensity values for the minima, located between the resonance peaks, increase with increasing frequency.
- (d) The conical horn gives a very considerable amplification for any frequency nearly the same as or greater than the fundamental frequency.

(The ordinate at the beginning of the curve is in fact no greater than the relative intensity without the horn at all.)

(e) The greater the frequency, the less the difference between maxima and minima relative intensities, the curve promising to approach one of smaller and smaller undulations, yet one representing a great amplification.

These conclusions indicate that if a horn, long compared to the wavelength of the least frequency involved, is used as a receiver, it promises amplification without highly marked resonance characteristics. Experimental tests of this conclusion have been made, and with satisfactory verification.¹

Utilization of Results.—In the utilization of the results of the foregoing experimental and theoretical discussion it must be remembered that the existence of the optimum angle and its alteration with the order of the overtone, prevents the experimental curve in Fig. 2 from being regarded as quantitatively typical. In fact, if the horn ratio had been approximately 7 : 1 instead of 5 : 1, the second resonance peak would have been higher than the first. In general, since there exists an optimum angle, the relative values of the amplification at the various resonance frequencies must depend upon the angle of the horn. It would be advantageous to have information concerning the effect of various frequencies, with a variation of the ratio and length of the horn, for then one could ascertain just what effects any conical horn would have, acting as a receiver for any sound of known frequencies. It is believed that the present results furnish a partial guide in the design of a conical horn for any purpose, particularly by way of suggestion as to what trials should be made to secure the desired horn.

Physical Aspects.—An explanation in physical terms will assist in understanding the causes for the characteristics of the curve in Fig. 2. Assume for the moment that the length of the conical horn is short compared with the wave-length, and that the horn opening is a part of a plane wall. The horn would thus reflect the sound with approximately the same effect as any other equal area of wall surface, or surfaces supposed to be perfect reflectors. We would then expect to find the horn acting as a reflector and not as a collector. This reflecting function of the conical horn persists irrespective of its dimensions, though of course as the length of horn becomes comparable to the wave-length of the sound, the reflection cannot be considered as in the same phase from the entire inner surface, but each element of area is a source of sound in such phase and amplitude that combined with the oncoming sound will result in a

¹ Stewart, *Phys. Rev.*, XIV., 2, 1919, p. 166.

zero value of the velocity potential over the wall surface. In the conical horn, then, reflection takes place throughout its entire length, this reflection being in all directions away from the inner surface just as if each element of that surface is a source of sound.

It is a consideration of this reflection which makes clear the reason for the lack of concentrating ability shown in that part of Fig. 2 to the left of the first resonance peak. Even when the radial length of the horn is approximately one-half of that for maximum resonance, the intensification produced by the horn is almost unnoticeable. In other words, the horn acts much like a plane reflector that subtends the same angle at the source and the energy is approximately the same at the vertex as if the opening were a hole in that plane.

Just as there is reflection from the walls of the incoming wave, so also there is reflection of the outgoing wave. In this latter reflection from the walls we find an explanation of the fact that the minima in Fig. 2 are not constant, showing zero amplification. Conditions for resonance demand reflection not only at a distant end but also at the opening where the incoming wave enters. In the conical tube, inasmuch as we have reflection of the outgoing wave throughout the entire length of the wall, there is always a certain amount of reflection at the point where the phase of the incoming wave and the just reflected portion of the outgoing wave are the same. There would thus be "resonance." According to this explanation there should be resonance for any frequency greater than the fundamental, for with every such frequency there is in at least one region of the inner surface a reflection of the outgoing wave with the above stated phase relations, thus furnishing the necessary conditions for resonance. Fig. 2 is in agreement with this conclusion in that the minima show a distinct amplification. As an illustration of the above explanation, suppose the horn has a length corresponding to the first minimum of Fig. 2. The reflection from the opening of the outgoing wave is opposite (approximately) in phase to the incoming wave. In a cylindrical tube this would be the cause of zero amplification. But in the conical horn there is reflection from the wall near the opening which, being reflection without change of phase, will be in phase (approximately) with the incoming wave. Resonance will therefore exist.

In this connection the writer wishes to emphasize that the usual view of the conical horn, *i.e.*, that it acts primarily as a concentrator, is misleading, and that the view of it as a resonator will, *in most applications be much more satisfactory and helpful*. Of course, both concentration and resonance are present and which predominates depends upon the dimensions of the horn and of the length of the wave.

In the usual theory, giving the resonating frequencies in conical tubes,¹ it is assumed that the incoming and outgoing waves are spherical within the tube and the results derived for the natural frequencies are approximately correct. But the assumption of sphericity is in effect an assumption of reflection from the walls throughout the length for otherwise there could not exist these imagined portions of spherical waves. In a portion of Webster's theory, to which reference has been made, plane waves are assumed in the case of a tube whose cross-section is always small, and spherical waves in the tubes not so limited, but there are applied hydrodynamic equations which permit reflections to exist. In the assumption of an approximately plane wave or a spherical wave reflections are virtually assumed, for these are necessary to maintain the stated shape of the wave. Thus it will be seen that existing theories of the action of conical horns give approximate results for the natural frequencies because they distinctly assume reflection throughout the length of the walls, though without specific mention.

There are further points which should be considered from a phenomenon viewpoint, especially since our theory gives us no direct assistance toward an explanation. Two of these are the reasons for an optimum angle and the variation of the optimum with frequency and with the order of overtone. It is believed by the writer that the existence of the optimum angle is occasioned by the fact that the horn has a diameter not vanishingly small as compared with a wave-length and hence the vibrations are not merely radial. Obviously then, certain angles would be expected to give the optimum resonance. A theoretical investigation of these nonradial vibrations is now being made and a further discussion of the optimum angle will be postponed.

Errors.—The experimental results exhibited in this report should be interpreted in the light of the conditions under which they were undertaken. The results are and must be inaccurate because the conditions that would make possible accurate measurements cannot be had. In these experiments the horn is really a frustrum of a cone with a flux of energy through the "listening" tube attached at the several vertices. Inasmuch as the effect of the tube upon the intensity at the point of attachment in any case depends upon the area, length, damping, and resonance characteristics of the tube, it is not possible to establish experimentally a universally applicable statement of variation of relative intensification of the sound as dependent upon the length and angle of the horn and frequency of the sound used. It is to be observed, however, that, with the area and the length of the rubber tubing used in these

¹ See Rayleigh and also Barton, loc. cit.

experiments, the flux of energy would seem not to be great enough to destroy the value of the relative intensities obtained regarded as an approximation to the truth, that the damping in the tubing was sufficiently great to make the resonance in itself very small indeed, and that, in fact, inasmuch as the length of the tubing was kept constant and different frequencies are not compared, the small resonance effect which might exist did not seriously modify the variation of relative intensity with the angle and length of the horn. It is believed by the writer that, when all the physical conditions are given appropriate consideration, one is justified in accepting the results herein described as furnishing a fair idea of the relative intensities at the vertices of conical horns.

In conclusion, I wish to acknowledge the valuable assistance of Mr. Eugene M. Berry.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

THE HIGH FREQUENCY SPECTRA OF LEAD ISOTOPES.

By C. D. COOKSEY AND D. COOKSEY.

SYNOPSIS.

Least Detectable Difference in Wave-length Dependent on Distance between Lines and Length of Reflected Ray.—Reasons are given for confining the work to a comparison of the $L\alpha_1$ lines of ordinary and uranio-lead. The least detectable difference in wave-length between two lines on the same plate is directly proportional to the least detectable distance between their axes and inversely to the length of the reflected ray between the crystal and plate. However, the nature of the $L\alpha_1$ line of lead is such as to make it more important to reduce as much as possible the least detectable distance between the lines, than it is to increase the length of the reflected ray.

Limit of Accuracy. Special Method of Suspending and Displacing Plateholder.—The spectrometer is described in detail in the preceding article. A special method of suspending the plateholder and giving it a known displacement is described, by which it is possible to fix the limit of accuracy directly. An X-ray bulb of the gas type was used with a specially designed anode which could be rotated so as to bring different parts of its face under the action of the cathode stream. The two kinds of lead were placed on different parts of the same anode. The uranio-lead was from a primary uraninite from India.

Upper Half of Line from One Lead and Lower Half from Other Photographed on Same Plate.—The α_1 line of one kind of lead was photographed on the upper half of the plate, and the same line from the other lead on the lower half, immediately below it. Full lines for reference were sometimes photographed near the half lines for measurement purposes. The limit of accuracy was found by giving the plate a known displacement between the taking of one half and the other, of a line from the same kind of lead.

$L\alpha_1$ Lines from Ordinary and Uranio-lead Do Not Differ in Wave-length by as much as 0.005 Per Cent.—It is found that a displacement between two half lines of $(10)^{-3}$ mm. can be readily detected and that the wave-lengths of the $L\alpha_1$ lines of the two kinds of lead can not differ by as much as $0.6 \times (10)^{-4}$ Å., or 0.005 per cent.

INTRODUCTION.

AN KNOWLEDGE of the relation between the spectrum of a substance and that of its isotope is important in that it may throw further light on the structure of the atom. Some work in this line has been done. Aronberg,¹ working with a grating spectrograph, has reached the conclusion that the wave-length of the line $\lambda 4058$ is greater by 0.0043 Å. for lead of radioactive origin than it is for ordinary lead. The work of Aronberg has recently been corroborated by Merton,² working with a

¹ Aronberg, *Astrophysical Jour.*, Vol. 47, p. 96, 1918.

² Merton, *Proc. Roy. Soc.*, No. A, 679, Vol. 96, p. 388, 1920.

Fabry and Perot *étalon*. The X-ray spectra of lead isotopes have been investigated by Duane and Shimizu,¹ and by Siegbahn and Stenström.² The former authors, using an ionization method, measured the three critical absorption wave-lengths in the L-series from a specimen of ordinary lead and one of radio-active lead, the atomic weights of which had been determined by Richards and found to differ by more than $\frac{1}{2}$ per cent. They conclude that these wave-lengths do not differ between the two kinds of lead by more than 0.1 per cent. or about 0.0008 Å. Siegbahn and Stenström used a photographic method and investigated all the lines in the L-series, and the strongest lines in the M-series of ordinary and radioactive lead. The spectra of the two kinds of lead were photographed on the same plate under the same conditions. By using a lead screen to cover one part or another of the plate at a time, the middle portion of the lines of one kind of lead were obtained with the upper and lower portions of the lines of the other kind of lead above and below. The authors conclude that the wave-lengths of the two kinds of lead do not differ by more than 0.0005 Å. They do not, however, in the article referred to, state how they estimated their limit of accuracy. The dispersion and width of slit used are not mentioned. For a description of their apparatus they refer to a number of the *Jahrbuch der Radioaktivität und Elektronik* for 1916 which, up to the present, has not come to hand on account of the war.

Some time ago Professor Uhler suggested to the senior author that it would be of interest to investigate the X-ray spectra of isotopes, and as we had a supply of uranio-lead at our disposal, it was decided to undertake the work. On account of delays caused by the war we did not receive notice of the work of Siegbahn and Stenström until our work was well under way. The question of the spectra of isotopes seemed, however, of sufficient importance to warrant a repetition of the work by independent investigators even though the methods used by each were nearly the same. At the same time we had reason to believe that we could attain a sufficient degree of accuracy to detect a difference of wave-length, if present, well within the limits set by the authors named. We therefore decided to complete the work.

The potential required to produce the K-series of lead is so high that it is doubtful if this series can be produced in the ordinary gas type tube. On the other hand the wave-lengths of the L-series are long enough to make the absorption of the air an appreciable factor. Not having a vacuum spectrometer at our disposal we decided to confine our investigation to the α_1 line of the L-series of lead.

¹ Duane and Shimizu, *Proc. Nat. Acad. Sci.*, No. 6, Vol. 5, p. 198, 1919.

² Siegbahn and Stenström, *Phys. Zeitschr.*, Vol. 18, p. 547, 1917.

The $L\alpha_1$ line of lead is very broad and more resembles a band than a line. This is probably a characteristic of the lines of the L-series in general. A line of this nature will always be broader than the slit even when the plate is placed exactly at the focus of the slit, and the higher the dispersion, the greater will be this difference. The smallest difference in position between two such lines, taken in a manner similar to that described by Siegbahn and Stenström,¹ which can be detected will depend on the narrowness, sharpness, and intensity of the lines. Since these fall off as the dispersion is increased there is nothing to be gained by carrying the dispersion beyond a certain point.

If a photographic plate is so placed on the spectrometer that its plane is normal to the collimating line, and at such a distance from the axis of rotation of the spectrometer that the portion of the plate that is to receive the spectral line is either at the focus of a single slit, or somewhere between the foci of two slits of equal width, then it is easily shown that the distance, δ , between the axes of two lines whose difference in wave-length, $\Delta\lambda$, is small, is given by:

$$\delta = \frac{r\Delta\lambda}{d \cos \theta \cos 2\theta},$$

where r is the distance from the axis of rotation to the plate, measured along the reflected ray, d is the grating space of the crystal, and θ is the glancing angle. For the $L\alpha_1$ line of lead and the (100) planes of calcite, θ is approximately $11.^\circ 18$ and d is $3.03 \text{ \AA.}$, which gives

$$\Delta\lambda = 2.8 \text{ \AA.} \left(\frac{\delta}{r} \right). \quad (1)$$

The smallest difference in wave-length that can be detected is therefore directly proportional to the smallest detectable value of δ , and inversely proportional to r . Stated in this way however, and for the line under consideration, δ is some direct function of r , and it becomes more important to reduce δ to a minimum than to increase r .

APPARATUS.

We used a spectrometer, a detailed description of which will be found in the preceding article. The plateholder was always mounted in such a manner that its plane was approximately normal to the line of collimation and at such a distance from the axis of rotation that the part of the plate where the spectral lines were produced was always either at the focus of one or other of the slits S_1 and S_2 , or between these foci.

For the special purpose of this experiment we mounted a screen

¹ *Loc. cit.*

directly in front of the plateholder, but not touching it. The screen consisted of a horizontal strip of brass about 1.7 mm. thick, of about the same length, and a little more than half the breadth of the opening in the front of the plateholder. The screen was supported by uprights, which stood on the same masonry pier with the spectrometer, but which were otherwise entirely independent of the latter. It could be raised entirely above the opening in the plateholder, or lowered so as to cover either the upper or lower half of this opening as desired.

For a purpose to be explained later, the plate holder, instead of being supported on the azimuth circle clamped to the T rail, could be hung on two parallel wires of equal length, one fastened at each end of the top of the plateholder. The wires were about 134 cms. long, and their upper ends were fastened to a beam above the spectrometer. When the plateholder was supported in this way its regular mounting was removed from the T rail. In place of this the plateholder had screwed to its base a brass plate with two vertical posts extending below it and about 10 cm. apart. Through the lower end of each post and normal to the plane of the plate extended a screw with a smooth rounded point. By suitably adjusting the supporting wires, a small component of the weight of the plateholder served to make these screws bear gently against a smooth glass plate clamped to the T rail. The frame of a spherometer was clamped to one end of the T rail so that the axis of the screw was parallel to the long axis of the photographic plate, and the point of the screw bore against a smooth glass plate fastened on the end of the plate holder. The spherometer screw had a pitch of 0.5 mm. and its circular scale had a least count of 0.001 mm. To facilitate turning the screw accurately through one division of the head without rocking it in its nut, a tangent screw was provided.

For diffraction grating we used the cleavage planes of a specimen of calcite crystal which had already been found to give very satisfactory lines.

The X-ray bulbs were of the gas type and were made especially for us by Mr. A. Greiner, vice-president of the Green and Bauer Company, Hartford, Conn.; and the skillful manner in which he followed our design greatly facilitated the experimental manipulation. The one from which we obtained the final results had a water-cooled anode which could be rotated or removed by means of a ground glass conical joint. The anode was a copper disc with a silver disc about 2 mm. thick screwed to its face. The plane of the anode was made as nearly normal as possible to the axis of revolution of the ground glass cone. The anode was so mounted that its face made an angle of about 45° with the cathode rays,

and so that different parts of its face came under the action of these rays when the ground joint was rotated. A short side tube was sealed to the main bulb so that the axis of the tube was normal to the plane containing the cathode rays and the axis of the ground joint. The end of this side tube was covered by a brass plate with a small hole in it. The hole, in turn, was covered with aluminum foil about 0.12 mm. thick. The line through this hole to the focal spot made a small angle with the plane of the anode.

The X-ray bulb was placed with the aluminium window close to the slit, S_1 , of the spectrometer, with the plane of the anode vertical, and so that the line from the focal spot through the aluminium window should be as nearly horizontal as possible. It was supported on three very rigid supports clamped to the anode tube, cathode tube, and an extra side tube respectively. These supports were fastened to the same table on which the spectrometer rested. This table consisted of a heavy granite slab cemented on a brick pillar isolated from the building.

The specimens of lead to be compared were both placed on the anode at the same time in the following manner. The position of the focal spot on the anode was marked by running the bulb; the ground joint was then turned through about 45° , and the new position of the focal spot marked. The positions of the ground joint corresponding to these two positions of the anode were marked on the outer end of the cone. The anode was then removed from the bulb, and the silver disc taken off. A cluster of small holes, as close together as possible, was drilled in the silver at both positions of the focal spot. The holes were about 1 mm. in diameter, and enough were drilled to more than cover the focal spot. One of the kinds of lead to be investigated was pressed into one cluster of holes, and the other kind into the other cluster. Great care was taken not to contaminate either lead with the other.

The pressure within the X-ray bulb, while running, was maintained as nearly constant as possible by a mercury diffusion pump so that the bulb would back a 12.7 cm. parallel spark gap. The current through the bulb was about 5 milliamperes, supplied by an "Ideal Interrupterless" X-ray current generator, purchased from the Kny-Scheerer Company of New York City.

The specimen of ordinary lead was cut from ordinary commercial sheet lead taken from the stock in the laboratory. The uranio-lead was very kindly prepared and furnished by Professor B. B. Boltwood with the statement that it was obtained from some lead chloride that had been separated from a primary uraninite from India; that it should be entirely free from ordinary lead, and should contain only a little "thorio-lead," if such a substance exists.

METHOD OF COMPARING SPECTRA.

The method employed to compare the spectra of the two leads was as follows. The plateholder was placed on its usual mounting on the T rail, and one half of its breadth covered by the brass screen. It was then exposed to the rays from one kind of lead, the crystal always being given a small rotation back and forth around the position which gave the maximum reflection. At the end of the exposure the anti-cathode was turned until the other kind of lead came under the focal spot, and the plate exposed again with its other half covered by the screen. On some plates a full line was taken on each side of the half lines and about 2 mm. from them. This was to give fiducial lines from which to measure. Plates were taken at various distances from the axis of rotation of the spectrometer and with various distances between this axis and the slit S_1 . The distance from the axis to slit S_2 was always 4.5 cms. Different widths of slit were tried, and sometimes one, and other times two slits were used.

The plane of the anode did not remain absolutely fixed when the ground joint was turned to bring one or the other lead under the focal spot, though the variation was very slight. However, when using two very narrow slits a long distance apart, it was necessary to realign the spectrometer after turning the anode. This could easily be done by rotating the base of the spectrometer about the long axis of S_1 as described in the previous article. The spectrometer was lined up for one kind of lead and the position of the base noted. The other kind of lead was then brought into position, and the spectrometer realigned for it. The first half line was then taken without changing the anode. Then, after the first kind of lead had been brought back into position, an auxiliary plate was introduced close to S_2 and a series of exposures of equal length taken on it for different recorded positions of the spectrometer base around the position which had been noted. The position which gave the most intense image of S_2 was then used in taking the second half line. Absolutely no part of the spectrometer, slits, crystal or plateholder, were ever touched from the time the exposure for the first half line was started till the exposure for the second half was completed, except to rotate the base of the spectrometer to adjust the line up. This was done by slow motion screws, the base resting at two points on *plane* surfaces and pivoted at a third point on a carefully made cone. The auxiliary plate was supported without touching any part of the spectrometer.

TEST FOR LIMIT OF ACCURACY.

We wished, if possible, to compare the spectra of the two kinds of lead to a much higher degree of precision than had previously been attained. We were therefore confronted with the problem of finding what was the smallest displacement that we could detect between two half lines taken in the manner described. To do this we proposed to give the photographic plate a known displacement after taking one half line and before taking the other, both halves being produced by the same kind of lead. Various means of supporting and displacing the plateholder were tried for this purpose and proved unsatisfactory. Finally, at the suggestion of Professor Uhler, we hung the plateholder on wires in the manner already described and displaced it with the spherometer screw.

RESULTS.

1. *Limit of Accuracy.*—Two plates, nos. 41 and 44, were taken with the plateholder suspended on the wires. Slit S_1 was wide open, and S_2 was 0.02 mm. wide. The value of r , equation (1), was 4.5 cms. The crystal was rotated back and forth through an angle of $1\frac{1}{2}^\circ$ on each side of the best setting. On No. 41 five exposures were taken using the uranio-lead only. The first exposure was a full line, the third and fourth were upper halves, and the second and fifth were lower halves. The plateholder was displaced by the screw in the same direction after each exposure; 2.000 mm. after the first, 0.002 mm. after the second, 2.000 mm. after the third, and 0.004 mm. after the fourth. On No. 44 only four exposures were taken, using the ordinary lead. The first and fourth were full lines, the second was a lower, and the third an upper half. The displacements after the first and third were both 2.000 mm., and that after the second was 0.001 mm.

Both the small displacements between adjacent half lines on plate No. 41 were quite obvious under a low power magnifying glass. The displacement of 0.001 mm. on No. 42 was not visible in this manner. The lines on both plates were sharp and black and not more than 0.025 mm. broad. Both plates were measured on a measuring engine, frequently tested and regularly used by Uhler and others for measuring plates obtained with the large Rowland spectrograph. The measurements were made along two lines normal to the spectral lines, one a small distance above the break between the halves and the other the same distance below. The measured values of the distances agreed very closely with the actual displacements. The values of the small displacements, obtained by subtracting the upper from the lower distances, in no case differed from the actual displacements by as much as 0.001 mm. We

therefore feel justified in assuming that with lines of the sort obtained on these two plates, a displacement of 0.001 mm. could not escape detection.

2. *Comparison of Leads.*—The following plates were taken with two exposures, giving the upper half of the line from one lead and the lower half from the other, on each, the plateholder remaining fixed during the two exposures, and the crystal rotated back and forth around the best setting.

Plate No. 6.—The width of S_1 was 0.04 mm., and S_2 was wide open. The distance from S_1 to the axis of rotation was 35 cms. and the value of r was 35 cms. The ordinary lead line was above, and the uranio-lead line below. The lines were very fuzzy, though dark, and about 0.1 mm. broad.

Plate No. 7.—The data for this plate are the same as for No. 6 except that S_1 was closed to 0.02 mm. and the positions of the lines from the two leads were reversed. The lines were not quite as broad as on plate No. 6, but were weaker and just as fuzzy.

Plates No. 29 and No. 30.—The width of both S_1 and S_2 was 0.02 mm. The distance of S_1 to the axis of rotation was 20 cms. and the value of r was 20 cms. The positions of the lines were reversed on the two plates. The lines were narrower than on the other two plates and were fairly strong, but were still too broad and fuzzy to permit of precise measurements.

None of these plates showed any displacement between the lines, but their axes were too indefinite to permit of their being compared with a high degree of precision. An attempt was made to set a superior limit to the displacement that could exist without being visible. These plates were placed on the measuring engine, and one cross hair of the microscope was set as nearly as possible parallel to, and on what appeared to be the axis of the pair of half lines considered as one continuous line. The engine screw was then turned the least amount that would throw the cross hair, beyond question, off the axis of the lines. In this manner it was estimated that, on the first two plates, there could not have been a displacement as great as 0.02 mm., and on the second two, as great as 0.01 mm. If these values are assigned to δ in equation (1), with the corresponding values of r , it is seen that the possible difference of wavelength between the lines of the two kinds of lead can not exceed $1.6 (10)^{-4}$ Å. on the first two plates, and one half this value on the second two.

In order to have lines which could be measured with as high a degree of accuracy as those obtained in the tests for the smallest detectable displacement, we took two more plates, No. 46 and 47. The conditions

under which these plates were taken were as follows. Both slits were 0.02 mm. wide. The distances from S_1 and S_2 to the axis of rotation were 15.6 cms. and 4.5 cms. respectively. The value of r was 4.9 cms. The crystal was rotated back and forth through an angle of $1\frac{1}{2}'$ on each side of the best setting. Four exposures were taken on each plate, namely: a full line A was taken for one kind of lead, the plateholder was displaced along the T rail about 2 mm., and the lower half, B , of the line from the same kind of lead was taken; the plate holder remaining fixed, the upper half, C , of the line from the other kind of lead was taken, and then the plateholder was displaced about 2 mm. further and a full line, D , from the same lead as C , was taken. The direction of the displacement of the plateholder was such as to place A on the long wave-length side of B and C , and D on the short.

TABLE I.

Plate No. 46.

Upper Half.			Lower Half.		
(AC).	(AD).	(CD).	(AB).	(AD).	(BD).
2.1355	4.2021	2.0666	2.1384	4.2038	2.0654
.1341	.2033	.0692	.1373	.2051	.0678
.1370	.2038	.0668	.1356	.2035	.0679
.1358	.2059	.0701	.1363	.2061	.0698
.1363	.2043	.0680	.1357	.2048	.0691
Mean 2.1357	4.2039	2.0681	2.1367	4.2047	2.0680

$$(AB) - (AC) = 0.0010$$

$$(CD) - (BD) = 0.0001$$

$$\text{Mean} = 0.0006$$

Plate No. 47.

Upper Half.			Lower Half.		
(AC).	(AD).	(CD).	(AB).	(AD).	(BD).
2.0020	4.0512	2.0492	2.0045	4.0534	2.0489
.0045	.0522	.0477	.0075	.0550	.0475
.0044	.0522	.0478	.0021	.0538	.0517
.0064	.0550	.0486	.0046	.0539	.0493
.0036	.0524	.0488	.0057	.0533	.0476
.0031	0.517	.0486			
Mean 2.0040	4.0525	2.0485	2.0049	4.0539	2.0490

$$(AB) - (AC) = 0.0009$$

$$(CD) - (BD) = -0.0005$$

$$\text{Mean} = 0.0002$$

On plate No. 46 the lines *A* and *B* were produced by uranio-lead and the lines *C* and *D* by ordinary lead. On plate No. 47, on the other hand, the lines *A* and *B* were produced by ordinary lead, and *C* and *D* by uranio-lead. The lines on both plates were well defined and sharp though not quite as black as on plates 41 and 44. The upper and lower halves of each plate were measured on the engine. These measurements are set forth in detail in Table I., the figures in each column being the various measurements of the distance between the two lines corresponding to the letters at the head of the column. The units are approximately millimeters.

It will be seen from the table that on plate No. 46 the measurements from both *A* and *D* assign to *C*, an ordinary lead line, a longer wave-length than *B*, a uranio-lead line. The measurements from *A* on plate No. 47, however, assign to *C*, a uranio-lead line, a longer wave-length than *B*, an ordinary lead line, while the measurements from *D* reverse this order. None of the displacements between the half lines deduced from the measurements of these plates are greater than 0.001 mm., and the means are much less. Obviously the conclusion to be drawn from these two plates is that there is no difference in wave-length between the two kinds of lead great enough to produce a shift in the line of as much as 0.001 mm. With the value of r used with these plates we feel justified in the conclusion that the difference in wave-length between ordinary and uranio lead for the α_1 line of the L-series can not be as great as 0.00006 Å. The wave-length of this line being about 1.18 Å., the difference, if it exists at all, must be less than 0.005 per cent.

In conclusion it gives us great pleasure to express our sense of obligation to Professor Uhler for his interest and many valuable suggestions in connection with the work, and to Professor Boltwood for supplying us with the specimen of uranio-lead used.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
March 21, 1920.

ON THE VARIATION OF THE FACTOR h IN THE EQUATION

$$\frac{1}{2}mv^2 = h\nu.$$

BY FERNANDO SANFORD.

SYNOPSIS.

Relation between the β - and γ -rays of Ra B; Theory.—The author has previously shown that by combining the equations for orbital motion of an electron about a central positive charge with the equation $\frac{1}{2}mv^2 = h\nu$, the magnitude of the central positive charge about which a radiating electron is revolving may be calculated, and that when this value of the central charge is equated with one derived from an equation of the form of Moseley's equation for the frequency of X-rays the wave-length of the shortest lines in the K-radiation of elements of atomic numbers from 35 to 58 may be calculated as accurately as they can be measured. Later measurements of wave-lengths of elements with higher atomic numbers show that the computed wave-lengths vary progressively from the measured wave-lengths as the atomic numbers increase. It is also known that Moseley's law for the frequency of X-rays varies in the same manner.

The equation from which the author calculates the wave-length is

$$\frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e^2} = 2e(N - 3.6).$$

Since the right-hand member of this equation shows that the nuclear charge in the case of K-radiation increases by $2e$ in passing from one element to the one of next higher atomic number, it is not probable that this law breaks down at atomic number 58, and that the nuclear charges of elements of higher atomic number increase by variable quantities. The failure of the equation to apply to elements of higher atomic number must then be looked for in the left-hand member. In this member the only variable quantities (since λ is measured) are m , the mass of the electron, and possibly h , the Einstein factor. These quantities appear in the form of the ratio h^2/m , and the author has calculated values of h^2/m which substituted in his equation will give the correct wave-lengths for elements of higher atomic number. These values of h^2/m are then plotted against their corresponding wave-lengths and a smooth curve is obtained. Values of h^2/m are then taken from this curve for a number of the γ -ray wave-lengths measured by Rutherford and Andrade. These wave-lengths are then assigned to certain groups of β -rays from Radium B on grounds which seem to the author permissible. The mass, m , and the quantity h corresponding to the speeds of these β particles were then computed. The values of h computed in this way seem to be derivable from m by the equation $h = 4.25 \log m + 2.50 \cdot 10^{-27}$. Using this value of h , wave-lengths were computed for the fourteen groups of β -rays from Radium B whose speeds were measured by Rutherford, and nine of them were found to agree satisfactorily with wave-lengths in the γ -radiation of Radium B and Radium C, while two more of the measured wave-lengths seem to correspond with the speeds of β -particles from Radium C.

Calculation of Orbital Velocities of Radiating Electrons by Substituting $4.25 \log m + 2.50 \cdot 10^{-27}$ for h in the equation $\frac{1}{2}mv^2 = h\nu$.

Using a series of values of v from $0.1 \cdot 10^{10}$ to $2.2 \cdot 10^{10}$ corresponding values of

m were calculated from Bucherer's equation for the transverse mass of an electron, viz., $m = m_0/\sqrt{1 - v^2/c^2}$. From these values v was computed using the equation $v = \frac{1}{2}mv^2/(4.25 \log m + 2.50)$, and these values of v were plotted on a large scale as abscissas using the values of v from which they were derived as ordinates. From this curve were taken values of v corresponding to all the highest frequencies of K-radiation as measured by Siegbahn and his colleagues and Duane and his colleagues. The values of v obtained in this way are then compared with values calculated from the equation $v = 2.054(N - 1.66)$ and are found to agree satisfactorily with no tendency to vary progressively with the higher numbers, as do the values calculated by Duane using h as a constant.

It is concluded that since h may be replaced in the equation $\frac{1}{2}mv^2 = h\nu$ by a function of m with a great improvement in the accuracy of the equation, there is no longer ground for speaking of an energy quantum in this equation.

THE writer has shown elsewhere¹ that by combining the equation for orbital motion of an electron about a central positive charge, viz., $mv^2/R = Qe/R^2$, with the equation $\frac{1}{2}mv^2 = h\nu$ it is possible to compute the central positive charge, Q , of a radiating atom, and from this value of Q and a corresponding value computed by an equation of the form given by Moseley to determine the wave-lengths of all known characteristic X-ray bands. Also, by combining the equation for Q with the formulæ for computing the wave-length of the lines in the various spectral series it was shown that the unit electrical charge may be computed from any line in any known spectral series of hydrogen or helium in terms of the wave-length and the serial number of the line. It is also possible by the use of the same equation to calculate the value of the Einstein factor h from the convergence charge of the Balmer series in hydrogen.

The above mentioned computations are not based upon any theory of atomic structure except that radiating electrons are in orbital revolution about a central positive charge, and that the frequency of revolution of the electron determines the wave-frequency of its radiation. In the opinion of the writer these assumptions are not incompatible with any of the commoner theories of atomic structure, but are admittedly in conflict with the Bohr theory of radiation.

The expression for the value of the central charge, Q , derived from the above equations is

$$Q = \frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi}.$$

By equating this value of Q with a value derived from an equation of the form of the Moseley equation we may get an expression for the

¹ See especially On The Nuclear Charges of Atoms, PHYSICAL REVIEW, IX., 383 (1917); The Astronomical Atom and the Spectral Series of Hydrogen, Astrophysical Journal, XLVIII., 1 (1910); The Helium Spectrum and the Unit Electrical Charge, Astrophysical Journal, XLIX., 337 (1919).

value of λ . Thus for the characteristic K-radiation $Q = 2e(N - 3.6)$ where N is the atomic number. Then for this radiation

$$\frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi} = 2e(N - 3.6).$$

This equation has been found to apply to the elements from $N = 35$ to $N = 58$ so that the wave-lengths of the K-radiation from these elements may be calculated from this formula as accurately as they may be measured.¹

In the PHYSICAL REVIEW of January, 1920, the present writer undertook to show that if the speed of the β -particles sent off from Ra-B and Ra-C is related by Einstein's law to the accompanying γ -radiation then before the expression $\frac{1}{2}mv^2$ may be substituted for Ve in that equation the factor h , as well as m , must be assumed to vary with the speed of the β -particles.

Since the article referred to was written, two papers which have an important bearing on this question have appeared. In the Physikalische Zeitschrift of June 1, 1919, Siegbahn and Jönssen have published new determinations of the wave-lengths of highest frequency in the K-radiation series of a considerable number of elements from cadmium to uranium, and in the Physical Review of December, 1919, Duane and Hang-Fuh-Huh and Duane and Shimizu have extended the wave-length measurements on the absorption frequencies of the K-radiation to elements of both lower and higher atomic numbers than those whose frequencies were determined by Blake and Duane. These measurements when taken with the wave-length determinations of Siegbahn and Stenström in 1916 give a series of measurements of the short wave-lengths in K-radiation for most of the elements from atomic numbers $N = 12$ to $N = 92$.

When these newly determined wave-lengths are computed by our formula it is found that the correspondence is equally as good as for the measurements of Blake and Duane up to an atomic number 58, but that from this point onward the calculated values are greater than the measured values. Siegbahn and Jönssen call attention to the fact that from this point onward their wave-lengths do not follow Moseley's law, and they attribute the variation from this law to the variable mass of the electron at high speeds.

In the papers in the PHYSICAL REVIEW, Duane and his colleagues calculate a value of v for each of the elements whose wave-lengths they have measured by using the equation $\frac{1}{2}mv^2 = h\nu$ and computing the value of m at each speed by the Bucherer equation, $m = m_0/\sqrt{1 - (v^2/c^2)}$.

¹ See PHYSICAL REVIEW, XV., 68 (January, 1920).

They found that their values of v computed in this way bore a straight line relation to the atomic numbers, as does the square root of the frequencies in Moseley's equation, but that for atomic numbers above 58 their calculated values of v vary slightly, but progressively, from the values given by the atomic number relation.

It will be seen that in the equation

$$\frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi} = 2e(N - 3.6)$$

a variation in m can affect only the left-hand member of the equation, and that the only other quantity in this member which may possibly vary (since λ is measured) is the quantity h , or, in other words, the possible variable quantity in this member is the fraction h^3/m . Since the right-hand member of the equation merely shows that Q increases by $2e$ in passing from any element to the one of next higher atomic number, there is no reason for thinking that this law changes and that the increase of charge becomes variable for elements of higher atomic number than 58.

If we may rely upon the constancy of the right-hand member of the equation, we may compute values of h^3/m for each of the given wave-lengths and perhaps decide whether the variation in this side of the equation is due wholly to m , or whether h is also probably variable.

In Table I. Blake and Duane's wave-length measurements are given in the third column, those of Siegbahn and Jönssen in the fourth column, the wave-lengths calculated from our formula in the fifth column and the values of h^3/m required to give the measured wave-lengths in the sixth column. It may be seen that h^3/m remains constant to within the errors of measurement to about $N = 58$, from which point on it decreases, while the calculated values of the wave-length also show a systematic variation from about the same value of N .

When these values of h^3/m are plotted as ordinates against their corresponding wave-lengths as abscissas a smooth curve is obtained from which values of h^3/m may be taken for any wave-length down to $\lambda = 0.1 \cdot 10^{-8}$. If corresponding values of v could now be found it would be possible to calculate the respective values of h . This I have attempted to do in the following way:

All of the wave-length calculations and the values of h^3/m given in Table I. are based upon the assumption that the speeds of electrons expelled under the stimulus of ultra-violet light or of X-rays are the orbital speeds which they possessed at the instant of breaking away from their respective atoms. If this be the true explanation of their velocities, it seems almost certain that the same explanation must account for the speeds of the β -particles expelled by radioactive bodies. This would

TABLE I.

Element.	<i>N</i> .	λ_{10^6} , B. & D.	λ_{10^6} , S. & J.	λ_{10^6} , Calc.	h^2/m , 10^{24} .
Br.....	35	.9179		.924	309.5
Rb.....	37	.8143		.817	310.5
Sr.....	38	.7696		.771	311
Yt.....	39	.7255		.727	309.5
Zr.....	40	.6872		.689	311
Nb.....	41	.6543		.654	311
Mb.....	42	.6180		.618	311
Ru.....	44	.5584		.558	311
Rh.....	45	.5324		.532	311.5
Pd.....	46	.5075		.507	312.5
Ag.....	47	.4850		.484	312
Cd.....	48	.4632	.4629	.463	312
In.....	49	.4434		.443	312.5
Sn.....	50	.4242	.4231	.424	312
Sb.....	51	.4065		.409	312
Te.....	52	.3896	.3877	.389	310.5
I.....	53	.3737	.3715	.373	310.5
Cs.....	55	.3444	.3436	.345	310.5
Ba.....	56	.3307	.3306	.332	310.5
La.....	57	.3188	.3186	.320	310.5
Ce.....	58	.3073	.3064	.308	310.5
Pr.....	59		.2946	.297	309
Nd.....	60		.2835	.286	308
Sa.....	62		.2636	.2665	307.5
Eu.....	63		.2543	.2585	306.5
Gd.....	64		.2456	.2495	306.5
Dy.....	66		.2294	.234+	305.5
Ho.....	67		.2214	.2265	304.5
Pt.....	78		.1578	.165	298.5
Au.....	79		.1524	.1605	296.5
Hg.....	80		.1479	.156	295
Tl.....	81		.1427	.152	292
Pb.....	82		.1385	.148	291.5
Bi.....	83		.1346	.1445	290
Th.....	90		.1127	.122	288
U.....	92		.1048	.1167	280

require that the Einstein photoelectric equation should hold for the energies of the β -rays and the frequencies of the accompanying γ -radiations from the same elements. Rutherford and his colleagues have given much time to the investigation of this question, and are of the opinion that such a relation does, indeed, hold and that some of the γ -rays from Ra-B and Ra-C are the true K- and L-radiation bands of these elements.

In an article on The Connexion Between β and γ Ray Spectra¹ Ruther-

¹ Phil. Mag., 28, 305-319.

ford gives the velocities relative to the velocity of light of fourteen groups of β -rays expelled from Ra-B and ten groups of β -rays from Ra-C. In an article by Rutherford and Andrade on The Spectrum of the Penetrating γ Rays from Radium B and Radium C in the same volume of the Philosophical Magazine is given a table of the wave-lengths of fourteen different lines in the γ -ray spectrum of these elements. The authors do not know to which element the various lines belong, but give their reasons for believing that the shortest wave-lengths are due to Radium C, and that the lines $\lambda = 1.59 \cdot 10^{-9}$ and $\lambda = 1.69 \cdot 10^{-9}$ are the characteristic K-radiation lines of Radium B.

In the data on the velocities of β -rays the groups of rays of speeds 1.905-2.100 are marked very strong, indicating that there are more β -rays of this speed than of any other unless it be 1.01. Since the lines of wave-length 1.59 and 1.69 are referred to as the strongest lines in the γ -ray spectrum, it seems reasonable to assign these lines to the group of β -rays whose speeds are from 1.9 to $2.10 \cdot 10^{-10}$.

If we calculate a value of m for a speed of $1.9 \cdot 10^{-10}$ we find $m = 1.16 \cdot 10^{-27}$. Since the wave-length 1.69 lies within the spectral region for which we have computed the value of h^3/m , we may take the value of this ratio from the curve, which was drawn for that region, where we find it to be $300 \cdot 10^{-34}$. Accordingly, $h^3 = 348 \cdot 10^{-81}$ and $h = 7.03 \cdot 10^{-27}$.

In order to express graphically the relation of h to m we must have this relation at one more point, at least, since we already have the initial values of the two quantities. By a slight extrapolation of our curve it was seen that the γ -ray line $\lambda = .99$ must give $h^3/m = 276$, approximately. For trial purposes this line was assigned to the highest speed β -particle from Radium-B, viz., $v = 2.469$. For this speed $m = 1.2 \cdot 10^{-27}$, and if $h^3/m = 276$, $h = 7.60$.

When these three values of h and m were compared they seemed to suggest that h varies as $\log m$. On plotting a curve with the values of h as abscissas and the values of $\log m$ as ordinates, a linear relation was seen to hold between the two quantities, so that for $h \cdot 10^{27}$ and $\log m \cdot 10^{28}$, $h = 4.25 \log m + 2.50 \cdot 10^{-27}$.

Assuming the validity of this equation it is possible to calculate m and h for each of the speeds given, and to compare the values of h^3/m obtained in this way with values taken from our curve for each of the wave-lengths measured in the spectra of Radium B and Radium C. This is done in Table II., where the velocities of the β -rays from Radium B are given in the first column, the corresponding values of m as computed from the equation $m = m_0 / \sqrt{1 - (v^2/c^2)}$ in the second column, h as derived from

the equation $h = 4.25 \log m + 2.50$ in the third column and h^3/m in the fourth column. In the fifth column are given values of λ corresponding to the values of h^3/m in the fourth column as taken from the curve drawn to show this relation. In the sixth column are given the values of λ as measured by Rutherford and Andrade in the γ -radiation of Radium B and Radium C.

TABLE II.

β Radiation from Radium B.				γ Radiation.	
ν .	m .	h .	h^3/m .	λ Calc.	λ Meas.
					.71
2.469	1.59	7.60	276	.99	.99
2.450	1.56	7.55	278	1.03	—
2.391	1.49	7.46	281	1.10	—
2.361	1.46	7.45	282	1.15	1.15
2.286	1.39	7.35	287	1.24	—
2.252	1.36	7.31	288	1.27	—
2.193	1.32	7.25	289.5	1.31	—
2.157	1.295	7.22	292	1.37	1.37
2.100	1.26	7.16	292	1.37	1.37
1.968	1.195	7.06	297	1.55	1.59
1.905	1.165	7.03	300	1.69	1.69
—	—	—	—	—	1.96
—	—	—	—	—	2.29
—	—	—	—	—	2.42
1.278	.995	6.74	307	2.62	2.62
1.242	.990	6.73	308	2.90	2.96
—	—	—	—	—	3.24
—	—	—	—	—	3.93
1.011	.956	6.66	309	4.28	4.28

A comparison of columns five and six of Table II. will show that nine of the fourteen measured wave-lengths correspond as closely as could be expected with the values calculated from the speeds of the β -particles on the assumption that $h = 4.25 \log m + 2.50$. For the three slowest groups of β -particles the values of h^3/m fall on a part of the curve where h^3/m varies but slightly with the wave-length, and where the values of λ derived from this curve must be more uncertain than for the faster β -particles.

The slowest speed given has been assigned to the wave-length 4.28 from the following considerations: The radiation of particles at this speed is marked "very strong," indicating that this group of β -particles corresponds to a characteristic radiation of the element. I have shown elsewhere¹ that the shortest wave-length of the characteristic L-radiation

¹ Electrical Charges of Atoms and Ions, Leland Stanford Junior University Publications, p. 70.

may be calculated from the equation

$$\lambda = \left[\frac{2.882 \cdot 10^{-12}}{e(N - 15.4)} \right]^2$$

for elements up to atomic number 71. This equation would give for Radium B ($N = 82$) a wave-length $\lambda = 4.33$, which is within the error of measurement the line 4.28.

The measured speeds of the β -particles from Radium C are with two exceptions much higher than those from Radium B, and the curve derived from X-ray wave-lengths may not safely be extrapolated far enough to give a probable estimate of their γ -ray wave-length. The slowest group of these particles has a velocity of $2.25 \cdot 10^{10}$, which would make its γ -ray wave-length about 1.27 from our computation. The next slowest group of β -particles has a speed of 2.604. This would give $m = 1.80$ and $h = 7.83$, making $h^2/m = 266$. This seems to correspond with the measured wave-length .71.

It seems not improbable that the γ -radiation from the other β -particles of Radium C was of too short wave-length to be measured by Rutherford and Andrade. With their spectrometer the wave-length taken as .71 was determined from a deviation of only 43', and the authors say, "It is surprising that the architecture of the crystal (rock salt) is sufficiently definite to resolve such short wave-lengths. This is especially the case when we consider that owing to the heat agitation of the atoms, the distance between the atoms must be continually varying over a range comparable with the wave-length of the radiation."

In a paper by F. Dessauer and E. Back published in *Berichte der Deutschen Physikalischen Gesellschaft*, May 30, 1919, the authors give the results of their attempt to find the shortest wave-length given by X-rays from any element. By using a potential of 170 k.v. and an exposure of 9 hours they found the apparent limit of X-rays at $\lambda = .57 \cdot 10^{-9}$. It is probable that all of the β -rays from Radium C except the two sets referred to would be accompanied by γ -rays of shorter wave-length than $.57 \cdot 10^{-9}$.

If these estimates of the relation between the speeds of β -rays and the wave-lengths of their accompanying γ -radiations are correct, it seems almost certain that the speeds of β -particles are the orbital speeds which they possessed at the instant of escaping from the atoms.

It also seems certain that for speeds as great as those of β -particles the product $\frac{1}{2}mv^2$ of the β -rays does not bear a constant ratio to the frequency of the accompanying γ -radiation.

It has already been mentioned that Duane and his colleagues have

calculated values of ν from the K-radiation of a large number of elements which values are related to the atomic numbers by an equation of the form of Moseley's equation for the square root of the frequencies. It will be seen from our fundamental equation for the atomic charge that this will necessarily be true if h^2/m is constant or bears a straight line relation to the atomic numbers. It was observed, however, that this law of Duane's holds only so far as the wave-length may be correctly calculated on the assumption of the constancy of h^2/m .

In order to test the same relation with values of ν calculated from our hypothesis, I have calculated values of m for a series of values of ν from $\nu = 0.1 \cdot 10^{10}$ to $\nu = 2.2 \cdot 10^{10}$, letting

$$m = \frac{9 \cdot 10^{-28}}{\sqrt{1 - \nu^2/c^2}}.$$

Then from the equation $\frac{1}{2}mv^2 = \nu(4.25 \log m + 2.50)$ I have calculated values of ν for each value of ν used in computing m . These values of ν were then plotted on a large scale as abscissas against the respective values of ν as ordinates. From this curve it is possible to obtain a value of ν accurate to the third significant figure for any value of ν from $\nu = 0.1 \cdot 10^{10}$ to $\nu = 40 \cdot 10^{10}$.

Values of ν corresponding to all the K-radiation frequencies measured by Siegbahn and Stenström, Siegbahn and Jönssen, Blake and Duane, Duane and Hang-Fuh-Huh and Duane and Shimizu were then taken from this curve and plotted against their respective atomic numbers. In the case of the Siegbahn and Stenström measurements the frequencies for the $K\beta$ lines were used throughout. It was found that the velocities calculated in this manner bear a straight line relation to the atomic numbers with no tendency to systematic departure on either side of the line for higher atomic numbers. This relation of velocity to atomic number may be expressed by the equation $\nu = 2.054(N - 1.66)$.

The relation between the values of ν calculated in the two ways is shown in Table III., in which the wave-length measurements of Siegbahn and his colleagues are given in the third column and those of Duane and his colleagues in the fourth column. The corresponding values of ν are given in the fifth column. Where two values of λ are available a mean value is used in computing ν . The sixth column gives ν from the equation

$$\nu = \sqrt{\frac{2\nu(4.25 \log m + 2.50)}{m}}$$

and the seventh column gives ν from the equation $\nu = 2.054(N - 1.66)$.

It will be seen that though the series runs to $N = 92$, the agreement

TABLE III.

$$v_1 = \sqrt{\frac{2\nu(4.25 \log m + 2.50)}{m}},$$

$$v_2 = 2.054(N - 1.66).$$

Element.	N.	λ Siegbahn and Others.	λ Duane and Others.	ν .	v_1 .	v_2 .
Mg.	12	9.477		.3165	.213	.2124
Al.	13	7.986		.3753	.233	.233
Si.	14	6.759		.4436	.2535	.2535
P.	15	5.808		.516	.274	.274
S.	16	5.018		.5978	.295	.2946
Cl.	17	4.394		.6826	.316	.3152
K.	19	3.449		.8700	.357	.3564
Ca.	20	3.086		.973	.377	.377
Sc.	21	2.778		1.080	.3975	.3972
Ti.	22	2.509		1.195	.418	.418
Va.	23	2.281		1.315	.437	.438
Cr.	24	2.079		1.443	.458	.458
Mn.	25	1.902	1.8892	1.582	.480	.4795
Fe.	26	1.743	1.7396	1.721	.500	.500
Co.	27		1.6018	1.86	.520	.521
Ni.	28		1.4890	2.015	.540	.541
Cu.	29		1.3785	2.175	.560	.562
Zn.	30		1.2963	2.315	.578	.582
Ga.	31		1.1902	2.521	.602	.603
Ge.	32		1.1146	2.693	.623	.623
As.	33		1.0435	2.875	.644	.644
Se.	34		.9790	3.065	.665	.664
Br.	35		.9179	3.268	.685	.685
Rb.	37		.8143	3.684	.726	.726
Sr.	38		.7696	3.898	.745	.746
Yt.	39		.7255	4.136	.767	.767
Zr.	40		.6872	4.365	.788	.788
Nb.	41		.6503	4.61	.808	.808
Mo.	42		.6180	4.855	.830	.829
Ru.	44		.5584	5.374	.871	.871
Rh.	45		.5330	5.63	.891	.890
Pd.	46		.5075	5.913	.912	.911
Ag.	47		.4850	6.185	.9325	.932
Cd.	48	.4629	.4632	6.48	.953	.952
In.	49		.4434	6.766	.973	.973
Sn.	50	.4231	.4242	7.08	.994	.994
Sb.	51		.4605	7.384	1.014	1.014
Te.	52	.3877	.3896	7.722	1.036	1.034
I.	53	.3715	.3737	8.065	1.057	1.055
Cs.	55	.3436	.3444	8.725	1.098	1.096
Ba.	56	.3306	.3307	9.08	1.118	1.116
La.	57	.3186	.3188	9.42	1.138	1.137
Ce.	58	.3064	.3068	9.80	1.160	1.157
Pr.	59	.2946		10.18	1.181	1.178

Element.	<i>N</i> .	λ Siegbahn and Others.	λ Duane and Others.	ν .	ν_1 .	ν_2 .
Nd.....	60	.2835	.2861	10.54	1.201	1.199
Sa.....	62	.2636		11.38	1.243	1.239
Eu.....	63	.2543		11.79	1.263	1.260
Gd.....	64	.2456		12.21	1.284	1.282
Tb.....	65		.2398	12.51	1.299	1.301
Dy.....	66	.2294	.2308	13.04	1.324	1.322
Ho.....	67	.2214		13.54	1.348	1.343
W.....	74		.1786	16.78	1.485	1.486
Os.....	76		.1683	17.82	1.531	1.528
Pt.....	78	.1578		19.01	1.572	1.568
Au.....	79	.1524	.1541	19.57	1.590	1.589
Hg.....	80	.1479	.1501	20.14	1.608	1.608
Tl.....	81	.1427		21.01	1.640	1.626
Pb.....	82	.1385	.1424	21.35	1.651	1.650
Bi.....	83	.1346		22.29	1.681	1.671
Th.....	90	.1127		26.61	1.814	1.815
U.....	92	.1048		28.60	1.866	1.835

between the two values of ν is very close and no progressive variation appears. In only one case, that of uranium, is the difference between the two values of ν as great as one per cent. From the equations here used the wave-length of the shortest K-radiation from uranium should be $\lambda = .1099 \cdot 10^{-8}$ instead of $.1048 \cdot 10^{-8}$, as given by Siegbahn and Jönssen. Dessauer and Back, in the paper already mentioned, attempted to find the wave-length of the $K\beta$ line of uranium, and concluded that it lies between $.104 \cdot 10^{-8}$ and $.154 \cdot 10^{-8}$. When extrapolated from their measured wave-length of the platinum line by Moseley's equation they found it to be between $.114$ and $.136 \cdot 10^{-8}$, but it is well known that Moseley's equation does not apply closely to elements of the higher atomic numbers. From Sommerfeld's formula this line would lie between $.131$ and $.146 \cdot 10^{-8}$.

The one other considerable deviation is in the case of thallium, for which our equation would give a wave-length $.1455 \cdot 10^{-8}$ instead of $.1427 \cdot 10^{-8}$, as determined by Siegbahn and Jönssen.

On the whole, the two values of ν are in very satisfactory agreement when we take into consideration that the most careful wave-length determinations in this region by different experimenters frequently differ in the third decimal place.

While it is probably too much to claim that the relations shown in Table II. and Table III. give an adequate test of the value of the expression here substituted for h in the equation $\frac{1}{2}mv^2 = h\nu$, they are, so far as is known to the writer, the only tests thus far made of the validity of

this equation. The equation $V_e = h\nu$ has frequently been tested by experiment, but the assumption that all the energy given to an electron in an electrostatic field appears in the form of kinetic energy of the electron has not been so tested. Whatever evidence may be derived from the relations here shown indicates that in this form h does not represent an energy quantum, but only a function of the mass of the electron.

A STUDY OF THE RESIDUAL IONIZATION IN A GAS WITH REFERENCE TO TEMPERATURE EFFECTS.

By C. H. KUNSMAN.

SYNOPSIS.

A Study of the Residual Ionization in a Gas with Reference to Temperature Effects.

—A form of electrometer was constructed which used the principle of the Wulf electrometer in separating the conduction of charge across the insulation from the conduction in the gas due to ionization. *The new features of this apparatus* were that it was possible (1) to make a quick and accurate test of the insulation properties under the same conditions as those under which the ionization tests were made, and (2) to subject the instrument to a wide range of temperatures.

Ions generated per C.C. per Second.—The data experimentally determined show that the number of ions generated per c.c. per second within an air tight chamber is 8.22 in the basement of the Physical Laboratory and 4.15 over the Pacific Ocean.

Residual Ionization not due to Thermal Impact.—The results of tests between -44.6° C. and 92.5° C. show no indications that the residual ionization is due in any part to a molecular impact of thermal agitation.

Effect of Temperature on Insulation System.—Observations made of changes in temperature on the insulation system, would seem to account for the apparent daily and seasonal variations of the residual ionization as previously reported.

Conclusions from Observations at Low Temperatures.—At temperatures from -30.5° C. to -44.6° C. an increase in conduction across the insulation system was noticed, which was comparable to the apparent increase in ionization as reported at high altitudes. It would therefore seem that it is not necessary to assume, as has been done by previous observers, that there is a highly radioactive cosmic layer in the upper atmosphere; or that the sun is a source of penetrating radiation sufficient to generate 90 ions per c.c. per second.

I. INTRODUCTION.

THE experiments described in this paper were undertaken with the purpose of studying the ionization of a gas as observed in an air-tight vessel. This was done with a twofold purpose: (1) to find an accurate method of measuring this conductivity in the gas, and (2) to make a thorough investigation of the effect of temperature on this conductivity.

C. T. R. Wilson¹ and Geitel² were the first to notice that gases in closed vessels conducted electricity and were therefore ionized. The more recent experiments of Simpson and Wright,³ King,⁴ Murray,⁵

¹ Wilson, Proc. Camb. Phil. Soc., II., 52, 1900.

² Geitel, Phys. Zeit., 2, 116, 1900.

³ Simpson and Wright, Roy. Soc. Lon. Proc., 85, 175, 1911.

⁴ King, Phil. Mag., 26, 610, 1913.

⁵ McLennan and Murray, Phil. Mag., 30, 430, 1915.

McLeod,¹ McLennan,² Wright,³ and the author⁴ showed conclusively that ions are being generated in an airtight metal chamber, as free from radio-active substances as possible, at the rate of from 8 to 9 ions on land, and over the ocean and large lakes from 4 to 5 ions per c.c. per second. These results include observations made over practically every continent and ocean. Some observations indicate a pronounced variation. Lassalle⁵ notices a diurnal variation in the Philippine Islands, while Gockel⁶ finds the value for the winter months of December and January lower by 1 to $1\frac{1}{2}$ ions per c.c. per second than for the months of June and July.

In connection with the ionization within a closed chamber, we are confronted with two problems: (1) to find the source of the residual ionization of from 4 to 5 ions which is obtainable when the chamber is screened naturally over large bodies of water, or artificially on land; (2) to find an explanation for the very large increase in the ionization at high altitudes, as observed in a balloon by Hess⁷ and Kolhörster.⁸ The latter reports an increase of from 4.3 ions at 2,000 meters to 80.4 ions at 9,000 meters, over the values obtained at the earth's surface. Treleaven⁹ concludes that the residual ionization of 4 ions is due to α , β , and possibly γ radiation emitted from the walls of the chamber. Murray⁶ comes to the same conclusion from observations of this effect when the walls of the chamber were of ice. Kingdon¹⁰ shows that a part of the residual ionization is due to thermal agitation and derives a formula which agrees with his experimental results.

The only other experimental works are those of Patterson¹¹ and Devik.¹² Patterson used a large iron chamber containing an electrode insulated by ebonite, and measured the current with an electrometer. He got a value of 61 ions, so it would be doubtful if he could observe a small temperature effect at ordinary temperatures. Devik attempted to measure the current at the instant of greatest compression in an adiabatic compression and found no effect at ordinary temperatures.

¹ McLennan and McLeod, *Phil. Mag.*, Oct., 740, 1913.

² McLennan, *Phys. Rev.*, 26, 526, 1908.

³ Wright, *Phil. Mag.*, 17, 295, 1909.

⁴ Kunsman, *Phys. Rev.*, 6, 493, 1915.

⁵ Lassalle, *Phys. Rev.*, 5, 135, 1915.

⁶ Gockel, *Phys. Zeit.*, 16, 350, 1915.

⁷ Hess, *Phys. Zeit.*, 14, 610, 1913.

⁸ (a) Kolkörster, *Phys. Zeit.*, 14, 1153, 1914; (b) *Ber. Deut. Phys. Ges.*, 13, 721, 1914.

⁹ Treleaven, *Phil. Mag.*, 30, 427, 1915.

¹⁰ Kingdon, *Phil. Mag.*, 32, 397, 1916.

¹¹ Patterson, *Phil. Mag.*, 6, 231, 1903.

¹² Devik, *Sitz. d. Heid. Akad. Wiss.*, 24, 1914.

The question has been theoretically investigated by Langevin and Rey,¹ and by Wolfke.²

Results obtained by the author seem to throw light on each of the following questions: (1) Do molecules of gases under ordinary temperatures collide in such a way as to produce ionization? (2) Is it necessary to assume a source of ionization in the upper atmosphere sufficient to produce ions at the rate of about 90 per c.c. per second?

II. APPARATUS.

Preliminary experiments on the measurements of the natural ionization at the State College of Pennsylvania, at the University of California,⁹ and on the Pacific Ocean between San Francisco and Los Angeles, were made with a Spindler and Hoyer¹⁰ aluminum leaf electroscope designed for radioactive tests when the gases tested could be drawn into the ionization chamber. Two chambers of different sizes were essential for the method used.

More extensive tests were made by means of an electrometer designed by the author which could be subjected to a wide range of temperature. The object of this was to make it possible to measure ionization down to the temperature of liquid air. The main feature of the instrument was that it provided an easy test for leak across the insulation system without altering the conditions under which the tests were made. Such a test of conduction over the insulation could not be made on the apparatus used by the observers previously mentioned.

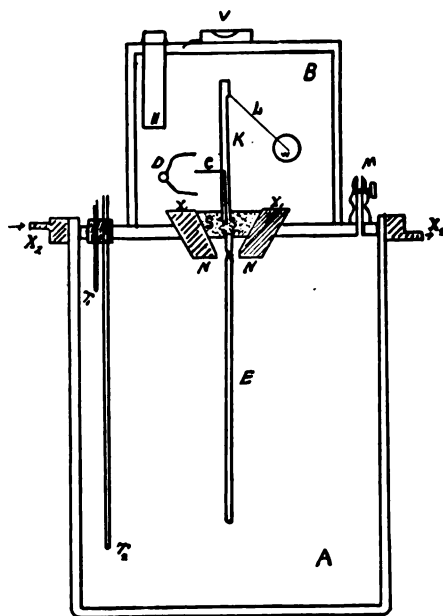


Fig. 1.

Practically the entire apparatus was built of as pure zinc as obtainable, since this metal seems more free from radioactive impurities than others. The ionization chamber *A* consisted of a zinc cylinder about 20 cm. long and 10 cm. in diameter, Fig. 1, the walls of the cylinder

¹ Langevin and Rey, *Le Radium*, 10, 142, 1913.

² Wolfke, *Le Radium*, 10, 265, 1913.

being $2\frac{1}{2}$ mm. thick. Into this screwed the upper part which consisted of a zinc box *B*, forming the walls of the electrometer. The sulphur plug *S* holds in place the zinc electrode *E*, and another zinc rod *K*, upon which the aluminum leaf *L* was attached. A very small rod *C* moves up or down through the center of the zinc rod *K*. The rod *C* is raised or lowered by an insulated rod *D* which passes through a ground glass joint attached to the electroscope case. The volume of *A* or *B* can thus be electrically connected or separated as desired. The electric charge is communicated to the insulated system by means of the same rod *D*. After the system is charged, rod *D* is disengaged from rod *C* and grounded, as is also the case of the electrometer. An external dial on the case and an indicator on the charging rod assure the rod *D* being turned to the same position after each time the instrument is charged.

The sulphur insulation was kept cool or warm as was necessary by means of a system *X*₁ through which water or alcohol was run. A similar cooling system *X*₂ kept the wax joints cool.

Chamber *B* was kept as dry as possible by means of metallic sodium, contained in a receiver *H*. Thermo-couples *T*₁ and *T*₂ made it possible to measure the temperature of the gas quickly and accurately. A stopcock *M* permitted the ionization chamber to be exhausted and filled with dry gas. A spirit level *V* made it possible to keep the instrument always in the same position. A very fine quartz fiber was attached to the end of the leaf and the position of the fiber observed through windows by means of a telescope. The quartz fiber greatly increased the accuracy of the observations.

The zinc electrode *E* could be easily removed and a small cup containing sodium could be sealed to the bottom of the water jacket *N*, so that the loss of charge due to the volume *B* together with the total leakage across the insulation support could be determined. The volume of chamber *A* was 1,277 c.c. The electrostatic capacity of the electrometer as determined with two spheres of different radii,¹ was 4.48 cm. with the cup at *N* in place, and 6.50 cm. with the cup removed and the central electrode replaced. The drop in potential for a given number of scale divisions was obtained from the calibration curve, Fig. 2.

III. THEORY.

If *C* is the electrostatic capacity of a body, which is charged to a potential *E*, and after a certain time some of the charge has disappeared, it is evident that we can express this loss, or change of quantity, of electricity as $\Delta Q = C\Delta E$; where ΔE is the drop in potential for a unit of

¹ Lichtenecker, Phys. Zeit., 13, 516, 1913.

time Δt . This loss of charge may be thought of as a neutralization of N ions, each carrying the unit charge, e . If n is the number of ions formed per c.c. per second in the volume V in the time Δt , we may write $N = nV\Delta t$. It follows that $\Delta Q = C\Delta E = NE = n\Delta teV$, or $n = C\Delta E/Ve\Delta t$.

Now ΔE is made up of two parts (*A*) a conduction of charge across the insulation support and (*B*) a loss of charge due to ions being formed in the gas, which neutralizes some of the charge on the insulated system. If two different volumes of gas are taken in such a way as not to alter the conduction of charge over the support, two equations can be set up of the form, $Q_1 = A + BV_1$ and $Q_2 = A + BV_2$; where Q_1 and Q_2 are the loss of charge per second with volumes V_1 and V_2 respectively and where $B = ne$. From this we get

$$n = \frac{C_1\Delta E - C_2\Delta E}{(V_1 - V_2)e},$$

which is the formula used for all calculations of n .

IV. FIRST INVESTIGATION.

In the first set of experiments dry, dust-free air was introduced in the ionization chamber of the Spindler and Hoyer electroscope, in which the amber insulation was replaced by sulphur. Observations were taken of the rate of fall of the leaf, or loss of charge, after the rate of loss became constant. An apparent soaking in of the electric charge was shown by the relatively rapid fall just after the instrument was charged. This effect lasted about two hours. Quite a pronounced increase in ionization was always noticed immediately after the instrument was filled with air. This effect largely disappeared in about 4 days, which leads one to conclude that it was due to radium emanation, its half period being 3.8 days. The rate of fall was observed for about 4 weeks for each ionization chamber, and a mean value of n calculated. The mean value of n as obtained in the basement of the physical laboratory was 8.68 ions. Variations in this number were observed. In the light of recent experiments, these variations can be attributed to a change in the temperature of the instrument and were not due to a change in ionization.

In a test made the same year (1915) with the same apparatus, the value of n on the Pacific Ocean, between San Francisco, and Los Angeles was found to be 4.15 ions. In the calculation of the results of the observations over the ocean, the leak over the support was calculated from data obtained on land. The differences between the observed loss of charge on the ocean, and the calculated leakage across the support, as observed on land, was attributed to the formation of ions.

V. SECOND INVESTIGATION.

The zinc electrometer was used in all experiments where the temperature of the gas was varied. The instrument was made airtight by covering all possible sources of leak with sealing wax and finally applying a number of coats of shellac. The ionization chamber was filled with air or carbon dioxide which was made as free from moisture as possible by drawing the gas through calcium chloride and phosphorus pentoxide tubes, and finally bubbled through two bottles of concentrated sulphuric acid. The thermocouples, previously annealed and calibrated from the temperature of liquid air to the melting point of lead, were sealed into a hard rubber plug with sealing wax. This plug was then screwed into the top wall of the ionization chamber, and covered with more sealing wax. The other junctions of the thermo-couples were kept in a bath of melting ice. The E.M.F. was measured on a potentiometer in microvolts. In practically all tests the instrument was charged in the even-

TABLE I.

Length of Test.	Chamber <i>A</i> Connected, Disconnected or Cup in Place.	Average Temp. of Gas °C.	Rate Scale Div. per Min.	Rate Volts per Min.	π (Calculated).	Remarks.
<i>Air.</i>						
14 dys.	Conn.	21 to 24°	.0915	.02955	8.22	Instrument surrounded with felt.
10 hrs.	Conn.	" "	.0931	.03007	8.55	Instrument surrounded with felt.
8 hrs.	Conn.	" "	.0896	.02914	(max.) 7.88	Instrument surrounded with felt.
5 dys.	Cup in place	" "	.0703	.02271	(min.)	Instrument surrounded with felt.
10 dys.	Disconn.	" "	.0336	.01085		Instrument surrounded with felt.
20 hrs.	Conn.	-12.5	.0721	.02603	6.20	Chamber immersed in ice and salt.
10 hrs.	Disconn.	-12.5	.0298	.01076		Chamber immersed in ice and salt.
48 hrs.	Conn.	22.0	.0815	.02632	6.37	Chamber immersed in brine.
5 hrs.	Disconn.	22.0	.0327	.01056		Chamber immersed in brine.
5 hrs.	Conn.	34.0	.0905	.02920	8.04	Chamber in heater.
<i>Carbon Dioxide.</i>						
7 dys.	Conn.	21 to 24°	.1104	.03566	11.95	Chamber in heater.
6 hrs.	Conn.	38.8°	.0985	.03171	9.58	Chamber in heater.

ing beyond the field of view of the telescope so that by morning it was within view. This did away with irregularities previously mentioned.

The residual ionization was determined at room temperature for air and carbon dioxide. The results for carbon dioxide are somewhat higher than those obtained by Treleaven. Since commercial carbon dioxide was used the increase may be attributed to radioactive impurities in the gas. By means of the formula for n , the given constants of the apparatus and the data given in Table I., the resulting values of n were calculated.

Tests were undertaken at low temperatures for two reasons: (1) if ionization is produced by molecular impact due to thermal agitation, such an effect could best be investigated by decreasing this molecular motion as much as possible, (2) there is no record of such tests being made at low temperatures.

The electrometer was clamped rigidly in a frame and a Dewar flask containing solid carbon dioxide and alcohol raised until the ionization chamber was completely surrounded with the cooling solution. During this change of temperature of the gas within the chamber, very pronounced variations of the rate of leak were observed. When the warm instrument first came in contact with the cold solution, the divergence of the leaf increased a few scale divisions. However, in a few minutes the leaf came back to its original position, and quite a rapid rate of fall of the leaf was observed. For some time, depending upon how rapidly the temperature was lowered, the rate of leak appeared to be normal. With a further decrease in temperature the rate of loss of charge became quite rapid, and with a further lowering of temperature the charge on the insulated system had disappeared entirely. Table II. shows how this leak depends upon the temperature.

The central electrode was then disconnected from the charged system and a similar set of results obtained, also given in Table II. This at once shows us that the increased conductivity takes place across the insulation. Repeated tests with additional precautions against air leaks were taken, and in every case the same effects were noticed. Separate insulation tests were made on sulphur, amber and hard rubber, in order to determine whether the increased conductivity at low temperatures was due to a property of the insulator or was due to the presence of water vapor. The insulators were attached to wires and were sealed into glass tubes. In some cases the tubes were exhausted and filled with air which had been passed through calcium chloride, phosphorous pentoxide, and concentrated sulphuric acid. No noticeable increase in the conduction was observed at the temperatures of -72° and -187° C., when the air was carefully dried. But in cases where water vapor

TABLE II.

Length of Observation.	Average Temp. of Gas ° C.	Rate Scale Div. per Min.	π *	Remarks. Carbon Dioxide in Chamber-connected.
30 min.	22.0	.1010	10.0	In alcohol bath at room temperature.
18 min.	-30.5	.1385	20.0	No warming bath used.
50 min.	-34.5	.1310	18.6	No warming bath used.
30 min.	-29.0	.1265	17.6	No warming bath used.
40 min.	-36.5	.1250	17.4	No warming bath used.
30 min.	-41.0	.1785	28.8	No warming bath used.
10 min.	-41.2	.2778	50.3	No warming bath used.
6 min.	-43.2	1.0820	222.5	No warming bath used.
1½ min.	-44.6	5.4800	1,162.0	No warming bath used.
¼ min.	-44.6	7.0500	1,498.0	No warming bath used.
4 min.				Small stream of alcohol used for warming bath.
7 min.	-38.9	1.0000	204.0	No warming bath used.
2 min.	-42.3	4.1000	867.0	No warming bath used.
6 min.				Small stream of alcohol used for warming bath.
7 min.	-38.8	.8560	181.5	No warming bath used.
10 min.	-39.2	.8550	181.0	No warming bath used.
20 min.	-40.1	.6250	124.5	No warming bath used.
25 min.	-36.2	.2320	40.5	No warming bath used.
				Air in Chamber-connected.
1 hr.	24.5	.0938	8.68	No warming bath used.
45 min. later	-41.7	charge lost		No warming bath used.
30 min.	+20.0	.1140		Instrument recharged (25 min. later).
8 min.	-31.1	.1250	17.4	Warming bath used.
14 min.	-31.1	.1570	25.1	Warming bath used.
11 min.	-35.2	.1820	29.7	Warming bath used.
10 min.	-33.4	.2000	33.4	Warming bath used.
11 min.	-25.8	.1630	25.5	Warming bath used.
37 min.	-28.1	.162		No warming bath.
15 min. later	-40.2	charge lost		No warming bath.
				Air in Chamber-disconnected.
2 hrs.	22.5	.0337		Room temperature.
45 min.	-21.4	.0248		Warming bath used.
36 min.	-30.1	.0694		No warming bath used.
20 min.	-29.6	.0588		No warming bath used.
8 min.	-34.5	.1250		No warming bath used.
5 min.	-36.4	.6450		No warming bath used.
1 min.	-41.0	3.2500		No warming bath used.
	-41.0	charge lost		No warming bath used.

* π calculated upon the supposition that the increased rate of loss of charge is due to ionization within the gas.

was present an increased conduction took place. The rate of increase depended upon the insulator and form of the electrode. Insulation tests were also made above room temperature. Sulphur proved to be

the best insulator under all conditions. Amber seemed to be especially sensitive to traces of moisture.

It was found that more even temperature conditions could be obtained at these low temperatures by allowing a small stream of alcohol at room temperature to flow through the system designed to keep the insulation cool when the gas was heated to higher temperatures.

As seen from Table II., there is a very marked increase in conduction across the insulation support when the average temperature of the gas is from -15° C. to -30° C. and at a temperature of about -41° C. the charge is lost entirely. The temperature of the sulphur, although below zero, as shown by the amount of ice crystals which form on the external surface of the instrument, is not as low as the temperature of the gas, since the insulation part of the instrument is not immersed in the cooling solution.

Kolhörster¹⁸ does not give the definite temperature of the electrometer for various altitudes, but states that when the temperature of the instrument had fallen to -10° C. (5 to 6 km. above sea level) and below -10° C. (6 to 9 km.) one would disturb the readings, if the electrometer was touched with the bare hands. In Table III., we have listed Kolhörster's results and probable atmospheric temperature for such elevations.

TABLE III.

Elevation Km. ¹⁰⁰ Above Sea Level.	* Value in 1913.	* Value in 1914.	† Temp. °C. ²¹
0	—	—	24.9
1	- 1.5	—	21.6
2	+ 1.2	—	16.6
3	+ 4.0	+ 4.3	10.3
4	+ 8.3	+ 9.3	3.9
5	+16.5	+17.2	- 2.5
6	+28.7	+28.7	- 8.5
7	—	+44.2	-14.3
8	—	+61.3	-20.9
9	—	+80.4	-27.2
10	—	—	-33.9

* Increase in ionization over value obtained on the earth's surface.

† Mean value of atmospheric temperature based upon sounding balloon observations at Fort Omaha, Nebraska; Indianapolis, Indiana; Huron, South Dakota; and Avalon, California, for the summer season.

²¹ Gregg, Month. Weather Rev., 46, 17, 1918.

On comparing Tables II. and III. we see very marked evidence that we are dealing with similar results due most probably to the same cause, and that there are no abnormal ionization effects at great elevations; nor

any additional source of penetrating radiation in that region. There is no need of introducing the theory of a very radioactive cosmic atmosphere at that elevation to explain these results. It also seems very improbable that the sun could be the source of a penetrating radiation sufficient to form 90 ions per c.c. per second within a closed vessel. And if the value of 90 ions represents the number generated at 9 km., might we not expect a much greater increase in ionization at 17 km. elevation where the temperature¹ is about $-84^{\circ}\text{C}.$? It does not seem at all unreasonable for us to believe that results would be obtained which would be in agreement with those in Table II.

In order to get such large results by experiments with a radioactive substance, it was necessary to place a considerable quantity of a very active material quite close to the chamber. This would lead us to believe that it is very improbable that there would be such quantities of radioactive material in the upper atmosphere as it would require to produce ionization at this rate. Then again experiments on the earth's surface,² show that the radioactive content of the atmosphere only contributes from 0.1 to 0.2 ions per c.c. per second, to the residual ionization. It would thus be necessary for the upper atmosphere to be from 450 to 900 times as radioactive as the atmosphere close to the earth's surface.

There seems to be no record of insulation tests being made at these altitudes. The author has found in measurements made with a similar Wulf electrometer obtained from Germany, that there is a very probable air leak when the change from one volume to the other is made, and in order to keep the gas as dry as possible, it is necessary to replace the metallic sodium from time to time. It would seem that the moisture in the closed gas has no noticeable effect on the insulation at ordinary temperatures, but when sufficiently low temperatures are reached minute ice crystals form on the insulation and conduct the charge to the walls of the instrument; again when the enclosed air is sufficiently warm, these crystals vaporize and normal conditions are established. One notices (see Table II.) that at one instance the rate had increased to 7 divisions per minute, and on allowing a small stream of alcohol at room temperature to flow through the water jacket for 4 minutes, the rate was decreased to one division per minute. A similar effect was noticed a little later in the test when a small stream of alcohol running 6 minutes caused the rate to change from 4.1 to 0.856 divisions per minute. No matter how rapidly the loss of charge takes place at low temperatures on bringing the temperature of the instrument back to room temperature the rate of loss of charge again becomes normal.

¹ Nature, 98, 21, 1916.

² Chauveay, *Le Radium*, 10, 18 and 70. 1913.

Other observations were made with the instrument surrounded by a mixture of ice and salt (Table I.) in this case the temperature of the gas was subject to only small variations during the test. Since a pronounced decrease in the loss of charge was noticed, tests were made when the chamber was surrounded with the same brine at room temperature. Differences were observed both when the chamber was connected and disconnected. A search for the difference led the author to calibrate the instrument at the corresponding temperatures. The result was two distinctly different calibration curves as given in Fig. 2. At room-temperature one scale division is equal to 0.232 volts; while at a temperature below zero, one scale division is equal to 0.361 volts. So that on reducing the respective scale divisions to volts per minute (Table I.) the rate of loss of charge is the same, when the gas is at -12.5° C. in the ice and salt solution, or when the gas is at room temperature and the ionization chamber is immersed in salt water. The difference between 8.22 ions at room-temperature (Table I.) and 6.37 ions with the chamber immersed in salt water is due to the screening effect of the salt water.

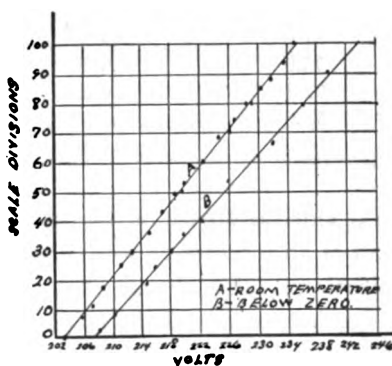


Fig. 2.

For investigations above room-temperature, the ionization chamber was placed in an electric heater formed by winding climax wire about a thin cylindrical sheet of asbestos, and packing this heating coil in magnesia. The heater was made just large enough to contain the ionization chamber. Both the sulphur insulation and the wax joints were kept cool by water jackets. From three fourths to two and one half amperes of current were passed through the heating coil. The results, as given in Table IV., show no increase with temperature below 92.5° C., but from 95° to 100° C. a pronounced increase is noticed, which is again evident when the volume is disconnected, at from 101° C. to 110° C. This shows an increased conduction across the insulation support, and is sufficient to account for the apparent increase in ionization in the gas noted by Kingdon from 80° to 100° C. His only method of testing the insulation was to exhaust the chamber. Thus different conditions were introduced whenever the insulation tests were made.

One notices from Table IV. that from 22° to 92.5° C. and from 22° to 101° C., with volumes connected and disconnected respectively, there

TABLE IV.

Air above Room Temperature.

Chamber Connected.		Chamber Disconnected.	
Temp. ° C.	Scale Div. per Min.	Chamber Temp. ° C.	Scale Div. per Min.
22.5	.091	22.0	.0333
38.0	.089	54.0	.0310
43.0	.088	76.0	.0308
52.0	.090	90.5	.0306
56.0	.087	95.0	.0300
68.0	.087	101.0	.0412
82.0	.079	109.5	.0450
90.0	.085		
92.5	.091		
95.0	.132		
100.0	.130		

is a decided decrease in the rate of loss in scale divisions per minute, instead of an increase as would be the case if there were an increase in the ionization. This change is no doubt largely due to a change in the dielectric constant of the sulphur.

The effect of temperature upon the insulated system can be easily shown by allowing solutions of different temperatures to flow through the water-jacket surrounding the sulphur insulation. When water at room temperature is allowed to flow through there is no change in the position of the leaf. When a solution above room temperature passes through, the leaf at once took a higher position on the scale. When a cold solution was allowed to flow through the water-jacket the leaf immediately took a lower place on the scale than that which it occupied at room temperature. When the temperature of the instrument was again at room temperature the leaf returned to its original position, except for the natural loss of charge which took place in the mean time. The greater part of this change is due to the change in the dielectric constant of the sulphur for various temperatures, as is shown by Schmidt.¹ A small variation may also be due to the change in the flexibility of the aluminum leaf with the change in temperature.

Bearing in mind that it requires a change of about 0.005 scale divisions per minute for an increase or decrease in the generation of one ion per c.c. per second, we find, on referring to Table I. and IV. that we cannot attribute the residual ionization to a thermal agitation of the molecules. Since the apparent variations in ionization can be attributed to temperature effects on the insulation system, it does not seem reasonable to

¹ Schmidt, *Ann. der Physik*; 44, 335, 1914.

attribute any part of the residual ionization to a temperature effect upon the gas.

VI. SUMMARY.

1. The mean value of the electrical conductivity of air in the basement of the physical laboratory at the University of California was 8.22 ions per c.c. per second when measured in a zinc chamber and 8.68 ions per c.c. per second when measured in a chamber where part of the walls were brass and the other part aluminum.

2. The mean value of the electrical conductivity as measured on the Pacific Ocean was 4.15 ions per c.c. per second.

3. This electrical conductivity, or residual ionization, is not due to molecular impact of thermal agitation.

4. The apparent increase in ionization as observed at high altitudes is solely a temperature effect, and is due to an increase in conduction over the insulation.

5. The effect of changes in temperature on the insulation system of an apparatus of this kind is sufficient to account for the apparent daily and seasonal variations of the residual ionization as reported by some observers, and is not due to a variation of the ionization of the gas within the chamber.

In conclusion I wish to express my thanks for the valuable guidance of Professor E. P. Lewis, under whose direction this work was done; and to Professor E. E. Hall for the laboratory facilities placed at my disposal, and also for helpful suggestions. Finally, acknowledgement is due Mr. W. R. Stamper for help in construction of apparatus.

PHYSICAL LABORATORY,
UNIVERSITY OF CALIFORNIA,
March 31, 1920.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE ONE HUNDRED AND FOURTH MEETING.

A MEETING of the American Physical Society was held in Denny Hall, University of Washington, Seattle, on June 18, at 9:30 o'clock in connection with a meeting of the Pacific Division, American Association for the Advancement of Science. About thirty members and visitors were in attendance,

The following papers were presented:

A Cable Loaded at One Point—A Special Case of the Catenary. S. H. ANDERSON. (By title.)

The Gibbs Thermodynamical Model. W. P. BOYNTON.

Transparency to Heat Radiation. S. L. BROWN.

Characteristics of Vacuum Tubes. S. L. BROWN AND C. F. NORMAND.

A Study of the Residual Ionization in a Gas with Reference to Temperature effects. C. H. KUNSMAN.

The Continuous Spectrum of Hydrogen in the Schumann Region. E. P. LEWIS.

Multiple Reflection from the Interior of a Ring. A. A. KNOWLTON AND G. A. WATT.

An Optical Illusion. A. A. KNOWLTON AND G. A. WATT.

Velocity of Sound from a Moving Source. R. B. ABBOTT AND J. W. COOK.

Voltage Wave Analysis with Indicating Instruments. LESLIE F. CURTIS.

The Dielectric Constant of Silk. F. J. ROGERS.

The Mathematical Structure of X-Ray Spectra. R. T. BIRGE.

The Spectroscopic Committee of the Division of Physical Sciences of the National Research Council. C. E. ST. JOHN.

Astronomers, mathematicians and physicists participated in a symposium on Einstein's Theory of Relativity June 17, at 2 o'clock, in Denny Hall. Dr. Eric T. Bell read a paper on the principle of general relativity, and there was a discussion led by Dr. J. H. Moore and Professor E. P. Lewis.

The Spectra of Compound Gases Flowing in Vacuum Tubes. W. H. BAIR.

On Friday evening President Suzzalo and the Committee on Arrangements gave a dinner to members of the affiliated societies.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

A CATENARY LOADED AT ONE POINT.

BY S. H. ANDERSON.

A FLEXIBLE string, hung between two points at *different* horizontal levels and loaded at some point between the two supporting points, assumes a conformation with a point of discontinuity at the position of the load, which divides the string into two arcs. It may be shown that each of these arcs is a portion of a catenary.

From a consideration of the forces acting upon *one* of these arcs a differential equation may be obtained the solution of which gives the two following equations:

$$(y + y_0) = \frac{y_0}{2} (e^{x/c} + e^{-(x/c)}) + \frac{s_0}{2} (e^{x/c} - e^{-(x/c)}), \quad (1)$$

$$(s - s_0) = \frac{y_0}{2} (e^{x/c} - e^{-(x/c)}) + \frac{s_0}{2} (e^{x/c} + e^{-(x/c)}), \quad (2)$$

in which x, y are the rectangular coördinates of any point, P , s is the length of the arc between the loaded point and P , and y_0, s_0 , and c are constants or parameters of the curve which depend for their values upon the weight of the string and the load, and are connected by the following equation

$$y_0 = \sqrt{s_0^2 + c^2}. \quad (3)$$

The interpretation of the three constants is as follows: If the arc is extended to a point where the tangent becomes horizontal, s_0 is the length of this auxiliary arc from the point of loading to the point where the tangent is horizontal; c is the vertical distance from this latter point to a horizontal line which may be called the directrix; and y_0 is the vertical distance from the point of loading to the directrix.

If in the above equations s_0 is put equal to zero, which will be the case when there is no load upon the string, (1) and (2) reduce to

$$(y + c) = \frac{c}{2} (e^{x/c} + e^{-(x/c)}), \quad (5)$$

$$s = \frac{c}{2} (e^{x/c} - e^{-(x/c)}), \quad (6)$$

which are the equations for the common catenary. This shows that the loaded catenary is a more general case of which the common catenary may be considered a special case.

The total tension at any point is given by

$$T = w(y + y_0). \quad (7)$$

The vertical component and the horizontal component are given by the following equations respectively:

$$Tv = w(d + s_0), \quad (8)$$

$$Tg = wc, \quad (9)$$

in which w is the weight of unit length of the string.

The equations for the other arc of the string are of the same form, but the parameters y_0 and s_0 are different, and c the same. Hence in general the form of the curve will be different. Let us call these parameters y_0' and s_0' . Then it may be shown that the load is given by

$$W = w(s_0 + s_0'). \quad (10)$$

This ideal case may be considered an approximation to the problem of the aerial tramway as is used in the logging "skyline" for transporting logs out of the forest. The chief factor of the practical problem that is neglected in the above treatment is the stiffness of the cable. However the length is generally very great as compared with the diameter so that the relative stiffness is very slight except at the ends where supported and at the carriage where the load is applied.

Perhaps the most valuable feature of the above analysis is the determination of the factors necessary to compute the load a given cable will carry. They are as follows: (1) the horizontal distance between the end points of the cable; (2) the vertical distance between these points; (3) the length of the cable, or the position (with reference to one of the end points) of the lowest point in the sag of the cable that the topography of the ground will allow; and (4) the breaking strength of the cable.

UNIVERSITY OF WASHINGTON.

GIBBS THERMODYNAMICAL MODELS.

BY W. P. BOYNTON.

GIBBS first suggested the representation of the thermodynamical properties of a substance by a three-dimensional model having for coördinates the volume, entropy and energy. Maxwell constructed such a model, and in his "Theory of Heat" presented a diagram of the principal lines upon it. The writer has also constructed a model intended to represent the model described by Maxwell. Finding difficulty in coördinating its form and the location of the lines upon it with the known properties of substances, he computed these lines for a substance following van der Waals' equation, with the ratio of its specific heats $1 \frac{2}{7}$, and constructed a model from these graphs. Lately he has recomputed for a van der Waals' substance having the ratio of the specific heats $1 \frac{2}{3}$, the maximum value, and constructed a model to correspond, and has also constructed a model, modified to show the properties of the solid state also, which is not included in the van der Waals' equation. These models must be at least qualitatively correct, and show in a general way what the surface must be like for an actual substance.

The moulds for the models have been preserved so that more copies may readily be made. The writer has also drawings and lantern slides showing contours and projections of principal lines upon the planes, and the appearance of the finished models.

UNIVERSITY OF OREGON,

EUGENE, OREGON,

May, 1920.

CHARACTERISTICS OF VACUUM TUBES.

BY S. L. BROWN AND C. E. NORMAND.

SEVERAL types of vacuum tubes for detection, amplification and generation of radio-frequency currents have been examined in order to compare types of tubes and different tubes of the same type. The different types of vacuum tubes examined include those from American, French and German manufacturers.

DATA SHOWING CHARACTERISTICS OF VACUUM TUBES AT THEIR RATED FILAMENT CURRENT AND PLATE VOLTAGE.

Type I: Western Electric, VT 1; filament current 1.1 amp., plate voltage 20 volts.
 Type II: General Electric, VT 11; filament current 1.1 amp., plate voltage 30 volts.
 Type III: General Electric, VT 14; filament current 1.4 amp., plate voltage 400 volts.
 Type IV: Marconi, Class I; filament current .6 amp., plate voltage 40 volts.
 Type V: French, Bis 3; filament current .6 amp., plate voltage 20 volts.
 Type VI: German, EVN 117; filament current .6 amp., plate voltage 50 volts.

Type.	Grid Voltage.	Plate Current.	Type.	Grid Voltage.	Plate Current.
I	-10 v.	00 ma.	IV	-20 v.	00 ma.
"	00	.75	"	-10	.05
"	10	2.65	"	00	.50
"	20	10.10	"	10	2.05
"	30	12.00	"	20	2.15
"	40	14.50	"	30	2.05
"	50	15.00	"	40	2.00
II	-10	00	V	-20	00
"	00	1.00	"	-10	00
"	10	2.05	"	00	.10
"	20	4.10	"	10	1.50
"	30	6.00	"	20	3.55
"	40	7.25	"	30	3.80
"	50	7.15	"	40	3.80
III	-50	00	"	50	3.70
"	-40	.45	"	60	3.65
"	-30	1.10	"	70	3.65
"	-20	3.25	VI	-20	00
"	-10	10.10	"	-10	.10
"	00	12.55	"	00	.50
"	10	12.80	"	10	1.00
"	20	13.00	"	20	2.10
"	30	13.10	"	30	3.55
"	40	13.15	"	40	4.00
"	50	13.20	"	50	4.10
"	60	13.20	"	60	3.80
"	70	13.20	"	70	3.40

UNIVERSITY OF TEXAS.

TRANSPARENCY TO TOTAL HEAT RADIATION.

BY S. L. BROWN.

THIS paper gives the results of measuring the percentage of total radiation transmitted by glass, rock-salt, mica, celluloid and paraffin for temperatures of radiating source from 200° C. to 1200° C. This work was undertaken to determine the possibility of selecting pyrometer screens which would intercept a known fractional part of the total radiation at a particular temperature. Results show that the per cent. of total radiation transmitted by ordinary sodium glass increases from about 6 per cent. for 200° C. as the temperature of the source to about 60 per cent. when radiating source is at 1200° C. The transparency of mica increases from 7 per cent. to 40 per cent. when the temperature of the radiating source is increased from 400° C. to 1200° C.

UNIVERSITY OF TEXAS.

A STUDY OF THE RESIDUAL IONIZATION IN A GAS WITH REFERENCE TO TEMPERATURE EFFECTS.

BY C. H. KUNSMAN.

INVESTIGATIONS were undertaken with the purpose of studying the ionization of a gas as observed in an airtight vessel. This was done with a twofold purpose: (1) to find an accurate method of measuring this conductivity in the gas, (2) to make a thorough investigation of the effect of temperature on this conductivity. A double ionization chamber one compartment containing the electroscope, made it possible to eliminate insulation losses by taking observation with the compartments connected or with the electroscope chamber above.

Tests were undertaken at low temperatures for two reasons: (1) if ionization is produced by molecular impact due to thermal agitation, such an effect could best be investigated by decreasing this molecular motion as much as possible, (2) there is no record of such tests being made at low temperatures. The electrometer was rigidly clamped in a frame and a Dewar flask containing solid carbon dioxide and alcohol raised until the ionization chamber was completely surrounded with the cooling solution. During this change of temperature of the gas within the chamber, very pronounced variations of the position of the leaf was observed. For some time, depending upon how rapidly the temperature was lowered, the rate appeared to be normal. With a further decrease in temperature the rate of loss of charge became quite rapid, and with a further lowering of temperature the charge on the insulation system disappeared entirely. The ionization chamber was then disconnected from the charged system and a similar set of results obtained. This at once shows us that the increased conductivity takes place across the insulation and is not due to a variation in the ionization in the enclosed gas.

Separate tests on various insulations including sulphur, indicate that the increased conductivity is due to traces of water vapor which have no notice-

able effect except at low temperatures. Sulphur proved to be the best insulator under all conditions and through a wide range of temperature.

For investigations above room-temperature, the ionization chamber was placed in an electric heater. Both the sulphur insulation and the wax joints were kept cool by water jackets. No increase in conductivity was noticed until the temperature of the enclosed gas was between 95° C. and 100° C., when a pronounced increase was noticed. This increase was again noticed when the ionization chamber was disconnected from the charged system at about the same temperature, which proves that the increased conductivity takes place across the insulation support and is not due to an increased conductivity in the gas.

From these experiments we may conclude:

1. The mean value of the electrical conductivity of air in the basement of the physical laboratory at the University of California was 8.22 ions per c.c. per second.
2. The mean value of the electrical conductivity as measured on the Pacific Ocean was 4.15 ions per c.c. per second.
3. This electrical conductivity, or residual ionization, is not due to molecular impact of thermal agitation.
4. The apparent increase in ionization as observed at high altitudes is solely a temperature effect, and is due to an increase in conduction over the insulation.
5. The effect of change in temperature on the insulation system of an apparatus of this kind is sufficient to account for the apparent daily and seasonal variations of the residual ionization as reported by some observers, and is not due to a variation of the ionization of the gas within the chamber.

UNIVERSITY OF CALIFORNIA,
May, 1920.

THE CONTINUOUS SPECTRUM OF HYDROGEN IN THE SCHUMANN REGION.

By E. P. LEWIS.

REFERENCES have frequently been made to a continuous spectrum of hydrogen associated with the spark discharge at high pressure or with vacuum tube discharges at low pressure, but the conditions most favorable to its appearance and its extent have not been discussed. Kayser rather takes it for granted that an intense discharge through a small capillary is the most favorable condition, and Schniederjost used that method, with poor success, for giving a background for absorption spectra. The writer pointed out some years ago that this spectrum could be best produced by using a rather large end-on tube, say one cm. in diameter, with a simple discharge of small current density, at a pressure of five mm. or more, and that it was greatly weakened by the use of a condenser and spark gap.¹ This spectrum extended to the limit of transmission of the quartz system, its gradual weakening apparently being

¹ Science, 41, 947, 1915.

determined rather by the increasing absorption of the quartz and the diminished sensitiveness of the photographic film than by failure of emission. There was no sign of resolution into lines, and it extended well into the visible region as well to in the opposite direction, although the superposition of the many-lined spectrum made it impossible to determine its exact limit. The writer has recently had constructed a fluorite vacuum spectrograph, which made it possible to extend observations into the Schumann region. As anticipated, it was found that the continuous spectrum extends with undiminished intensity well into this region. At a wave-length of about 1800 it begins rather suddenly to diminish in intensity, and disappears about 1750. Faint bands attributed by Schumann and by Lyman to carbon monoxide make observations in this region somewhat uncertain. A little below 1700 the line spectrum of hydrogen reappears, with no trace of a continuous background. Lyman refers to an absorption band of unknown origin in this region, but the fact that the continuous spectrum does not reappear with the lines shows that the limit is not far from the point specified. There may be some theoretical interest attached to the fact that this continuous spectrum completely fills the region between the Balmer series and the Schumann line spectrum, with no lines whatever superimposed on it.

This continuous spectrum offers the best background yet obtained for the study of absorption spectra in the extreme ultra-violet. Its perfect uniformity and continuity makes it far superior to the aluminum spark under water. Good photographs of the absorption spectrum of benzol have been obtained with an exposure of fifteen minutes. The absorption appears to be complete beyond about 1900.

Lyman makes a casual reference to a continuous spectrum, and Schumann used it in the study of absorption spectra, but neither appears to have investigated the most favorable conditions for its appearance.

Helium and neon resemble hydrogen in giving continuous spectra far into the ultra-violet, which are not, however, so intense as that of hydrogen. The continuous spectra given by parts of some nebulae may be due to hydrogen and helium.

UNIVERSITY OF CALIFORNIA,
May, 1920.

MULTIPLE REFLECTIONS FROM THE INTERIOR OF A RING.

BY A. A. KNOWLTON AND G. A. WATT.

LIGHT from a point source reflected at a concave cylindrical mirror of large aperture gives rise to the well known caustic. If the reflecting surface forms a complete ring multiple reflections occur which produce caustics of higher orders. Slides prepared from photographs showing these curves for the second and third order were shown. The equation for the n th caustic has been obtained by a simpler method than that used by Cayley, who treated the theory of such reflections.

REED COLLEGE,
May, 1920.

AN OPTICAL ILLUSION.

By A. A. KNOWLTON AND G. A. WATT.

IF one stands with his back toward a window and looks into an ordinary flask nearly filled with clear water and held slightly above the level of his eyes he appears to see, near the bottom of the flask a bubble of oil or other highly refractive substance. This is in fact an image of the plane surface of the water distorted by reflection in the spherical mirror of large aperture and by refraction. Examples of this phenomena were shown and the cause of the illusion discussed.

REED COLLEGE,
May, 1920.

VELOCITY OF SOUND FROM A MOVING SOURCE.

By R. B. ABBOTT AND J. W. COOK.

OBSERVATIONS have been made which do not show any change in the velocity of sound due to a spark source moving *8550 cm. per second*. The source consisted of a spark gap mounted on the end of a vane rotating 30 R.P.S. The spark was made to occur at the same point once in a revolution. Opposite points on the spherical wave front were located by two telephone transmitters and one receiver connected to a differential transformer. A minimum sound in the receiver indicated that the wave front arrived at the two transmitters simultaneously. The radii of the spherical waves measured varied from 190 to 260 centimeters. Normal room temperature and pressure prevailed.

The conclusions are that the velocities of sound disturbances produced by a moving spark are not appreciably affected by the velocity of the gap.

Whatever increment of velocity may have been given to the sound by the motion of the gap, was damped out too quickly to affect the measurements beyond the errors involved.

UNIVERSITY OF CALIFORNIA,
May 25, 1920.

VOLTAGE WAVE ANALYSIS WITH INDICATING INSTRUMENTS.

By LESLIE F. CURTIS.

AN investigation and instrument are described which were inspired by the lack of a suitable workable standard of wave form. The use of telephone condensers, air core inductances, and a sensitive hot-wire galvanometer to determine the order and amplitude of the harmonic components of a wave—leaving only their phase positions undetermined—is described. The method of analysis is based on the well-known use of the wave meter, but adapted to low frequencies and with this important difference—a fundamental frequency is always present, may not be neglected, and must be treated simultaneously

with the other harmonics. The equations used are developed and explained under the headings "Order" and "Amplitude" of the Harmonics. The sensitiveness of indication is discussed and applied to several types of circuits. A convenient arrangement of the constants of the circuit in an instrument called by the author an "Harmonimeter" and giving direct readings of the order of the harmonics and easily calculated values of their amplitude is illustrated. An example of wave form measurement is given in which the results are checked by oscillogram.

UNIVERSITY OF WASHINGTON,
SEATTLE, WASH.

THE DIELECTRIC CONSTANT OF SILK.

BY F. J. ROGERS.

SILK does not appear in any of the usual tables of dielectric constants, probably no such measurements have ever been made.

In general, when two dielectrics are electrified by contact and friction, the one having the larger constant becomes positive. If this holds true with silk and other substances then its dielectric constant must be rather large. This is what suggested the measurement of this constant for silk.

The usual condenser method was adopted. As a silk fabric is not a continuous solid, some method must be used which will allow for the space between the condenser plates which is not filled with silk. This was done in two ways. First, several thicknesses of silk were used and the thickness measured while weighted between the condenser plates. The ratio of silk to air was then computed from the weight of the silk, the total volume, and the density of the silk. The density of the silk was determined by the simple expedient of weighing it in air and in water. The density proved to be surprisingly large, but as the silk fibers are solid and of a horny nature this ought to have been expected. In the second method the silk was dipped in melted paraffin and the "paraffined" silk used between the condenser plates. In this case to insure contact the lower plate was amalgamated zinc with a large excess of mercury and the upper plate was liquid mercury. In both cases it was assumed that the dielectric contact of the mixed dielectric was a simple additive function of the two dielectric constants for silk and air and for silk and paraffin.

The simple bridge method was used for measuring the capacity of the condenser having silk as a dielectric. It formed part of one of the condenser arms of the bridge and its capacity was determined by the substitution of variable calibrated air condenser. Alternating E.M.F. of 60 cycles was supplied to the bridge, and a quadrant electrometer was used as an indicating instrument. The electrometer needle was maintained at a high alternating E.M.F. of the same frequency and in quadrature with that supplied to the bridge.

The experimental results obtained were as follows:

	Specimen.		
	White Satin.	Black Satin.	White Silk.
Density.....	1.51	1.515	
Dielectric constant.....	4.62	5.07	4.62
Silk and air exp.....	4.68	5.07	4.68
Dielectric constant.....	4.58	4.86	4.58
Silk and paraffin exp.....			

STANFORD UNIVERSITY,
May, 1920.

THE MATHEMATICAL STRUCTURE OF X-RAY SPECTRA.

BY R. T. BIRGE.

THE mathematical relations of X-ray spectra may be divided into two classes, (1) those concerning the same spectral line as it appears in the spectra of successive elements (Moseley's $\sqrt{\nu} : N$ curves) and (2) those concerning the various lines of any one element. Using the very accurate data now available, the author has tested many of the previously proposed relations, as well as some new ones. The work is still in progress.

Under (1), the Siegbahn 1919 data for nine $K\alpha_1$ lines, extending from $N = 17$ to $N = 29$, plus that for $N = 74$, have been fitted to an ordinary power series equation. A fourth degree equation (five undetermined constants) was found entirely satisfactory, the average $O-C$ values being 1/300 per cent., with no evidence of systematic divergence. The agreement for the smaller atomic numbers is especially good, and so the extrapolation to hydrogen and helium can be made with some confidence. The value for $N = 1$ is 2535 Å., with a possible error of not more than a few hundred Å. (Uhler obtained 400 Å. or less.) The theoretical "resonance" line is 1216 Å. For helium the extrapolated value is 470.7 Å.,—the resonance line 567 Å. Theoretical considerations indicate that H and He have no $K\alpha_1$ line, and so the disagreement is to be expected. The author's curve has an intercept $N = +0.23$.

The general equation of the second degree (a special form of which was used by Uhler) was found unsatisfactory. Duane has proposed a linear relation between velocity and atomic number, for K critical absorption, but his formula for kinetic energy, from which the velocity was obtained, is incorrect, and therefore the results must be considered purely empirical.

It is noteworthy that the $\sqrt{\nu} : N$ curves are smooth, even for the lowest atomic numbers, and that each curve seems to begin when there are only two electrons in the ring (or shell) from which the emitting electron comes. None of the graphs are linear, although in some cases the slope varies only about 10 per cent.,—in others 40 per cent. to 50 per cent.

The only satisfactory relations, under the second type of regularity, are given by Sommerfeld in a chart.¹ Even these are only partially satisfactory

¹ Zeitschrift für Physik, Vol. 1, page 137, 1920.

since $K\alpha_1 + L\alpha_1 + M\alpha_1 + N\alpha_1 + \text{etc.}$ does *not* equal K absorption. In fact, it would be necessary to change all Stenström's M series data (both emission and absorption) by about 15 per cent., in order to obtain a general agreement. The combination principle appears strictly accurate in certain restricted types of relations, but quite inaccurate in general.

The fact that certain $\sqrt{\nu} : N$ curves of the L series *cross* one another proves that there must be various *independent* configurations of the atom. Thus far, in the theoretical X-ray work, only *one* set of related orbits has been considered. For ordinary spectral series there are *four* such sets of orbits (fundamental, principal, sharp, and diffuse.)

D. L. Webster's relations between emission lines and critical absorption are extremely significant, but the fact that the L_2 critical absorption curve¹ *crosses* the γ_2 emission curve, raises a vital objection, on quantum theory, to the association of γ_2 with L_2 absorption. There are other similar cases.

Overn's eight equal frequency ratios for Tungsten are interesting, but unfortunately only two (β_1/α_1 and γ_1/β_1) can be tested for other elements. The two ratios are *not* equal except for Tungsten, the curves (ratio against N) crossing at $N = 74$. Can it be possible that all eight curves will be found to cross at the *same* point? Additional experimental data is much to be desired.

UNIVERSITY OF CALIFORNIA.

THE SPECTROSCOPIC COMMITTEE OF THE DIVISION OF PHYSICAL SCIENCES OF THE NATIONAL RESEARCH COUNCIL.

BY CHARLES E. ST. JOHN.

IT is not necessary to recall to this body the part played by the National Research Council in organizing the scientific resources of the country for the purpose of winning the great war. It was, however, a revelation to the country as a whole of the strength and weakness of the pre-war conditions in scientific work. The great success with which men trained in pure science attacked the various practical problems of the war increased the estimation in which scientific research was held and has made it possible to carry on under peace conditions some of the benefits of organization and to increase the fostering agencies of research in pure science. In line with such considerations and the executive orders issued by the President outlining the purposes of the Council, the National Academy reorganized and put upon a permanent basis the National Research Council whose purpose is "to promote research in mathematical, physical, and biological sciences, and in the application of these sciences to engineering, agriculture, medicine, and other useful arts, with the object of increasing knowledge, of strengthening the national defense and of contributing in other ways to the public welfare." In paragraph 3 of the order issued by the President an important point is expressed as follows: "to promote coöperation in research, at home and abroad, in order to secure concentration of effort, minimize duplication, and stimulate progress; but in all

¹ Duane's 1920 data, not yet published.

coöperative undertakings to give encouragement to individual initiative as fundamental to the advancement of science."

In furtherance of the purposes of the Council the Division of Physical Sciences, among others, has been organized of which Dr. C. E. Mendenhall is chairman. Other members of the Division are representatives of the related societies. In accordance with the provisions of the Council, the Division of Physical Sciences has recently established a General Spectroscopic Committee of ten members divided into an Eastern and a Western group. Before entering into the committee's plans in detail it may be of interest to refer to the financial situation.

The Rockefeller Foundation has entrusted the Council with the expenditure of \$500,000 within a period of five years for providing fundamental research in physics and chemistry, primarily in educational institutions of the United States. This purpose is to be carried out through establishing a system of fellowships to be awarded to American citizens whose training is equivalent to that represented by the doctor's degree.

Important results are hoped from this undertaking and such a degree of success in accomplishing the ends in view that conditions in American centers of research will be permanently improved. Some of the results expected are:

1. Opening of a scientific career to a larger number of investigators and their more thorough training in research, thus meeting an urgent need of our Universities and industries.
2. Increase of knowledge relating to the fundamental principles of physics and chemistry.
3. Creation of more favorable conditions for research in this country.

In addition to the fellowship funds placed at the disposal of the Division of Physical Sciences, the Rockefeller Foundation has also appropriated funds to meet the expenses involved in conferences of special committees and in the honoraria of investigators to whom the preparation of special reports, surveys, or monographs is committed. In fact the funds necessary to set the machinery in action are provided and it now remains to be seen how men of science, young and old, rise to the opportunities. It may well be that the next few years will be crucial ones in developing physical science in the United States and it behooves us all to aid in all ways we are able, either by joining in or promoting interests in research, by preparing others for it, or by helping to create an understanding and hence a sympathetic attitude on the part of the general public toward increase in our knowledge of the fundamental facts and principles of physics, irrespective of their immediate practical applications.

The Eastern group of the Spectroscopic Committee consists at present of Lyman, Chairman, Gale, Randall, Saunders, and Uhler, the Western Group of Anderson, Babcock, King, Lewis, and St. John.

The activities of the committees are as yet but partly organized, but they hope so serve as clearinghouses of information as the means of correlating the work so that needless duplication may be avoided, and of bringing into closer relations men who are working along similar lines in different parts of the

country. It is already evident that men are going to appreciate the contacts with the outside which the committees supply, and that the committees will be able to render assistance in approaching the administrators especially of our state universities in the interests of fundamental research, helping to create a more favorable atmosphere and to bring about a more generous provision for research.

At present a canvass of the universities is under way to ascertain the equipment and opportunities available and the lines of investigation in progress. Such a body of data will be of great service to the committee when responding to requests for suggestions as to lines of investigation that will fit into general plans or that need to be undertaken.

It is expected that subcommittees, membership not confined to the main committee, will soon be organized for discussion and report on special fields which are best suited for or are in need of such a survey. The consideration in view would be as follows:

1. They would be stimulating and valuable to the men themselves.
2. The groups might undertake to interest others in their special fields.
3. The reports would be of value to all physicists in helping them to get into touch with various phases of spectroscopic work.

Correlated with the work of the general committee is that of the International Commission for Standards of wave-length. The American Committee reporting at the Washington meeting preliminary to the Brussels conference was unanimous in the opinion that one of the greatest needs of the physicist at the present time is a complete determination of the wave-lengths of the elements with the utmost accuracy possible. Such an undertaking will require the united work of many investigators but its value will be commensurate to the cost in time and effort. It is well suited to coöperative efforts, as each investigator or small group of investigators can have a definite field and feel the satisfaction that he is contributing an element of value to the larger work. Experience has shown that such investigations, besides accomplishing the important ends directly in view, can hardly fail to yield valuable results in related fields, in particular in connection with problems of radiation, and in the relations between spectra of terrestrial and cosmic sources.

There is much said of correlation and coöperation but it must ever be borne in mind that the greatest asset in research is individual initiative and that in all our efforts at coöperation the aim should be to discover and recognize early the fortunate possessor of this rare and precious gift and the utmost care be taken that in our desire for efficiency of organization the machinery does not hinder its free and full development.

MT. WILSON OBSERVATORY.

THE PHYSICAL REVIEW.

ARCING VOLTAGES IN MERCURY VAPOR AS A FUNCTION OF THE TEMPERATURE OF THE CATHODE.

BY T. C. HEBB.

SYNOPSIS.

Low Voltage Arcs in Mercury Vapor.—One of the objects of this work was to study the production of the low-voltage arcs in a more uniform mercury-vapor atmosphere than had been used before. As a result of improved apparatus much more consistent results have been obtained. It has been shown that there is a linear relation between the striking voltage and the current through the cathode for the larger currents. This in turn has been shown, in the case of platinum coated with lime, to mean that the striking voltage forms a linear relation with the temperature of the cathode. The results further suggest that the difference between the potential at which ionization takes place and the accepted ionization potential is directly proportional to the absolute temperature. *Results with Tungsten Cathodes.*—In line with the above it has been shown that tungsten cathodes produce lower arcs than lime-coated platinum cathodes. A striking voltage as low as 3.2 volts was obtained, the lowest for platinum coated with CaO having been in the vicinity of 4.9 volts. *Effect of Thickness of Oxide Deposit on the Cathode.*—It has been shown that the thickness has an effect on the value of the striking voltage. A thinly coated platinum cathode produced an arc at a potential as low as 6.0 volts whereas a thickly coated one produced an arc as low as 4.9 volts. *Effect of Hot Anode.*—A hot anode used with a thinly coated platinum cathode has been found to produce a lower arc than when the anode was not heated. *Discussion of Results.*—The results are briefly discussed but no definite theory is offered to explain them.

IN two previous papers¹ I have shown that mercury vapor can be ionized at potentials as low as about 5 volts. These results which were somewhat contradictory to those of McLennan and Henderson² have since been substantiated by McLennan.³ As the type of apparatus used in my previous experiments was considered unsatisfactory for the better understanding of the relations between the various factors involved

¹ PHYS. REV., Vol. 9, p. 686, 1916, and Vol. 11, p. 170, 1918.

² Proc. Roy. Soc., A, Vol. 91, 1915.

³ Proc. Phy. Soc. Lon., Vol. 31, Dec., 1918.

in the production of the low ionization, I have constructed and used the following apparatus.

The glass tube *A*, Fig. 1, about 2.5 cm. in diameter, was cemented with Khotinsky cement to the base *BB* which stood on the shelf *SS*. Through the plug *P* three holes were bored—one to admit the barometer tube *t* and the other two to admit two electrodes for conveying the current to the cathode *c*. The anode, *a*, was attached to an aluminum cylinder which was inserted in the side tube *m*. Electrical connection was made between the anode and the mercury in the tube *R* by a platinum wire which passed down the tube *U*. The apparatus could be evacuated through the tube *U* as indicated in the figure. The upper part of the apparatus was surrounded by an asbestos furnace containing a glass window. The furnace was heated electrically. By the use of this apparatus a uniform mercury-vapor atmosphere could be obtained around the anode and cathode. The pressure of this vapor could be

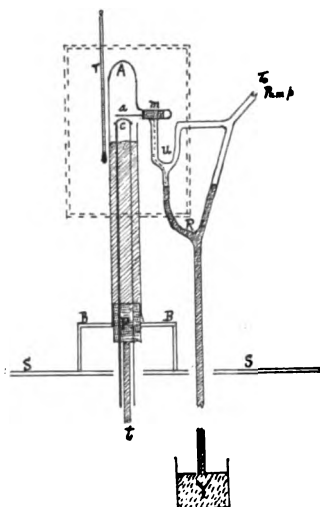


Fig. 1.

obtained by raising the reservoir *Y* until the mercury stood in the tube *U*. The apparatus was evacuated by means of a Toeppler pump and the pressure of the remaining gas was measured with a McLeod gauge. During the process of "heating up" of the apparatus and of the cathode a great deal of gas was given off. Pumping was continued, however, until the pressure was of the order of .001 cm. In order to obviate the collection of this gas in the apparatus *A*, the mercury in *R* was not allowed to shut off the tube *U*. As is evident, the evaporation of mercury in the tube *A* and its passage out through *U* rapidly clears the tube *A* of any foreign gases. The thermometer *T* had its bulb placed below the

level of the surface of the mercury in *A* and very near the outside of the tube.

The cathode, unless otherwise stated, was of platinum foil about .0025 cm. in thickness. Various widths were used but the length was about 2.0 cm. Either CaO or SrO was placed on it. The anode *A* was either platinum foil or platinum wire.

It was soon found that much more consistent results could be obtained with this apparatus than with the previous type. With my earlier apparatus I found that it was necessary to have the pressure, as read

by a McLeod gauge, at a particular value in order to get the lowest arcs and that a greater or less pressure increased the value of the arcing voltage. With the present form I found that the pressure of the mercury was of less importance. It still had to be above a certain minimum value but, aside from that, no great care had to be exercised. If the thermometer read anywhere between 170 and 220° C. the results were practically the same. This gave a vapor pressure of the order of 1-3 cm. of mercury.

As in the previous work so here it was found that the electronic density was important. Low temperature cathodes would not give low arcs. As the temperature of the cathode rose, however, the striking voltage dropped rapidly until a further increase had only a small effect on the striking voltage. This is brought out by Fig. 2 which gives the results

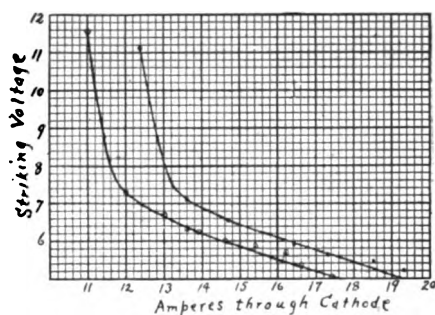


Fig. 2.

for two different cathodes rather heavily coated with CaO. I mention the thickness of the CaO coating as I am of the opinion that it makes a difference. In my previous work on the mercury arc I used heavy coats of BaO on the cathode.

It will be noticed that the lower parts of the graphs approximate more or less to straight lines. The straightness of this part of the graphs I have examined closely and find that in a great many cases it seems to be straight within the limits of observation. Had I taken care to glow well the cathodes used in Fig. 2 before I made those observations the lower part of the graphs, I have no doubt, would have been straight.

In order to obviate the correction and uncertainty due to the drop in the cathode I arranged a revolving commutator so that the cathode was heated intermittently, and the potential between the anode and cathode was applied only when the cathode current was off. In the use of the commutator the greatest care was exercised to prevent the potential being applied even for an instant while the cathode current

was flowing. The current was made and broken 60 times per second and the potential was applied a like number of times. This method is not favorable for studying the weak arcs produced with low temperature cathodes hence that end of the graph is not shown. Fig. 3 gives the results obtained with such a commutator when the cathodes were rather heavily coated with CaO. The two graphs are for separate cathodes.

The graphs of Fig. 3 suggest that there is no significance, as far as these experiments go, in the potential difference of 4.9 volts. As a matter of fact it was only on rare occasions that I got an arc as low as 4.9 volts with the present apparatus. And it should be stated that in

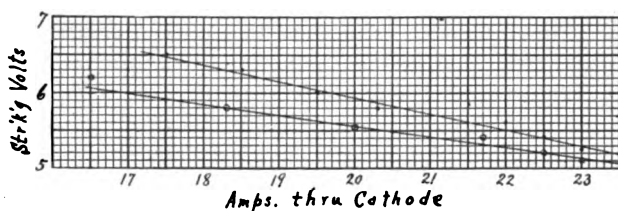


Fig. 3.

the experiments recorded in Figs. 2 and 3 the cathodes were heated until they melted.

The graphs in Fig. 3 also suggest the idea that if the current could be still further increased the striking voltage would keep on decreasing in a linear manner with the current. I attempted to follow this line of reasoning by using tungsten coated with CaO but owing, apparently, to a chemical change between the tungsten and the CaO I was unsuccessful. With tungsten alone, however, I found that I could get the arc to strike at voltages much lower than I had obtained before. I am not prepared at present to say what is the lowest voltage at which an arc in mercury vapor using a tungsten cathode can be made to strike but I have got it as low as 3.2 volts. This was obtained with the commutator so that there is no uncertainty from cathode drop. Further I found that there was a linear relation between the current through the cathode and the striking voltage as was the case for platinum cathodes. It should be stated that the tungsten used was in the form of a strip as nearly as possible to the size of the platinum used. Fig. 4, graph *a*, shows the results for one cathode. Although only three observations were made before the cathode melted, it will be seen that they lie very close to a straight line. With the commutator I have found it impossible to make observations on arcs when I used low-temperature cathodes. Fig. 4, graph *b*, however, gives the results with a tungsten cathode heated without the use of the commutator. In order to approximate

the correction for cathode drop the cathode was constricted at the center so that only that portion of it was highly heated. It will be noticed that the upper portion of the graph is very steep, as was the case for platinum, and that then the graph becomes straight. The lowest striking potential observed was 3.9 volts but the cathode melted at about 30 amperes before other observations were made. If the straight

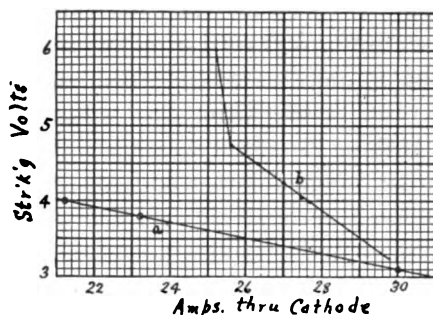


Fig. 4.

portion of the graph is produced, however, it reaches about 3.2 volts at 30 amperes.

I have already suggested that the thickness of the oxide deposit on the cathode affected the value of the striking voltage. I had always followed the practice, unusual I think, of coating the cathode rather heavily. This was done by putting small pieces of the nitrate on the cathode and then heating them. In endeavoring to determine the effect of impurities in the oxide on the value of the striking voltage I was led to use much thinner deposits. I found immediately that there was a considerable change. In fact, it was found that the lowest striking voltage for thinly coated cathodes was in the neighborhood of 6.0 volts. This is shown in Fig. 5 which gives the data for two separate cathodes.

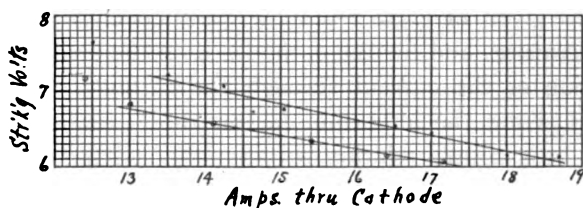


Fig. 5.

In both cases the current was increased until the cathode melted. The two curves also show the variation caused by different treatment of the cathodes. In the case of the upper graph the cathode was used for the

first time, whereas in the case of the lower graph the cathode had previously been heated to a high temperature and well freed of gas. In neither case was the commutator used but as only a small spot of CaO was used a fairly close correction for drop in the cathode could be made.

Fig. 6 gives a comparison between the results obtained with two cathodes, one more heavily coated with oxide than the other. The two cathodes were initially of as nearly the same dimensions as they could be made. The commutator was used in both cases. The thinly coated one melted at about 25 amperes whereas the more heavily coated one melted at 27 amperes. In the latter case when the current was first brought to 27 the striking potential was 5.5 volts but as time elapsed the striking potential dropped lower and lower until it reached the minimum of 4.8 volts. Shortly afterwards the cathode melted.

In some cases with coated cathodes the low striking voltage is only temporary. In one particular case, for instance, while studying the effect of vapor pressure on the striking of the arc, the arc suddenly

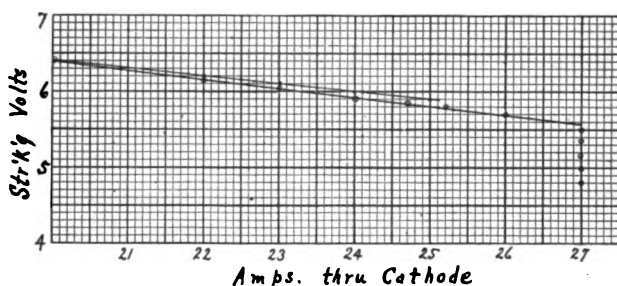


Fig. 6.

began to strike lower and lower until it reached 4.8 volts. The next day I obtained with the same apparatus a minimum of 5.8 volts just before the cathode melted.

The graphs given in Figs. 2-6 show quite conclusively that for the higher temperatures and for the cathodes used the striking voltage is a linear function of the current through the cathode. As the cathode current itself is not the important factor, these results suggested that there might be a simple relation between the striking voltage and the temperature of the cathode. Rough experiments in which I used the melting points of various salts and metals convinced me that the temperature of the cathode was, at least roughly, a linear function of the cathode current. It may be of interest to state in passing that the same apparently holds quite closely for a platinum wire in free air. This will be seen if the results of Langmuir given on page 413, Vol. 34, 1912, of the

PHYSICAL REVIEW, be plotted. The article referred to is on the conduction and convection of heat in gases.

To measure the temperature of the cathode more accurately I determined the resistance of a portion of it at two known temperatures, and then made use of the fact pointed out by Langmuir that the temperature-resistance graph for platinum is straight above 1100°C . As I found that I could use the melting points of NaCl and K_2SO_4 most satisfactorily, I used these two temperatures, viz., 801°C . and 1070°C ., and drew a straight line through them. This, of course, gives too low a value of the temperature for points above 1100°C . To correct for this, approximately, I drew another straight line between 1070°C . and a point which was 20°C . above the first line at 1900°C . This correction would vary somewhat with the platinum used but seemed from a study of the temperature-resistance curve for platinum to be somewhere near the truth.

The resistance of the portion of the cathode used was determined by measuring the current through the cathode with a standard 0.1 ohm resistance and a potentiometer, and the drop across the portion used with a potentiometer. For the latter purpose two fine pin holes were made in the central part of the cathode about 0.5 cm. apart. Through these holes fine wedge-shaped strips of platinum foil were inserted until they were tight. The ends of these strips were cemented with platinum chloride to platinum wires and these wires led to the potentiometer set. The part of the cathode between the potential leads seemed to have a fairly uniform temperature. Small bits of fused sodium chloride and potassium sulphate were then used to find, in the usual way, the resistance of the cathode at their melting points. The results obtained for one cathode are given below:

Substance.	Amperes.	Volts.	Resistance.
NaCl	4.652	.2422	.05206
"	4.636	.2400	.05177
"	4.675	.2432	.05204
"	4.656	.2433	.05226
			Mean .05203
Substance.	Amperes.	Volts.	Resistance.
K_2SO_4	5.710	.3513	.06152
"	5.737	.3529	.06151
"	5.700	.3511	.06160
"	5.714	.3515	.06152
			Mean .06154

With these two resistances and their respective temperatures the temperature-resistance graph was obtained as indicated above.

Having obtained the above data a small thin spot of CaO was placed at the center of the cathode. This no doubt changed the temperature-resistance characteristics of the cathode, but the method was deemed better than the one in which the graph was obtained after the CaO was placed on the cathode. Of course, it was impossible to use the commutator in determining the striking voltage for any particular temperature and hence an uncertainty is introduced in correcting for cathode drop. As the spot of CaO was placed at the center, the correction was always taken as equal to one half the drop over the cathode. The following table contains the data obtained for the cathode whose resistance-temperature data was given above. The cathode was well heated before these observations were made.

Amps. Thro. Cathode.	P. D. Volts.	Resistance	Temp. of Cathode.	Obs'd Str'k'g Voltage.	Cathode and Lead Drop.	Corrected Str'k'g Volt.
6.197	.4433	.0716	1630 K	6.25	1.81	7.16
6.519	.4757	.0729	1670 "	5.90	1.95	6.83
7.046	.5313	.0754	1740 "	5.50	2.15	6.58
7.703	.6036	.0783	1818 "	5.15	2.40	6.35
8.210	.6632	.0807	1890	4.85	2.59	6.15
8.576	.7017	.0818	1925	4.70	2.72	6.06

The cathode melted shortly after the last observation was made. Both amperes and volts were observed with the aid of the potentiometer sets and are more accurate than necessary. The resistance has been calculated with a slide rule.

The relation between the temperature of the cathode and the striking voltage is shown in Fig. 7, lower curve. The upper curve is from data

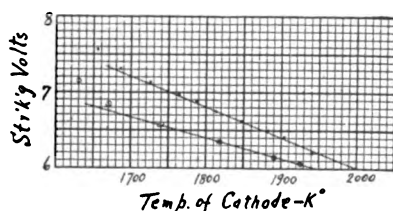


Fig. 7.

obtained with an entirely different cathode. Both graphs show that there is a linear relation between the temperature of the cathode and the striking voltage of the arc. I have not data enough yet to say whether or not these two graphs should be coincident but there is a good possibility that such should be the case as the errors arising from cathode-drop correction, temperature determination and effect of CaO thickness might easily, I think, explain the difference. If these two lines be extended to cut the $T = 0$ axis they do so at $V = 11.2$ and $V = 14.0$ volts respectively. These two values are sufficiently close to the ionization potential of mercury vapor, viz., 10.5, to be, at least, suggestive.

possibility that such should be the case as the errors arising from cathode-drop correction, temperature determination and effect of CaO thickness might easily, I think, explain the difference. If these two lines be extended to cut the $T = 0$ axis they do so at $V = 11.2$ and $V = 14.0$ volts respectively. These two values are sufficiently close to the ionization potential of mercury vapor, viz., 10.5, to be, at least, suggestive.

I have already pointed out that the lowest striking voltage for mercury vapor with a platinum cathode thinly coated with CaO is in the neighborhood of 6.0 volts. The data I have at present suggest a slightly higher value. And I have also pointed out that the striking voltage for mercury vapor with a tungsten cathode near its melting point is 3.2 volts. If these two voltages be plotted against their corresponding absolute temperatures, viz. 2028 and 3540, and the straight line drawn through them be produced to cut the $T = 0$ axis a value of 9.8 volts is obtained. This value is not very different from the ionization potential of mercury vapor.

Now these data, admittedly imperfect, show that there is a linear relation between the striking voltage of the arc in mercury vapor and the temperature of the cathode for the higher temperatures. They further suggest that the decrease in the ionization potential is directly proportional to the absolute temperature, *i.e.*, that the relation between the striking voltage and the absolute temperature of the cathode can be expressed by the equation $V = 10.5 - kT$, where k is a constant. If the values for V and T at the melting point of tungsten be substituted in this equation a value of k is obtained viz., .0021 volts/degree.

These results do not seem to admit of any simple explanation although at first sight they appear to do so. The fact that the decrease in the potential at which ionization takes place appears to vary directly with the absolute temperature of the cathode suggests that this decrease is due to the velocity of emission of the electrons. Although it has been shown that the velocity of emission of the electrons is directly proportional to the absolute temperature, yet it has also been shown that the velocity is that of thermal agitation and hence would not be equivalent to more than a fraction of a volt at the temperatures used.

As the mercury vapor in contact with the cathode must be very nearly at the temperature of the cathode, it is evident that the absolute temperature of the former would be directly proportional to the decrease in the striking voltage. But the same objection holds for this case as held for the previous one. The increase in the energy of thermal agitation of the molecules per degree is only about one sixteenth of the energy increase necessary to explain the phenomena.

A few experiments were performed to ascertain if possible whether the cause of the low arc depended on the temperature of (1) the cathode, (2) the vapor, or (3) the anode. For this purpose the minimum striking voltage, *i.e.*, the striking voltage just before the cathode melts, was determined for cathodes of widths varying from approximately 0.5 mm. to 7.0 mm. These experiments led me to believe that there was no

relation between width of cathode and the striking voltage. The values obtained for all cathodes with thin spots of CaO on them were in the neighborhood of 6.0 volts. As the wider cathodes required very much larger currents to heat them, this would appear to show that it is the temperature of the cathode which is important and not that of the vapor or anode. As, however, no temperature determinations were made these experiments are not conclusive.

Some further experiments were made with a hot anode. For this purpose two commutators connected to the same revolving shaft were used. By this means the anode and cathode could be heated simultaneously and the voltage between the anode and cathode could be applied when no current was passing through either anode or cathode. Further by opening the appropriate switch the anode could be kept cool while the cathode was still heated. The only experiments I have performed with this arrangement of apparatus were those in which I used platinum electrodes. The cathode contained a small thin spot of CaO at its center and was separated 2-3 mm. from a clean platinum anode. Under these conditions I found that with the anode hot the striking voltage was in the vicinity of one volt lower than when the anode was cold. The following data may make it clearer:

Striking Voltage with Anode Cold.	Striking Voltage with Anode Hot.
6.8	5.4
6.5	5.2
6.3	5.15
6.25	5.05

It may be necessary to point out that the different striking voltages with the anode cold were obtained by passing different currents through the cathode.

From these data it will be seen that the heated anode causes a lowering of the striking voltage of about 1.3 volts. Further experiments on these very interesting phenomena developed the fact that although the striking voltage was lowered by the hot anode the amount of lowering was increased by shutting off the current through the anode. Parenthetically, I may add that I think that this effect is due to the change in density of the mercury vapor between the anode and cathode. This state of affairs, however, lasted only a short time and then the striking voltage gradually rose to its old value. Thus with the anode cold the arc struck at 6.7 volts in a certain experiment. The anode was then heated and the arc struck lower (I have not recorded the value in my notebook), but it struck still lower, viz., at 5.05 volts, immediately after the anode current was shut off. The striking voltage then rose slowly

to 6.7 volts. It was found that the amount the striking voltage was lowered increased with the temperature of the anode. Also, within limits, a short period of heating did not cause as low an arc as a longer period. Continued heating beyond a certain maximum of time did not further affect the striking voltage. Continued low heating did not accomplish as great a drop as continued high heating. The lowest striking voltage I obtained during the above experiments was 4.95 volts. It will be noticed, therefore, that a platinum cathode with a thin patch of CaO on it can, with the aid of a hot anode, produce an arc about as low as I have obtained with a thickly coated cathode without a hot anode. Whether these two phenomena are produced by the same causes or not, I am unable to say.

At this point I was forced to drop the experiments for a considerable time but it was thought that the results would be of interest even in this unfinished state.

The results embodied in this paper do not appear to me to have any simple interpretation. As long as the striking of the arc was obtained at a value equal to or higher than 4.9 volts it seemed possible that Van der Bijl's theory of multiple collision in terms of the Bohr atom might be the correct explanation. But even this explanation has been shown by Compton¹ to be doubtful. That the density of the electron stream is very important I have shown previously. This is also shown by Fig. 2. As the temperature of the cathode rises from red heat the striking voltage drops rapidly at first as would be expected if density of electron emission is important. But for the lower part of the graph entirely different phenomena seem to be operating. For this part of the graph it seems possible that space charge might be influencing the results but, if so, apparently in no simple manner. When the arc strikes, the operating voltage is still lower, which is also in line with the idea that space charge is active in these phenomena. In this connection I may state that I have had an arc operate with tungsten cathodes as low as 1.7 volts.

In order to explain the striking of the arc at potential differences considerably lower than 4.9 volts and possibly for the explanation of low-voltage arcs it would seem as if one would be forced to assume inelastic collision and gradual accumulation of energy in the atom.

There are other factors, however, which, I believe, have to be considered, such as contact potential difference and impurities. I have previously shown that if sodium or potassium are present with mercury the arc strikes and operates lower than with pure mercury and that the

¹ *Phys. Rev.*, Feb., 1920.

lines of the sodium or potassium may not appear. I have also tried putting a little sodium hydroxide or potassium hydroxide with the CaO on the cathode. The sodium hydroxide causes the arc to strike at 5.0 volts whereas the potassium hydroxide causes it to strike lower than 5.0 volts. In the first case the D lines of sodium appear with the mercury lines, in the latter case only the mercury lines appear.

The results with the hot anode suggest that something in the nature of an impurity has been evaporated from the anode, and hence changed the contact potential. The fact that it takes some minutes for the striking voltage to regain its former value when the anode current is shut off would lead one to think that some deposit is accumulating on the anode. It may be, however, that the extra heat has changed the internal energy of the mercury atoms. I have tried, also, to picture a redistribution of current in the cathode due to the heat from the anode. As the hot lime has a negative temperature-resistance coefficient it might be possible for the calcium oxide to carry more of the current when heated by the anode and hence be at an even higher temperature than the platinum of the cathode. If such is possible this would also explain the lower arcs obtained with thickly coated cathodes. However, even if this were found to be true it would not be an explanation of the production and operation of these low arcs.

UNIVERSITY OF BRITISH COLUMBIA, VANCOUVER, B. C.

THE CALCULATION OF DETECTING AND AMPLIFYING PROPERTIES OF AN ELECTRON TUBE FROM ITS STATIC CHARACTERISTICS.

By G. BREIT.

SYNOPSIS.

Calculation of the Detecting Efficiency of an Electron Tube.—The detecting efficiency of an electron tube is calculated. In the calculation it is assumed that experimental knowledge of the static characteristics of the tube is available. The constants of the circuits used with the tube are also taken as known. From these quantities the average change in the plate current for a given amplitude impressed grid voltage is derived. The impressed grid voltage is taken to be of the form $A \cos \omega t$ where A is constant. The case when A varies slowly is also discussed.

Calculation of the Input Impedance of an Electron Tube.—The input impedance of an electron tube is calculated for the case of both positive and negative grid voltages, no assumption being made as to the mathematical form of the tube characteristics.

Generalisation of the Concept of Internal Resistance and Amplification Factor.—The concept of the complex internal resistance is introduced in treating the amplification due to a single tube. This quantity is defined by

$$\bar{\xi}_{\omega} = \frac{1}{\frac{1}{r_p} + \frac{1}{\left(\frac{1}{jC_{20}\omega}\right)}}$$

where r_p is the internal resistance, C_2 the grid plate capacity, $j = \sqrt{-1}$, and $\omega/2\pi$ = frequency. Similarly the amplification factor is generalized. (See (4.3)).

Condition for the Vanishing of Incoming Signal.—The condition for the vanishing of the incoming signal has been worked out. For the case of zero grid current this condition is just satisfied by tubes obeying Van der Bijl's relation.

It is also seen that the condition is not satisfied by the values of plate circuit constants predicted from the simple theory neglecting internal capacities.

THE first rigorous calculation of the detecting action of an electron tube is to my knowledge due to Carson.¹ Carson treats the special case of an electron tube used with sufficiently high negative grid voltage to make the grid current zero. He does not take into account the possibility of having a different value for the resistance of the plate circuit at high and low frequencies, as is seen from his formula. He also confines himself to the consideration of an electron tube for which Van der Bijl's relation holds for the plate current in terms of the plate and grid voltages.

The following treatment concerns itself with an electron tube having

¹ See Carson, Proceedings of the Institute of Radio Engineers, Vol. 7, April, 1919.

three electrodes: (1) A cathode consisting usually of a wire heated to incandescence by an electric current. (2) An anode which is usually made of a flat or cylindrical metallic plate and (3) a wire control or grid which is placed between the cathode and the anode.

In what follows I shall refer to the cathode as the *filament*, to the anode as the *plate*, and to the intermediary electrode as the *grid* of the tube.

The filament of the tube has two terminals which are used to heat the filament. It is clear that when the filament is heated by the passage of an electric current through it the potential of its two terminals is different. For this reason when the filament is used as a cathode we cannot speak of the potential of the cathode as a whole. However, if the filament current is known and if the potential of one of its terminals is known the

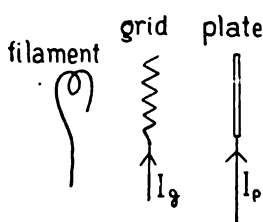


Fig. 1.

potential of all the remaining points of the filament is determined uniquely. We thus choose arbitrarily as our standard the potential of one of the terminals of the filament and we shall refer to it as the *potential of the filament*. The grid and plate may in all practical cases be regarded as equipotential surfaces. The terms "grid potential" and "plate potential" are thus

clear. The difference between the potential of the grid and that of the filament we shall call E_g ; similarly the difference between the potential of the plate and that of the filament will be denoted by E_p .

Experiments with direct currents show that two quantities E_g , E_p determine completely the currents which flow from the grid and from the plate inside the tube. If these currents, reckoned positive in the directions shown on Fig. 1, be called I_g , I_p respectively we can write,

$$I_g = f(E_g, E_p),$$

$$I_p = \varphi(E_g, E_p),$$

where f and φ are functions of the two arguments E_g and E_p . The form of these functions can be determined mathematically with fair precision for some tubes. The general action of the tube, however, can be predicted without having an accurate knowledge of the mathematical law connecting I_g , I_p with E_g , E_p .

We shall assume in what follows that these functions f and φ have within the range considered a first, second and third derivative, all of these being finite and single valued, and the function itself with its first two derivatives with respect to each argument being in addition continuous.

We shall also assume that even for unsteady currents the equations (1)

give values of I_g and I_p which represent the current carried by conduction by the ions inside the tube provided E_g , E_p denote the instantaneous values of the grid and plate potentials. (The filament potential is taken as zero.)

Before proceeding with the mathematical development of the problem I must point out the limitations put on this development by the physical assumptions made. There is no doubt that all of the functions considered are finite and single valued. The differentiability of the functions can be doubted in some cases especially in that of the grid current. Thus it is known that there is an essential distinction between the behavior of the grid current for positive and negative grid potentials. The grid current for negative potentials grid is zero to all intents and purposes. Thus at $E_g = 0$ one would be unable to define the derivative: $\partial I_g / \partial E_g$.

Fortunately this difficulty is only formal in character because a sufficiently close examination of the static characteristics shows that there is some grid current even for negative grid potentials. So that $\partial I_g / \partial E_g$ always exists. Thus we will be safe to apply our theory provided we shall recognize by means of our instruments currents and potentials so small that $\partial I_g / \partial E_g$ is continuous.

Another possibility of a break down in the theory is offered by our second assumption. In fact it is conceivable that the space inside the electron tube may have a resonant frequency. This frequency may be fixed by the dimensions or shape of the space between the electrodes. If such is the case I_g , I_p are no longer single-valued functions of E_g , E_p when unsteady currents are considered. In fact one would expect to have here a dependence of I_g , I_p either on $\partial E_g / \partial t$, dE_p / dt or on the previous history of E_g , E_p .

Such resonant effects have been described by R. Whiddington¹. They come into play, however, at frequencies very much higher than those ordinarily employed in wireless telegraphy. Also these effects are practically absent in high vacuum tubes. Thus here again we have a wide range within which our theory applies. In many cases electron tubes are used as detectors in direction finder circuits. If such is the case they are connected across the terminals of the tuning condenser the connections being as in Fig. 2. In this type of connection the electromotive force is induced in the coil. The reader will see without difficulty that this circuit is a special case of a slightly more general one drawn in

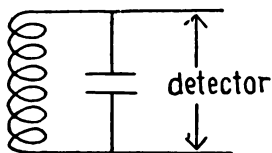


Fig. 2.

¹ Radio Review, I, 53, 1919.

Fig. 3. Here ϵ represents the place where the electromotive force is induced; Z_g' takes the place of the direction finder coil; Z_g'' replaces the tuning condenser. For generality Z_g was introduced into the circuit so as to include the possibility of a blocking condenser and grid leak. The capacities c_1 , c_2 represent the effective capacities between the electrodes. There is, of course, a third capacity between the filament and plate. But this can be incorporated as part of the circuit Z_p . The diagram of Fig. 3 includes most of the circuits which are used with electron tubes. The only essential type of connection left out is that in which there is inductive coupling between the plate and grid circuits.

NOTATION.

We shall distinguish between the behavior of the various circuits Z_g , Z_g' , Z_g'' , Z_p at different frequencies. Thus we shall not assume that a definite *resistance* can be assigned to any one of these, because experiments show that such an assumption is illegitimate. Since it is a difficult matter to give *in general* the relation between the properties of an electric circuit at various frequencies we shall content ourselves by taking the impedance [meaning thereby $R + jX$ ($j = \sqrt{-1}$), R

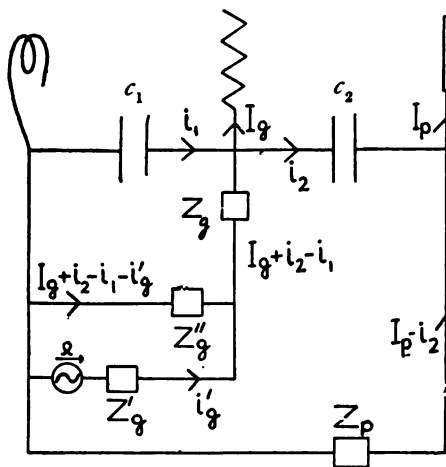


Fig. 3.

denoting the resistance, X the reactance] of a circuit as a function of the frequency which is known either experimentally or by computation. These complex values of the impedance at frequency $\omega/2\pi$ we shall write $Z_{g\omega}$, $Z_{g'\omega}$, $Z_{g''\omega}$, $Z_{p\omega}$, etc. The real part of these expressions (*i.e.*, the resistance) will be written $R_{g\omega}$, $R_{g'\omega}$, $R_{g''\omega}$, $R_{p\omega}$ and the imaginary part (*i.e.*, the reactance) will similarly be denoted by $X_{g\omega}$, $X_{g'\omega}$, $X_{g''\omega}$, $X_{p\omega}$.

We will not be concerned in this problem with transient phenomena.

If therefore the impressed electromotive force is periodic or if it consists of a sum of several periodic electromotive forces all the currents are periodic or are sums of periodic functions the periods of the electromotive forces and the currents being equal.

It is convenient for the mathematical treatment to represent the currents and potentials as Fourier series.

We shall first treat the case when the electromotive force impressed (e) is of the form $A \cos \omega t$. Then all the currents and potentials must be Fourier series in ωt . Thus any one of these quantities is of the form

$$\begin{aligned} b_0 + b_1 \cos \omega t + b_2 \cos 2\omega t + \cdots + a_1 \sin \omega t + a_2 \sin 2\omega t + \cdots \\ = b_0 + \Sigma (b_m \cos m\omega t + a_m \sin m\omega t) \\ = b_0 + \Sigma B_m e^{jm\omega t}, \end{aligned}$$

where

$$B_m = \sqrt{a_m^2 + b_m^2} e^{-j \tan^{-1} a_m/b_m}$$

and where the real part only of the expression on the right is to be taken. Using this complex notation we can write

$$i_1 = \Sigma i_1, m\omega e^{jm\omega t}$$

and similarly for all other currents of Fig. 3.

We shall now express the quantities

$$E_g, E_p \text{ in terms of } I_g, I_p \text{ and } e.$$

We have

$$\begin{aligned} \bar{i}_{g'w} \bar{Z}_{g'w} - \bar{e}_w &= \bar{Z}_{g''w} (\bar{I}_{gw} + \bar{i}_{2w} - \bar{i}_{1w} - \bar{i}_{g'w}) \\ &= \frac{\bar{i}_{1w}}{jC_{1w}} - \bar{Z}_{gw} (\bar{I}_{gw} + \bar{i}_{2w} - \bar{i}_{1w}) \\ &= \bar{Z}_{pw} (\bar{I}_{pw} - \bar{I}_{2w}) - \frac{\bar{i}_{2w}}{jC_{2w}} - \bar{Z}_{gw} (\bar{I}_{gw} + \bar{i}_{2w} - \bar{i}_{1w}), \end{aligned}$$

where the currents are as on the figure the positive directions being those of the arrows. (The currents are written to satisfy the first of Kirchhoff's laws.) Solving these equations we find

$$\begin{aligned} \bar{i}_{1w} &= \frac{-\bar{e}_w \bar{Z}_{g''w} + \bar{I}_{gw} \bar{S}_{2w}}{\frac{\bar{Z}_{g'w} + \bar{Z}_{g''w}}{jC_{1w}} + \bar{S}_{2w}} \left[1 - \frac{\bar{S}_{2w}}{\bar{S}_{2w} + \left(\bar{Z}_{pw} + \frac{1}{jC_{2w}} \right) (\bar{Z}_{g'w} + \bar{Z}_{g''w} + \bar{S}_{2w} jC_{1w})} \right] \\ &\quad + \bar{I}_{pw} \frac{\bar{Z}_{pw} \bar{S}_{2w}}{\left(\frac{\bar{Z}_{g'w} + \bar{Z}_{g''w}}{jC_{1w}} + \bar{S}_{2w} \right) \left(\frac{1}{jC_{2w}} + \bar{Z}_{pw} + \frac{\bar{S}_{2w}}{\bar{Z}_{g'w} + \bar{Z}_{g''w} + \bar{S}_{2w} jC_{1w}} \right)} \\ (I) \quad \bar{i}_{2w} &= \frac{\frac{\bar{e}_w \bar{Z}_{g''w} - \bar{I}_{gw} \bar{S}_{2w}}{\bar{Z}_{g'w} + \bar{Z}_{g''w} + jC_{1w} \bar{S}_{2w}} + \bar{Z}_{pw} \bar{I}_{pw}}{\frac{1}{jC_{2w}} + \bar{Z}_{pw} + \frac{\bar{S}_{2w}}{\bar{Z}_{g'w} + \bar{Z}_{g''w} + \bar{S}_{2w} jC_{1w}}}, \end{aligned}$$

where $\bar{S}_{2w} = \bar{Z}_{g'w} \bar{Z}_{g''w} + \bar{Z}_{gw} \bar{Z}_{g'w} + \bar{Z}_{gw} \bar{Z}_{g''w}$. The complex expression for $\bar{E}_{gw}$ is obtained as $\bar{i}_1/jC_{1w}$ while that for $\bar{E}_{pw}$ is $\bar{i}_{1w}/jC_{1w} + \bar{i}_{2w}/jC_{2w}$.

Performing the calculation we find

$$(1.1) \quad \begin{aligned} \bar{E}_{g\omega} &= \bar{G}_{1\omega} \bar{e}_{\omega} + \bar{G}_{g\omega} \bar{I}_{g\omega} + \bar{G}_{p\omega} \bar{I}_{p\omega}, \\ \bar{E}_{p\omega} &= \bar{P}_{1\omega} \bar{e}_{\omega} + \bar{P}_{g\omega} \bar{I}_{g\omega} + \bar{P}_{p\omega} \bar{I}_{p\omega}, \end{aligned}$$

where

$$(1.2) \quad \begin{aligned} \bar{G}_{1\omega} &= \frac{\bar{Z}_{g''\omega} \left(\bar{Z}_{p\omega} + \frac{1}{jc_2\omega} \right)}{\bar{A}_{\omega}}, & \bar{G}_{g\omega} &= - \frac{\bar{S}_{2\omega} \left(\bar{Z}_{p\omega} + \frac{1}{jc_2\omega} \right)}{\bar{A}_{\omega}}, \\ \bar{G}_{p\omega} &= - \frac{\bar{S}_{2\omega} \bar{Z}_{p\omega}}{\bar{A}_{\omega}}, \\ \bar{P}_{1\omega} &= \frac{\bar{Z}_{g'\omega} \bar{Z}_{p\omega}}{\bar{A}_{\omega}}, & \bar{P}_{g\omega} &= - \frac{\bar{S}_{2\omega} \bar{Z}_{p\omega}}{\bar{A}_{\omega}}, \\ \bar{P}_{p\omega} &= \frac{- \bar{Z}_{p\omega} \left[\bar{S}_{2\omega} \frac{c_1 + c_2}{c_2} + \frac{\bar{Z}_{g'\omega} + \bar{Z}_{g''\omega}}{jc_2\omega} \right]}{\bar{A}_{\omega}}, \end{aligned}$$

where

$$\bar{A}_{\omega} = \bar{S}_{2\omega} \left(1 + \frac{c_1}{c_2} + j \bar{Z}_{p\omega} c_1 \omega \right) + (\bar{Z}_{g'\omega} + \bar{Z}_{g''\omega}) \left(\bar{Z}_{p\omega} + \frac{1}{jc_2\omega} \right).$$

We called the impressed electromotive force $A \cos \omega t$. To each value of A there corresponds a definite plate current. It is clear that if $A = 0$ the plate current is steady. Similarly the grid current is steady. Let these values of grid and plate current be I_{g_0} , I_{p_0} respectively. For a value of A distinct from zero

$$I_g = I_{g_0} + \Delta I_g,$$

$$I_p = I_{p_0} + \Delta I_p,$$

where ΔI_g , ΔI_p are Fourier Series in ωt . We shall thus write

$$\Delta I_p = b_0 + b_1 \cos \omega t + b_2 \cos 2\omega t + \dots + a_1 \sin \omega t + a_2 \sin 2\omega t + \dots$$

$$\Delta I_g = \beta_0 + \beta_1 \cos \omega t + \beta_2 \cos 2\omega t + \dots + a_1 \sin \omega t + a_2 \sin 2\omega t + \dots$$

Let us now write

$$\bar{G}_{1\omega} = G_{1\omega} e^{j \tan^{-1} \frac{G_{1\omega}''}{G_{1\omega}'}} ,$$

$$\bar{G}_{g\omega} = G_{g\omega} e^{j \tan^{-1} \frac{G_{g\omega}''}{G_{g\omega}'}} ,$$

then denoting by ΔE_g , ΔE_p the changes in the grid and plate voltage from the value corresponding to $A = 0$ we have:

$$\Delta E_g = G_{g_0} \beta_0 + G_{p_0} b_0 + G_{1\omega} A \cos \left(\omega t + \tan^{-1} \frac{G_{1\omega}''}{G_{1\omega}'} \right)$$

$$\begin{aligned}
 & + G_{g\omega} \left[\beta_1 \cos \left(\omega t + \tan^{-1} \frac{G_{g\omega}''}{G_{g\omega}'} \right) + \alpha_1 \sin \left(\omega t + \tan^{-1} \frac{G_{g\omega}''}{G_{g\omega}'} \right) \right] \\
 & + G_{p\omega} \left[b_1 \cos \left(\omega t + \tan^{-1} \frac{G_{p\omega}''}{G_{p\omega}'} \right) + a_1 \sin \left(\omega t + \tan^{-1} \frac{G_{p\omega}''}{G_{p\omega}'} \right) \right] \\
 & + G_{g2\omega} \left[\beta_2 \cos \left(2\omega t + \tan^{-1} \frac{G_{g2\omega}''}{G_{g2\omega}'} \right) + \alpha_2 \sin \left(2\omega t + \tan^{-1} \frac{G_{g2\omega}''}{G_{g2\omega}'} \right) \right] \\
 & + G_{p2\omega} \left[b_2 \cos \left(2\omega t + \tan^{-1} \frac{G_{p2\omega}''}{G_{p2\omega}'} \right) + a_2 \sin \left(2\omega t + \tan^{-1} \frac{G_{p2\omega}''}{G_{p2\omega}'} \right) \right] \\
 & + \dots, \\
 (1.3) \quad \Delta E_p &= P_{g0} \beta_0 + P_{p0} b_0 + P_{1\omega} A \cos \left(\omega t + \tan^{-1} \frac{P_{1\omega}''}{P_{1\omega}'} \right) \\
 & + P_{g\omega} \left[\beta_1 \cos \left(\omega t + \tan^{-1} \frac{P_{g\omega}''}{P_{g\omega}'} \right) + \alpha_1 \sin \left(\omega t + \tan^{-1} \frac{P_{g\omega}''}{P_{g\omega}'} \right) \right] \\
 & + P_{p\omega} \left[b_1 \cos \left(\omega t + \tan^{-1} \frac{P_{p\omega}''}{P_{p\omega}'} \right) + a_1 \sin \left(\omega t + \tan^{-1} \frac{P_{p\omega}''}{P_{p\omega}'} \right) \right] \\
 & + P_{g2\omega} \left[\beta_2 \cos \left(2\omega t + \tan^{-1} \frac{P_{g2\omega}''}{P_{g2\omega}'} \right) + \alpha_2 \sin \left(2\omega t + \tan^{-1} \frac{P_{g2\omega}''}{P_{g2\omega}'} \right) \right] \\
 & + P_{p2\omega} \left[b_2 \cos \left(2\omega t + \tan^{-1} \frac{P_{p2\omega}''}{P_{p2\omega}'} \right) + a_2 \sin \left(2\omega t + \tan^{-1} \frac{P_{p2\omega}''}{P_{p2\omega}'} \right) \right] \\
 & + \dots.
 \end{aligned}$$

But our assumption as to the nature of the relationship between I_p , I_g with E_p , E_g shows that for sufficiently small values of ΔE_p , ΔE_g

$$\begin{aligned}
 \Delta I_p &= \frac{\partial I_p}{\partial E_p} \Delta E_p + \frac{\partial I_p}{\partial E_g} \Delta E_g + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_p^2} (\Delta E_p)^2 \\
 &\quad + \frac{\partial^2 I_p}{\partial E_p \partial E_g} \Delta E_p \Delta E_g + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_g^2} (\Delta E_g)^2, \\
 \Delta I_g &= \frac{\partial I_g}{\partial E_p} \Delta E_p + \frac{\partial I_g}{\partial E_g} \Delta E_g + \frac{1}{2} \frac{\partial^2 I_g}{\partial E_p^2} (\Delta E_p)^2 \\
 &\quad + \frac{\partial^2 I_g}{\partial E_p \partial E_g} \Delta E_p \Delta E_g + \frac{1}{2} \frac{\partial^2 I_g}{\partial E_g^2} (\Delta E_g)^2,
 \end{aligned}$$

where the first derivatives are to be evaluated at $I_p = I_{p0}$; $I_g = I_{g0}$ while the second derivatives are to be evaluated for some suitably chosen values of E_p , E_g in the intervals

$$(E_{p0}, E_{p0} + \Delta E_p)(E_{g0}, E_{g0} + \Delta E_g).$$

If ΔE_p , ΔE_g are sufficiently small the values of the second derivatives are sensibly constant and can be taken simply as the values at (E_{p0}, E_{g0}) .

We can, therefore, regard all the derivatives as constant coefficients.

If the mathematical expressions for I_p , I_g are known the derivatives will be obtained by ordinary differentiation. If no mathematical expression is available the derivatives are still easily obtainable from the graphs of the functions.

FIRST APPROXIMATION. INTERNAL RESISTANCE. AMPLIFICATION
CONSTANT. INPUT IMPEDANCE.

As a first approximation we can disregard the squares of ΔE_p , ΔE_g . Then

$$(2) \quad \begin{aligned} \Delta I_p &= \frac{\partial I_p}{\partial E_p} \Delta E_p + \frac{\partial I_p}{\partial E_g} \Delta E_g, \\ \Delta I_g &= \frac{\partial I_g}{\partial E_p} \Delta E_p + \frac{\partial I_g}{\partial E_g} \Delta E_g. \end{aligned}$$

Let us analyze ΔI_p , ΔI_g , ΔE_p , ΔE_g into a Fourier series, say $\Sigma \bar{\Delta I}_{pw}$, $\Sigma \bar{\Delta I}_{gw}$, $\Sigma \bar{\Delta E}_{pw}$, $\Sigma \bar{\Delta E}_{gw}$ where $\bar{\Delta I}_{pw}$ is of the form $A e^{j\omega t}$ ($j = \sqrt{-1}$) and where it is understood that only the real parts of the equations are to be considered. Since the equations (2) are linear we can write

$$\begin{aligned} \bar{\Delta I}_{pw} &= \frac{\partial I_p}{\partial E_p} \bar{\Delta E}_{pw} + \frac{\partial I_p}{\partial E_g} \bar{\Delta E}_{gw}, \\ \bar{\Delta I}_{gw} &= \frac{\partial I_g}{\partial E_p} \bar{\Delta E}_{pw} + \frac{\partial I_g}{\partial E_g} \bar{\Delta E}_{gw}. \end{aligned}$$

Combining these with (1.1) we have

$$\begin{aligned} \bar{I}_{gw} \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{gw} + \frac{\partial I_p}{\partial E_g} \bar{G}_{gw} \right) + \bar{I}_{pw} \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{pw} + \frac{\partial I_p}{\partial E_g} \bar{G}_{gw} - 1 \right) &= -\bar{e}_w \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{1w} + \frac{\partial I_p}{\partial E_g} \bar{G}_{1w} \right), \\ \bar{I}_{gw} \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{gw} + \frac{\partial I_g}{\partial E_g} \bar{G}_{gw} - 1 \right) + \bar{I}_{pw} \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{pw} + \frac{\partial I_g}{\partial E_g} \bar{G}_{pw} \right) &= -\bar{e}_w \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{1w} + \frac{\partial I_g}{\partial E_g} \bar{G}_{1w} \right). \end{aligned}$$

Hence

$$(2.1) \quad \begin{aligned} \bar{I}_{pw} &= e_w \frac{\left(\frac{\partial I_g}{\partial E_p} \bar{P}_{gw} + \frac{\partial I_g}{\partial E_g} \bar{G}_{gw} - 1 \right) \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{1w} + \frac{\partial I_p}{\partial E_g} \bar{G}_{1w} \right) - \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{gw} + \frac{\partial I_p}{\partial E_g} \bar{G}_{gw} \right) \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{1w} + \frac{\partial I_g}{\partial E_g} \bar{G}_{1w} \right)}{\Delta}, \\ \bar{I}_{gw} &= e_w \frac{\left(\frac{\partial I_p}{\partial E_p} \bar{P}_{pw} + \frac{\partial I_p}{\partial E_g} \bar{G}_{pw} - 1 \right) \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{1w} + \frac{\partial I_g}{\partial E_g} \bar{G}_{1w} \right) - \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{pw} + \frac{\partial I_g}{\partial E_g} \bar{G}_{pw} \right) \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{1w} + \frac{\partial I_p}{\partial E_g} \bar{G}_{1w} \right)}{\Delta}, \end{aligned}$$

where

$$\begin{aligned} \Delta &= \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{g\omega} + \frac{\partial I_p}{\partial E_g} \bar{G}_{g\omega} \right) \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{p\omega} + \frac{\partial I_g}{\partial E_g} \bar{G}_{p\omega} \right) \\ &\quad - \left(\frac{\partial I_p}{\partial E_p} \bar{P}_{p\omega} + \frac{\partial I_p}{\partial E_g} \bar{G}_{p\omega} - 1 \right) \left(\frac{\partial I_g}{\partial E_p} \bar{P}_{g\omega} + \frac{\partial I_g}{\partial E_g} \bar{G}_{g\omega} - 1 \right) \\ &= -1 + \frac{\partial I_g}{\partial E_p} \bar{P}_{g\omega} + \frac{\partial I_g}{\partial E_g} \bar{G}_{g\omega} + \frac{\partial I_p}{\partial E_p} \bar{P}_{p\omega} + \frac{\partial I_p}{\partial E_g} \bar{G}_{p\omega} \\ &\quad + \begin{vmatrix} P_{g\omega} & G_{g\omega} \\ P_{p\omega} & G_{p\omega} \end{vmatrix} \cdot \begin{vmatrix} \frac{\partial I_p}{\partial E_p} & \frac{\partial I_p}{\partial E_g} \\ \frac{\partial I_g}{\partial E_p} & \frac{\partial I_g}{\partial E_g} \end{vmatrix}. \end{aligned}$$

Before proceeding with the second approximation of the problem we shall interpret the equations given. Let us consider the special case when $Z_{g\omega} = Z_{p\omega} = 0$. Then (see 1.2) $\bar{S}_{2\omega} = 0$,

$$A_{\omega} = \bar{Z}_{g''\omega} \left(\bar{Z}_{p\omega} + \frac{1}{jc_2\omega} \right)$$

$$\bar{G}_{1\omega} = 1, \quad \bar{G}_{g\omega} = 0, \quad \bar{G}_{p\omega} = 0,$$

$$(2.2) \quad \bar{P}_{1\omega} = \frac{\bar{Z}_{p\omega}}{\bar{Z}_{p\omega} + \frac{1}{jc_2\omega}}, \quad \bar{P}_{g\omega} = 0, \quad \bar{P}_{p\omega} = -\frac{\bar{Z}_{p\omega}}{\bar{Z}_{p\omega} + \frac{1}{jc_2\omega}}.$$

From (2.1) and (2.2) we derive:

$$(2.3) \quad \begin{aligned} \bar{I}_{g\omega} &= \bar{e}_{\omega} \frac{\frac{\partial I_g}{\partial E_g} + \frac{\partial I_g}{\partial E_p} \frac{jc_2\omega \bar{Z}_{p\omega}}{1 + jc_2\omega \bar{Z}_{p\omega}} + \frac{\partial(I_p, I_g)}{\partial(E_p, E_g)} \frac{\bar{Z}_{p\omega}}{1 + jc_2\omega \bar{Z}_{p\omega}}}{1 + \frac{\bar{Z}_{p\omega}}{1 + jc_2\omega \bar{Z}_{p\omega}} \frac{\partial \bar{I}_p}{\partial E_p}}, \\ \bar{I}_{p\omega} &= \bar{e}_{\omega} \frac{\frac{\partial I_p}{\partial E_p} \frac{jc_2\omega \bar{Z}_{p\omega}}{1 + jc_2\omega \bar{Z}_{p\omega}} + \frac{\partial I_p}{\partial E_g}}{1 + \frac{\bar{Z}_{p\omega}}{1 + jc_2\omega \bar{Z}_{p\omega}} \frac{\partial \bar{I}_p}{\partial E_p}}. \end{aligned}$$

Also from (1') and (2.2)

$$(2.4) \quad \bar{i}_{2\omega} = \frac{jc_2\omega(\bar{e}_{\omega} + \bar{Z}_{p\omega} \bar{I}_{p\omega})}{1 + jc_2\omega \bar{Z}_{p\omega}}.$$

Hence

$$\bar{I}_{g\omega} + \bar{i}_{2\omega} = \bar{e}_{\omega} \left[\frac{jc_2\omega}{1 + jc_2\omega \bar{Z}_{p\omega}} + \frac{\frac{\bar{Z}_{p\omega}^2}{\left(\bar{Z}_{p\omega} + \frac{1}{jc_2\omega} \right)^2} \frac{\partial I_p}{\partial E_p} + \frac{\bar{Z}_{p\omega}}{\bar{Z}_{p\omega} + \frac{1}{jc_2\omega}} \frac{\partial I_p}{\partial E_g}}{1 + \frac{\bar{Z}_{p\omega}}{1 + jc_2\omega \bar{Z}_{p\omega}} \frac{\partial \bar{I}_p}{\partial E_p}} \right]$$

$$\begin{aligned}
 & \left[\frac{\partial I_g}{\partial E_g} + \frac{\partial I_g}{\partial E_p} \frac{\bar{Z}_{pw}}{\bar{Z}_{pw} + \frac{1}{jc_2\omega}} + \frac{\partial(I_p, I_g)}{\partial(E_p, E_g)} \frac{\bar{Z}_{pw}/jc_2\omega}{\bar{Z}_{pw} + \frac{1}{jc_2\omega}} \right] \\
 & + \frac{1 + \frac{\bar{Z}_{pw}}{1 + jc_2\omega \bar{Z}_{pw}} \frac{\partial I_p}{\partial E_p}}{\frac{1 + \bar{Z}_{pw} \left(\frac{\partial I_p}{\partial E_p} + \frac{\partial I_p}{\partial E_g} + \frac{\partial I_g}{\partial E_g} + \frac{\partial I_g}{\partial E_p} \right) + \frac{1}{jc_2\omega} \frac{\partial I_g}{\partial E_g} + \frac{\bar{Z}_{pw}}{jc_2\omega} \frac{\partial(I_p, I_g)}{\partial(E_p, E_g)}}{\bar{Z}_{pw} + \frac{1}{jc_2\omega} + \frac{\bar{Z}_{pw}}{jc_2\omega} \frac{\partial I_p}{\partial E_p}} \\
 & = \bar{e}_\omega
 \end{aligned}$$

The ratio $\bar{e}_\omega/(\bar{I}_{g\omega} + \bar{i}_{2\omega})$ may be called the *intrinsic* input impedance of the tube because (neglecting the current flowing through the capacity C_1) $\bar{I}_{g\omega} + \bar{i}_{2\omega}$ is the total current of frequency $\omega/2\pi$ which flows from the outside circuit into the tube. I call the quotient $\bar{e}_\omega/(\bar{I}_{g\omega} + \bar{i}_{2\omega})$ the *intrinsic* input impedance of the tube in distinction to the *true* input impedance which is $\bar{e}_\omega/(\bar{I}_{g\omega} + \bar{i}_{2\omega} - \bar{i}_{1\omega})$. There is really no advantage in knowing the true input impedance of the tube because the tube is always put in parallel with some other circuit, and the capacity C_1 can, therefore, be considered as part of that other circuit. This procedure as well as the fact that we incorporate the capacity from filament to plate in Z_p simplifies our expressions.

Let $\bar{Z}_{i\omega}$ denote the *intrinsic input impedance*. Then

$$\begin{aligned}
 (3) \quad \bar{Z}_{i\omega} = & \frac{\bar{Z}_{pw} + \frac{1}{jc_2\omega} + \frac{\bar{Z}_{pw}}{jc_2\omega} \frac{\partial I_p}{\partial E_p}}{1 + \bar{Z}_{pw} \left(\frac{\partial I_p}{\partial E_p} + \frac{\partial I_p}{\partial E_g} + \frac{\partial I_g}{\partial E_g} + \frac{\partial I_g}{\partial E_p} \right)} \\
 & + \frac{1}{jc_2\omega} \frac{\partial I_g}{\partial E_g} + \frac{\partial(I_p, I_g)}{\partial(E_p, E_g)} \frac{\bar{Z}_{pw}}{jc_2\omega}
 \end{aligned}$$

An inspection of this equation shows that both the numerator and denominator are linear in $\bar{Z}_{pw}$. This fact admits of an interesting interpretation. For it is clear that the relation between $\bar{R}_{pw}$ and $\bar{X}_{pw}$ ($\bar{Z}_{pw} = \bar{R}_{pw} + j\bar{X}_{pw}$) which will necessitate the real or the imaginary part of $\bar{Z}_{i\omega}$ to be constant is such as to make the pts. $(\bar{R}_{pw}, \bar{X}_{pw})$ referred to cartesian axes lie on a circle. Thus the lines of constant input resistance in the $\bar{R}_{pw}, \bar{X}_{pw}$ plane, or constant input reactance are circles. Moreover all of these circles must pass through one point, viz., the point corresponding to $\bar{Z}_{i\omega} = \infty$. Again since in the $\bar{Z}_{i\omega}$ plane the lines of input resistance are orthogonal to the lines of input reactances the same must be true in the $\bar{Z}_{pw}$ plane. This gives a general picture of the variation of $\bar{Z}_{i\omega}$ with $\bar{Z}_{pw}$. I have treated this matter in more detail

for the special case of negative grid voltage.¹ It is seen, therefore, that so far as the general properties of the relationship between the plate circuit and the input impedance are concerned there is no difference between the case of positive and negative grid voltage.

A more thorough investigation of the expression (3) will show, however, an essential distinction between the two cases. This is given by the value of $\bar{Z}_{i_0}$ for $\omega = 0$. In fact this value is

$$(3.1) \quad \bar{Z}_{i_0} = \frac{1 + \bar{Z}_{p_0} \frac{\partial I_p}{\partial E_p}}{\frac{\partial I_g}{\partial E_g} + \frac{\partial(I_p, I_g)}{\partial(E_p, E_g)} \bar{Z}_{p_0}}.$$

If the grid voltage is sufficiently negative I_g is constant and therefore $\bar{Z}_{i_0} = \infty$. In other words, there is no appreciable current absorbed from the external circuit when the frequency of the alternating current becomes sufficiently low and when $E_g < 0$.

On the contrary if $E_g > 0$ then no matter how low the frequency used may be a sufficient amount of current is absorbed from the external circuit by the tube to oblige us to replace the tube for purposes of computation by the fictitious resistance $\bar{Z}_{i_0}$ given in (3.1).

In most tubes $\partial I_p / \partial E_p$, $\partial I_g / \partial E_g$, $\partial I_p / \partial E_g$ are positive while $\partial I_g / \partial E_p$ is negative. Thus the Jacobian $\partial(I_p, I_g) / \partial(E_p, E_g)$ is positive and $\bar{Z}_{i_0}$ is positive.

The subject of input impedance has been worked out in detail for the case of negative grid voltage by Dr. J. M. Miller.² My formula (3) agrees with Dr. Miller's result if $I_g = \text{const.}$ and if in Dr. Miller's formula C_1 and C_2 are put equal to zero. In fact both of these become

$$\bar{Z}_{i_0} = \frac{\frac{\bar{Z}_{p_0}}{jC_2\omega} + r_p \left(\bar{Z}_{p_0} + \frac{1}{jC_2\omega} \right)}{(K + 1)\bar{Z}_{p_0} + r_p},$$

provided we set

$$\frac{\partial I_p}{\partial E_p} = \frac{1}{r_p}, \quad \frac{\partial I_p}{\partial E_g} = K \frac{\partial I_p}{\partial E_p}.$$

The reasons why the internal resistance and the amplification constant K are to be identified with the expressions given will be discussed later.

In order to compute the input impedance in the general case it is convenient to call

¹ See Bureau of Standards Radio Laboratory Report 5-V.

² See Bureau of Standards Scientific Paper No. 351.

$$A_1 = \left(\frac{\partial I_p}{\partial E_p} + \frac{\partial I_p}{\partial E_g} + \frac{\partial I_g}{\partial E_p} + \frac{\partial I_g}{\partial E_g} \right) / \frac{\partial I_p}{\partial E_p};$$

$$B = \frac{\partial(I_p, I_g)}{\partial(E_p, E_g)} / \frac{\partial I_p}{\partial E_p};$$

$$C = \frac{\partial I_g}{\partial E_g} / \frac{\partial I_p}{\partial E_p}; \quad r_p = \frac{1}{\frac{\partial I_p}{\partial E_p}}.$$

Then

$$\bar{Z}_{i\omega} = \frac{r_p \left(\bar{Z}_{p\omega} + \frac{1}{jC_2\omega} \right) + \frac{\bar{Z}_{p\omega}}{jC_2\omega}}{r_p + A_1 \bar{Z}_{p\omega} + \frac{C}{jC_2\omega} + B \frac{\bar{Z}_{p\omega}}{jC_2\omega}}.$$

Writing

$$\bar{Z}_{i\omega} = R_{i\omega} + jX_{i\omega}, \quad \bar{Z}_{p\omega} = R_{p\omega} + jX_{p\omega},$$

$$R_{i\omega} = \frac{\alpha\gamma + \beta\delta}{\gamma^2 + \delta^2}; \quad X_{i\omega} = \frac{-\alpha\delta + \beta\gamma}{\gamma^2 + \delta^2}, \quad \text{where}$$

$$(3.2) \quad \alpha = R_{p\omega} r_p + \frac{X_{p\omega}}{C_2\omega}, \quad \beta = r_p X_{p\omega} - \frac{r_p + R_{p\omega}}{C_2\omega},$$

$$\gamma = r_p + A_1 R_{p\omega} + B \frac{X_{p\omega}}{C_2\omega}, \quad \delta = A_1 X_{p\omega} - \frac{C}{C_2\omega} - B \frac{R_{p\omega}}{C_2\omega}.$$

It is thus seen that a knowledge of the four derivatives $\partial I_p/\partial E_p$, $\partial I_p/\partial E_g$, $\partial I_g/\partial E_g$, $\partial E_g/\partial E_p$ of C_2 and of $R_{p\omega}$, $X_{p\omega}$ is sufficient to enable one to calculate the input resistance and input reactance. We can thus consider the problem of input impedance as solved for both positive and negative grid voltages.

COMPUTATION OF CURRENT IN PLATE CIRCUIT.

In many cases it is necessary to know the quantity $I_{p\omega} - I_{2\omega}$ because this, to a first approximation is the current in Z_p . Such is, for example the case of a transformer-coupled amplifier. Now from (2.3) and (2.4,

$$(4) \quad \bar{I}_{p\omega} - i_{2\omega} = \frac{-jC_2\omega \bar{e}_\omega}{1 + jC_2\omega \bar{Z}_{p\omega}} + \frac{\bar{I}_{p\omega}}{1 + jC_2\omega \bar{Z}_{p\omega}}$$

$$= \frac{\bar{e}_\omega}{1 + jC_2\omega \bar{Z}_{p\omega}} \left[-jC_2\omega + \frac{\frac{\partial I_p}{\partial E_g} + \frac{\partial I_p}{\partial E_p} \frac{jC_2\omega}{1 + jC_2\omega \bar{Z}_{p\omega}}}{1 + \frac{\bar{Z}_{p\omega}}{1 + jC_2\omega \bar{Z}_{p\omega}} \frac{\partial I_p}{\partial E_p}} \right]$$

$$= \frac{\bar{e}_\omega \left[-jC_2\omega + \frac{\partial I_p}{\partial E_g} \right]}{1 + jC_2\omega \bar{Z}_{p\omega} + \bar{Z}_{p\omega} \frac{\partial I_p}{\partial E_p}}.$$

This simple relation connects the current through Z_p with the voltage from the filament to the grid of the tube. It is seen that $\partial I_g/\partial E_p$, $\partial I_g/\partial E_g$ do not come into this expression as could be expected. If $c_2\omega$ is sufficiently small

$$(4.1) \quad \bar{I}_{p\omega} - \bar{i}_{2\omega} = \bar{e}_\omega \frac{\frac{\partial I_p}{\partial E_g}}{1 + \bar{Z}_{p\omega} \frac{\partial I_p}{\partial E_p}} = \frac{K \bar{e}_\omega}{r_p + \bar{Z}_{p\omega}},$$

where

$$K = \frac{\partial I_p}{\partial E_g} / \frac{\partial I_p}{\partial E_p} \quad r_p = \frac{1}{\frac{\partial I_p}{\partial E_p}}.$$

A simple interpretation can be given to this last expression. In fact it shows that if C_2 is negligible the current through Z_p can be imagined as due to an electromotive force $K\bar{e}_\omega$ acting in series with $\bar{Z}_{p\omega}$ and a pure resistance, r_p , inside the tube. The positive direction of this electromotive force is visibly in the direction from filament to plate outside the tube. For this reason we can call K the amplification factor and r_p the internal resistance of the tube. We note the fact that Van der Bijl's relation is not used in making these definitions.

If C_2 is not negligible we can still write (4) as:

$$\bar{I}_{p\omega} - \bar{i}_{2\omega} = \frac{\bar{e}_\omega \frac{\frac{\partial I_p}{\partial E_g} - jC_2\omega}{jC_2\omega + \frac{\partial I_p}{\partial E_p}}}{\bar{Z}_{p\omega} + \frac{1}{\frac{\partial I_p}{\partial E_p} + jC_2\omega}}$$

and writing

$$(4.2) \quad \bar{\xi}_\omega = \frac{1}{\frac{\partial I_p}{\partial E_p} + jC_2\omega},$$

$$\bar{k}_\omega = \frac{\frac{\partial I_p}{\partial E_g} - jC_2\omega}{jC_2\omega + \frac{\partial I_p}{\partial E_p}},$$

$$(4.3) \quad \bar{I}_{p\omega} - \bar{i}_{2\omega} = \frac{\bar{k}_\omega \bar{e}_\omega}{\bar{Z}_{p\omega} + \bar{\xi}_\omega}.$$

Thus $\bar{\xi}_\omega$ may be spoken of as the complex internal resistance of the tube and $\bar{k}_\omega$ as its complex amplification factor at frequency $\omega/2\pi$.

It is of interest to note the connection between $\bar{z}_\omega$ and r_p . For obviously

$$\bar{z}_\omega = \frac{1}{\frac{1}{r_p} + 1/jC_2\omega}.$$

This shows that the complex internal resistance can be obtained by imagining the capacity C_2 connected in parallel with the internal resistance r_p .

SECOND APPROXIMATION.

Having calculated our quantities to the first order we can proceed to the calculation of b_0 and β_0 . From equations (1.3) we find that the constant terms of $(\Delta E_g)^2$, $(\Delta E_g(\Delta E_p))$, $(\Delta E_p)^2$ are neglecting quantities of higher order than the second. In $(\Delta E_g)^2$:

$$\begin{aligned} & \frac{G_{1\omega}^2 A^2}{2} + \frac{G_{g\omega}^2}{2} (\beta_1^2 + \alpha_1^2) + \frac{G_{p\omega}^2}{2} (b_1^2 + a_1^2) \\ & + A\beta_1(G_{g\omega}'G_{1\omega}' + G_{g\omega}''G_{1\omega}'') + A\alpha_1(G_{g\omega}''G_{1\omega}' - G_{g\omega}'G_{1\omega}'') \\ & + Ab_1(G_{p\omega}'G_{1\omega}' + G_{p\omega}''G_{1\omega}'') + Aa_1(G_{p\omega}''G_{1\omega}' - G_{p\omega}'G_{1\omega}'') \\ & + (-\alpha_1a_1 + \beta_1b_1)(G_{g\omega}'G_{p\omega}' + G_{g\omega}''G_{p\omega}'') \\ (5) \quad & + (\beta_1a_1 - \alpha_1b_1)(G_{p\omega}''G_{g\omega}' - G_{p\omega}'G_{g\omega}'') = 2R_{gg}. \end{aligned}$$

In $2\Delta E_g\Delta E_p$:

$$\begin{aligned} & A^2(P_{1\omega}'G_{1\omega}' + P_{1\omega}''G_{1\omega}'') + A\beta_1[G_{1\omega}'P_{g\omega}' + G_{1\omega}''P_{g\omega}''] \\ & + P_{1\omega}'G_{g\omega}' + P_{1\omega}''G_{g\omega}'' + A\alpha_1[P_{g\omega}''G_{1\omega}' - P_{g\omega}'G_{1\omega}''] \\ & + P_{1\omega}'G_{g\omega}'' - G_{g\omega}'P_{1\omega}''] + Ab_1(G_{1\omega}'P_{p\omega}' + G_{1\omega}''P_{p\omega}'') \\ & + P_{1\omega}''G_{p\omega}'' + P_{1\omega}'G_{p\omega}') + Aa_1(P_{p\omega}''G_{1\omega}' - P_{p\omega}'G_{1\omega}'') \\ & + G_{p\omega}''P_{1\omega}' - G_{p\omega}'P_{1\omega}'') + (P_{g\omega}'G_{g\omega}' + P_{g\omega}''G_{g\omega}'')(\beta_1^2 - \alpha_1^2) \\ & + (P_{p\omega}'G_{p\omega}' + P_{p\omega}''G_{p\omega}'')(b_1^2 - a_1^2) \\ & + (\alpha_1b_1 - a_1\beta_1)(G_{g\omega}''P_{p\omega}' - P_{p\omega}''G_{g\omega}' + P_{g\omega}''G_{p\omega}' - P_{g\omega}'G_{p\omega}'') \\ & + (\beta_1b_1 - \alpha_1a_1)(G_{g\omega}'P_{p\omega}' + G_{g\omega}''P_{p\omega}'' + P_{g\omega}'G_{p\omega}'') \\ (5.1) \quad & + P_{g\omega}''G_{p\omega}'') = 2R_{gp}. \end{aligned}$$

$$\begin{aligned} \text{In } (\Delta E_p)^2 &= \frac{P_{1\omega}^2 A^2}{2} + \frac{P_{g\omega}^2}{2} (\beta_1^2 + \alpha_1^2) + \frac{P_{p\omega}^2}{2} (b_1^2 + a_1^2) \\ & + A\beta_1(P_{g\omega}'P_{1\omega}' + P_{g\omega}''P_{1\omega}'') + A\alpha_1(P_{g\omega}''P_{1\omega}' - P_{g\omega}'P_{1\omega}'') \\ & + Ab_1(P_{p\omega}'P_{1\omega}' + P_{p\omega}''P_{1\omega}'') + Aa_1(P_{p\omega}''P_{1\omega}' - P_{p\omega}'P_{1\omega}'') \\ & + (\beta_1b_1 - \alpha_1a_1)(P_{g\omega}'P_{p\omega}' + P_{g\omega}''P_{p\omega}'') \\ (5.2) \quad & + (\beta_1a_1 - \alpha_1b_1)(P_{p\omega}''P_{g\omega}' - P_{p\omega}'P_{g\omega}'') = 2R_{pp}. \end{aligned}$$

These expressions are calculable if we content ourselves with using values of $a_1, b_1, \alpha_1, \beta_1$ as obtained by our first approximation.

In most practical cases the expressions for R_{ss}, R_{sp}, R_{pp} simplify considerably, but for the sake of generality I preferred giving their complete expression.

Identifying now constant terms on both sides of the equations

$$\begin{aligned}\Delta I_p &= \frac{\partial I_p}{\partial E_p} \Delta E_p + \frac{\partial I_p}{\partial E_s} \Delta E_s + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_s^2} (\Delta E_s)^2 + \frac{\partial^2 I_p}{\partial E_p \partial E_s} \Delta E_p \Delta E_s \\ &\quad + \frac{\partial^2 I_p}{\partial E_p^2} (\Delta E_p)^2, \\ \Delta I_s &= \frac{\partial I_s}{\partial E_p} \Delta E_p + \frac{\partial I_s}{\partial E_s} \Delta E_s + \frac{1}{2} \frac{\partial^2 I_s}{\partial E_s^2} (\Delta E_s)^2 + \frac{\partial^2 I_s}{\partial E_p \partial E_s} \Delta E_p \Delta E_s \\ &\quad + \frac{\partial^2 I_s}{\partial E_p^2} (\Delta E_p)^2,\end{aligned}$$

we find

$$\begin{aligned}b_0 \left(1 - P_{ps} \frac{\partial I_p}{\partial E_p} - G_{ps} \frac{\partial I_p}{\partial E_s} \right) - \beta_0 \left(P_{ss} \frac{\partial I_p}{\partial E_p} + G_{ss} \frac{\partial I_p}{\partial E_s} \right) &= C_p, \\ -b_0 \left(P_{ps} \frac{\partial I_s}{\partial E_p} + G_{ps} \frac{\partial I_s}{\partial E_s} \right) + \beta_0 \left(1 - P_{ss} \frac{\partial I_s}{\partial E_p} - G_{ss} \frac{\partial I_s}{\partial E_s} \right) &= C_s,\end{aligned}$$

where

$$\begin{aligned}C_p &= \frac{\partial^2 I_p}{\partial E_p^2} R_{pp} + \frac{\partial^2 I_p}{\partial E_p \partial E_s} R_{ps} + \frac{\partial^2 I_p}{\partial E_s^2} R_{ss}, \\ C_s &= \frac{\partial^2 I_s}{\partial E_p^2} R_{pp} + \frac{\partial^2 I_s}{\partial E_p \partial E_s} R_{ps} + \frac{\partial^2 I_s}{\partial E_s^2} R_{ss}.\end{aligned}\tag{5.3}$$

Solving these two simultaneous equations in b, β , we obtain

$$\begin{aligned}b_0 &= \frac{C_p \left(1 - P_{ss} \frac{\partial I_s}{\partial E_p} - G_{ss} \frac{\partial I_s}{\partial E_s} \right) + C_s \left(P_{ss} \frac{\partial I_p}{\partial E_p} + G_{ss} \frac{\partial I_p}{\partial E_s} \right)}{\left(1 - P_{ps} \frac{\partial I_p}{\partial E_p} - G_{ps} \frac{\partial I_p}{\partial E_s} \right) \left(1 - P_{ss} \frac{\partial I_s}{\partial E_p} - G_{ss} \frac{\partial I_s}{\partial E_s} \right)} \\ &\quad - \left(P_{ps} \frac{\partial I_p}{\partial E_p} + G_{ps} \frac{\partial I_p}{\partial E_s} \right) \left(P_{ss} \frac{\partial I_p}{\partial E_p} + G_{ss} \frac{\partial I_p}{\partial E_s} \right), \\ \beta_0 &= \frac{C_s \left(1 - P_{ps} \frac{\partial I_p}{\partial E_p} - G_{ps} \frac{\partial I_p}{\partial E_s} \right) + C_p \left(P_{ps} \frac{\partial I_s}{\partial E_p} + G_{ps} \frac{\partial I_s}{\partial E_s} \right)}{\left(1 - P_{ps} \frac{\partial I_p}{\partial E_p} - G_{ps} \frac{\partial I_p}{\partial E_s} \right) \left(1 - P_{ss} \frac{\partial I_s}{\partial E_p} - G_{ss} \frac{\partial I_s}{\partial E_s} \right)} \\ &\quad - \left(P_{ps} \frac{\partial I_p}{\partial E_p} + G_{ps} \frac{\partial I_p}{\partial E_s} \right) \left(P_{ss} \frac{\partial I_p}{\partial E_p} + G_{ss} \frac{\partial I_p}{\partial E_s} \right).\end{aligned}\tag{5.4}$$

The first of these expressions enables us to calculate the average change in plate current due to the electromotive force, $A \cos \omega t$, impressed in

series with Z_g' . In a similar manner the second equation gives the average change in the grid current. The amplitude A enters in these equations only through C_p , C_g which are homogeneous linear functions of R_{gg} , R_{gp} , R_{pp} . Since α_1 , β_1 , a_1 , b_1 , are as a first approximation proportional to A it is readily seen from (5), (5.1), (5.2) that R_{gg} , R_{gp} , R_{pp} are proportional to A^2 . Hence our expressions for b_0 , β_0 are proportional to A^2 . In general, of course, there is no reason why this law should be strictly true. But it is true that the smaller A the more nearly b_0 , β_0 are proportional to A^2 .

One of the fundamental assumptions in our discussion was that the electromotive force $A \cos \omega t$ has been impressed on the circuit so long that the effect of the initial conditions has been obliterated. The time necessary for this is generally very small. Let us change A slowly enough to justify us in considering all of the currents at any instant as having values identical with the values which they would have if A had had the value which it has at that instant during an infinite time before the instant under consideration. If such is the case the value of b_0 at any instant is proportional to the value which A has at that instant. Thus, if A should vary as $1 + K \cos pt$ where p is sufficiently small to secure conditions discussed above, the average change in the plate current will vary as $1 + (K^2/2) + 2K \cos pt + (K^2/2) \cos 2pt$.

SPECIAL CASES.

The expressions (5.4) apply to the most general case under consideration and enable us to calculate b_0 , β_0 when the E.M.F. impressed in series with Z_g' is known. We have shown, however, how the action of the tube on the currents in the circuits Z_g , Z_g'' , Z_g' can be computed by endowing the tube with an *input impedance*. Since our expressions for R_{pp} , R_{pg} , R_{gg} involve only the squares of first order terms in A we can for convenience of computation divide the problem into two parts. The first will concern itself with a calculation of the voltage from F to G account being taken only of first powers in A . The second will deal with the calculation of b_0 assuming voltage as known. Using formula (3) we have no difficulty in solving the first phase of the problem.

In order to deduce a solution of the second phase from our general expressions we imagine a case when $\bar{Z}_{g'u} = \bar{Z}_{gu} = 0$. Then equations (1.2) simplify as in (2.2) and $\bar{I}_{gu}$, $\bar{I}_{pu}$ are given as in (2.3). Also from (5), (5.1), (5.2) we have

$$\begin{aligned}
 2R_{gg} &= \frac{A^2}{2}, \\
 (5.5) \quad 2R_{gp} &= A^2 P_{1g}' + A(b_1 P_{pg}' + a_1 P_{pg}''), \\
 2R_{pp} &= \frac{P_{1g}^2}{2} A^2 + \frac{P_{pg}^2}{2} (a_1^2 + b_1^2) + A b_1 (P_{pg}' P_{1g}' + P_{pg}'' P_{1g}'') \\
 &\quad + A a_1 (P_{pg}'' P_{1g}' - P_{pg}' P_{1g}'').
 \end{aligned}$$

But it follows from expressions (2.2) that

$$P_{1g}' = + C_2 \omega P_{pg}'' \text{ and } P_{1g}'' = - C_2 \omega P_{pg}'.$$

Hence

$$P_{pg}' P_{1g}' + P_{pg}'' P_{1g}'' = 0$$

and

$$P_{pg}'' P_{1g}' - P_{pg}' P_{1g}'' = C_2 \omega P_{pg}^2.$$

Thus

$$2R_{pp} = \frac{P_{1g}^2}{2} A^2 + \frac{P_{pg}^2}{2} (a_1^2 + b_1^2) + A a_1 C_2 \omega P_{pg}^2. \quad (5.5)$$

We note that α_1, β_1 do not occur in these expressions. Now (2.3) can be written

$$\begin{aligned}
 \bar{I}_{pg} &= \bar{e}_\omega \frac{-\frac{\partial I_p}{\partial E_p} j C_2 \omega \bar{P}_{pg} + \frac{\partial I_p}{\partial E_g}}{1 - \bar{P}_{pg} \frac{\partial I_p}{\partial E_p}} = \bar{e}_\omega \frac{K - j C_2 \omega \bar{P}_{pg}}{r_p - \bar{P}_{pg}} \\
 &= \bar{e}_\omega \frac{(K + C_2 \omega P_{pg}'') - j C_2 \omega P_{pg}'}{r_p - P_{pg}' - j P_{pg}''} \\
 &= \bar{e}_\omega \left\{ \frac{(K + C_2 \omega P_{pg}'')(r_p - P_{pg}') + C_2 \omega P_{pg}' P_{pg}''}{(r_p - P_{pg}')^2 + P_{pg}''^2} \right. \\
 &\quad \left. + j \frac{P_{pg}''(K + C_2 \omega P_{pg}'') - C_2 \omega (r_p - P_{pg}') P_{pg}'}{(r_p - P_{pg}')^2 + P_{pg}''^2} \right\} \\
 &= \bar{e}_\omega \frac{[K(r_p - P_{pg}') + r_p C_2 \omega P_{pg}''] + j[KP_{pg}'' + C_2 \omega P_{pg}^2 - r_p C_2 \omega P_{pg}']}{(r_p - P_{pg}')^2 + P_{pg}''^2}.
 \end{aligned}$$

In our case

$$\bar{e}_\omega = A (\cos \omega t + j \sin \omega t).$$

Therefore

$$\begin{aligned}
 b_1 &= A \frac{K(r_p - P_{pg}') + r_p C_2 \omega P_{pg}''}{(r_p - P_{pg}')^2 + P_{pg}''^2}, \\
 a_1 &= A \frac{r_p C_2 \omega P_{pg}' - C_2 \omega P_{pg}^2 - K P_{pg}''}{(r_p - P_{pg}')^2 + P_{pg}''^2}.
 \end{aligned}$$

Also

$$\begin{aligned}
 a_1^2 + b_1^2 &= A^2 \frac{(K + C_2 \omega P_{pw}'')^2 + C_2^2 \omega^2 P_{pw}^2}{(r_p - P_{pw}')^2 + P_{pw}''^2}, \\
 A(b_1 P_{pw}' + a_1 P_{pw}'') \\
 &= A^2 \frac{-KP_{pw}^2 + Kr_p P_{pw}' + 2r_p C_2 \omega P_{pw}' P_{pw}'' - C_2 \omega P_{pw}'' P_{pw}^2}{(r_p - P_{pw}')^2 + P_{pw}''^2} \\
 &= A^2 \left[-C_2 \omega P_{pw}'' + \frac{r_p^2 C_2 \omega P_{pw}'' - KP_{pw}^2 + Kr_p P_{pw}'}{(r_p - P_{pw}')^2 + P_{pw}''^2} \right].
 \end{aligned}$$

We thus derive from (5.5) and (5.5)' that

$$R_{gg} = \frac{A^2}{4}, \quad R_{gp} = \frac{A^2 r_p^2 C_2 \omega P_{pw}'' - KP_{pw}^2 + Kr_p P_{pw}'}{2(r_p - P_{pw}')^2 + P_{pw}''^2},$$

because $P_{pw}' = C_2 \omega P_{pw}''$,

$$\begin{aligned}
 R_{pp} &= \frac{A^2}{4} C_2^2 \omega^2 P_{pw}^2 + \frac{A^2}{4} P_{pw}^2 \frac{(K + C_2 \omega P_{pw}'')^2 + C_2^2 \omega^2 P_{pw}^2}{(r_p - P_{pw}')^2 + P_{pw}''^2} \\
 (5.6) \quad &+ \frac{A^2}{2} C_2 \omega P_{pw}^2 \frac{r_p C_2 \omega P_{pw}' - C_2 \omega P_{pw}^2 - KP_{pw}''}{(r_p - P_{pw}')^2 + P_{pw}''^2} \\
 &= \frac{A^2}{4} P_{pw}^2 \frac{K^2 + r_p^2 C_2^2 \omega^2}{(r_p - P_{pw}')^2 + P_{pw}''^2}.
 \end{aligned}$$

Now from (5.4) and (5.3) it follows that

$$b_0 = \alpha_{pp} R_{pp} + \alpha_{pg} R_{pg} + \alpha_{gg} R_{gg},$$

where

$$\Delta' \alpha_{pp} = \frac{\partial^2 I_p}{\partial E_p^2} \lambda + \frac{\partial^2 I_g}{\partial E_p^2} \mu, \quad \text{where} \quad \lambda = 1 - P_{g0} \frac{\partial I_g}{\partial E_p} - G_{g0} \frac{\partial I_g}{\partial E_g},$$

$$\Delta' \alpha_{pg} = \frac{\partial^2 I_p}{\partial E_p \partial E_g} \lambda + \frac{\partial^2 I_g}{\partial E_p \partial E_g} \mu, \quad \mu = P_{g0} \frac{\partial I_p}{\partial E_p} + G_{g0} \frac{\partial I_p}{\partial E_g},$$

$$\Delta' \alpha_{gg} = \frac{\partial^2 I_p}{\partial E_g^2} \lambda + \frac{\partial^2 I_g}{\partial E_g^2} \mu,$$

and

$$\begin{aligned}
 \Delta' &= \left(1 - P_{p0} \frac{\partial I_p}{\partial E_p} - G_{p0} \frac{\partial I_p}{\partial E_g} \right) \left(1 - P_{g0} \frac{\partial I_g}{\partial E_p} - G_{g0} \frac{\partial I_g}{\partial E_g} \right) \\
 &\quad - \left(P_{g0} \frac{\partial I_p}{\partial E_p} + G_{g0} \frac{\partial I_p}{\partial E_g} \right) \left(P_{p0} \frac{\partial I_p}{\partial E_p} + G_{p0} \frac{\partial I_g}{\partial E_p} \right).
 \end{aligned}$$

Using the values of R_{pp} , R_{pg} , R_{gg} obtained in (5.6) we have

$$\begin{aligned}
 b_0 &= \alpha_{gg} \frac{A^2}{4} + \alpha_{pp} \frac{A^2}{2} \frac{r_p^2 C_2 \omega P_{pw}'' - KP_{pw}^2 + Kr_p P_{pw}'}{(r_p - P_{pw}')^2 + P_{pw}''^2} \\
 (5.7) \quad &+ \alpha_{pp} \frac{A^2}{4} P_{pw}^2 \frac{K^2 + r_p^2 C_2^2 \omega^2}{(r_p - P_{pw}')^2 + P_{pw}''^2}.
 \end{aligned}$$

This expression shows the nature of the dependence of the rectified component of the plate current on the constants of the plate circuit. In fact the quantity

$$\bar{P}_{pw} = - \frac{\bar{Z}_{pw}}{1 + jc_2 \omega \bar{Z}_{pw}} = - \frac{1}{\frac{1}{\bar{Z}_{pw}} + (jc_2 \omega)^{-1}}$$

can be seen to be the negative of the impedance which would be obtained by connecting the capacity C_2 across the plate circuit impedance $\bar{Z}_{pw}$.

If P_{pw} is very large the predominating terms in (5.7) are

$$\alpha_{gg} \frac{A^2}{4} - \frac{K\alpha_{gp}}{2} A^2 + \alpha_{pp} \frac{A^2}{4} (K^2 + r_p^2 C_2^2 \omega^2).$$

It is seen, therefore, that in general there is no reason why b_0 should become zero when $P_{pw} = \infty$. It is also seen that on account of the term $r_p^2 C_2^2 \omega^2$ multiplying into α_{pp} even if it were possible to make $b_0 = 0$ by $P_{pw} = \infty$ for one value of ω this in general would not be true for a different ω . The condition which is necessary and sufficient to make $b_0 = 0$ when $P_{pw} = \infty$ for the case when $r_p^2 C_2^2 \omega^2$ is negligible in comparison to K^2 is $\alpha_{gg} - 2K\alpha_{gp} + \alpha_{pp}K^2 = 0$.

CASE OF NEGATIVE GRID VOLTAGE.

It is of interest to see how our equations reduce in the case of negative grid voltage when $I_g = \text{const.} = 0$. In this case

$$\Delta' = 1 + \frac{Z_{p0}}{r_p}, \quad \alpha_{pp} = \frac{\frac{\partial^2 I_p}{\partial E_p^2}}{1 + \frac{Z_{p0}}{r_p}}, \quad \alpha_{pg} = \frac{\frac{\partial^2 I_p}{\partial E_p \partial E_g}}{1 + \frac{Z_{p0}}{r_p}}, \quad \alpha_{gg} = \frac{\frac{\partial^2 I_p}{\partial E_g^2}}{1 + \frac{Z_{p0}}{r_p}}.$$

Hence

$$\left(1 + \frac{Z_{p0}}{r_p}\right) b_0 = \frac{A^2}{4} \left[\frac{\partial^2 I_p}{\partial E_g^2} + 2 \frac{\partial^2 I_p}{\partial E_p \partial E_g} \frac{r_p^2 C_2 \omega P_{pw}'' - K P_{pw}'^2 + K r_p P_{pw}'}{(r_p - P_{pw}')^2 + P_{pw}''^2} + \frac{\partial^2 I_p}{\partial E_p^2} \frac{P_{pw}^2 (K^2 + r_p^2 C_2^2 \omega^2)}{(r_p - P_{pw}')^2 + P_{pw}''^2} \right].$$

It is convenient to write $\bar{Z}_w = -\bar{P}_{pw} = R_w + jX_w$; R_w, X_w being both real. Then

$$\left(1 + \frac{Z_{p0}}{r_p}\right) b_0 = \frac{A^2}{4} \left[\frac{\partial^2 I_p}{\partial E_g^2} - 2 \frac{\partial^2 I_p}{\partial E_p \partial E_g} \frac{r_p^2 C_2 \omega X_w + K Z_w^2 + K r_p R_w}{(r_p + R_w)^2 + X_w^2} + \frac{\partial^2 I_p}{\partial E_p^2} \frac{Z_w^2 (K^2 + r_p^2 C_2^2 \omega^2)}{(r_p + R_w)^2 + X_w^2} \right].$$

Similarly of course (5.7) could be written

$$(5.7)' \quad b_0 = \frac{A^2}{4} \left[\alpha_{gg} - 2\alpha_{gp} \frac{r_p^2 C_2 \omega X_w + K Z_w^2 + K r_p R_w}{(r_p + R_w)^2 + X_w^2} + \alpha_{pp} \frac{Z_w^2 (K^2 + r_p^2 C_2^2 \omega^2)}{(r_p + R_w)^2 + X_w^2} \right].$$

It is of interest to investigate the conditions for which b_0 can be made equal to zero or else to reverse sign by a proper choice of R_w and X_w . Clearing fractions

$$b_0 = \frac{A^2}{4} \frac{(R_w^2 + X_w^2)(\alpha_{gg} - 2K\alpha_{gp} + \alpha_{pp}(K^2 + r_p^2 C_2^2 \omega^2)) + 2r_p R_w(\alpha_{gg} - K\alpha_{gp}) - 2\alpha_{gp} r_p^2 C_2 \omega X_w + r_p^2 \alpha_{gp}}{(r_p + R_w)^2 + X_w^2}.$$

If $\alpha_{gg} - 2K\alpha_{gp} + \alpha_{pp}(K^2 + r_p^2 C_2^2 \omega^2) \neq 0$ the denominator cannot become infinite without making the numerator infinite and of the same order. Hence excluding the case mentioned the condition is that the equation

$$(R_w^2 + X_w^2)(\alpha_{gg} - 2K\alpha_{gp} + \alpha_{pp}(K^2 + r_p^2 C_2^2 \omega^2)) + 2r_p(\alpha_{gg} - K\alpha_{gp})R_w - 2\alpha_{gp} r_p^2 C_2 \omega X_w + r_p^2 \alpha_{gp} = 0 \quad (6.0)$$

be satisfied by a possible pair of values of R_w and X_w . The equation written is the equation of a circle in Cartesian coördinates. This circle is real if

$$-r_p^2 \alpha_{gp}(\alpha_{gg} - 2K\alpha_{gp} + \alpha_{pp}(K^2 + r_p^2 C_2^2 \omega^2)) + r_p^2(\alpha_{gg} - K\alpha_{gp})^2 + \alpha_{gp}^2 r_p^4 C_2^2 \omega^2 > 0 \text{ or } \alpha_{gp}^2 > \alpha_{gg} \alpha_{pp}. \quad (6.1)$$

This therefore is the condition which is necessary in order that it be possible to reverse the sign of b_0 by merely changing the plate circuit. If the grid voltage is negative this condition becomes

$$\left(\frac{\partial^2 I_p}{\partial E_p \partial E_g} \right)^2 > \frac{\partial^2 I_p}{\partial E_g^2} \frac{\partial^2 I_p}{\partial E_p^2}.$$

It is remarkable that for tubes obeying Van der Bijl's relation

$$\left(\frac{\partial^2 I_p}{\partial E_p \partial E_g} \right)^2 = \frac{\partial^2 I_p}{\partial E_g^2} \cdot \frac{\partial^2 I_p}{\partial E_p^2}.$$

This corresponds to the case when circle (6.0) collapses to a point. Thus b_0 can always be reduced to zero but cannot be made to reverse sign. As a matter of fact the circle (6.0) reduces in this case to

$$R_w^2 + \left(X_w - \frac{K}{C_2 \omega} \right)^2 = 0.$$

Thus $b_0 = 0$ when $R_w = 0$,

$$X_w = \frac{K}{C_2 \omega}.$$

This explains why for very high values of R_w , b_0 has been observed to be diminished but not quite to zero.

Proceeding now with the special case of tubes obeying Van der Bijl's relation we have

$$b_0 = \frac{A^2 \frac{\partial^2 I_p}{\partial E_p^2} \left[R_w^2 + \left(X_w - \frac{K}{C_2 \omega} \right)^2 \right]}{4 \left(1 + \frac{Z_{p0}}{r_p} \right) [(r_p + R_w)^2 + X_w^2]}.$$

In particular if C_2 is very small

$$R_w = R_{pw}, \quad X_w = X_{pw}$$

and

$$b_0 = \frac{A^2 \frac{\partial^2 I_p}{\partial E_p^2}}{4 \left(1 + \frac{Z_{p0}}{r_p} \right) [(r_p + R_{pw})^2 + X_{pw}^2]}.$$

JOHNS HOPKINS UNIVERSITY,

March, 1920.

THE DETECTING EFFICIENCY OF THE SINGLE ELECTRON TUBE.

BY E. O. HULBURT AND G. BREIT.

SYNOPSIS.

Definition of Detecting Efficiency.—As a definition of the detecting efficiency $\lim \frac{A^2}{b_0}$ is taken. Here b_0 is the average change in the rectified component of the output plate current and A is the amplitude of the input grid potential.

Derivation of Theoretical Formula.—A theoretical formula is derived for the detecting efficiency. (See formula 12).

Measurement of Detecting Efficiency.—Measurements of detecting efficiency were carried out by means of a condenser potential-divider and a sensitive quadrant electrometer.

Verification of Theory by Experiment.—By the measurements performed the theory has been verified qualitatively. An attempt to verify the theory quantitatively showed the necessity of taking into account the capacities between the tube elements.

Application to Armstrong's Tuned Plate Circuit.—The sudden drop in signal strength observed with Armstrong's circuit has been explained by the above theory.

I. INTRODUCTORY.

IN a previous paper by the authors,¹ it has been pointed out that the detecting efficiency means, in general words, the efficiency of an amplifier to render a weak signal intelligible. It was shown that the detecting efficiency depended upon the relation between the input grid potential and the resulting output plate current. In the case of the high frequency transformer-coupled amplifier the detecting efficiency was defined by the relation $\lim_{A \rightarrow 0} \frac{b_0}{A^2}$ where A and b_0 are the amplitudes, respectively, of the input grid potential, and of the rectified component of the output plate current. Experimental methods of measuring b_0 and A were devised, and the detecting efficiency was thus determined by direct measurement. In order to arrive at a complete understanding of the detecting efficiency, it is necessary to know how the detecting efficiency depends upon the characteristics of the tubes and upon the constants of the circuits of the amplifier. The present paper takes up this phase of the problem for a simple case and describes an investigation, both theoretical and experimental, of the detecting efficiency of a single electron tube. A theoretical analysis, in which certain simplifying

¹ "The Detecting Efficiency of the Electron Tube Amplifier." (As yet unpublished.)

assumptions were made, was carried out which yielded an expression for the detecting efficiency of the tube in terms of the constants of the tube and the circuits. Experiments were then performed to test the theory.

2. THEORETICAL.

In the current literature on electron tubes Van der Bijl's¹ relation is usually taken as the starting point in the derivation of the concept of the internal resistance of the tube. This procedure is unnecessarily special, for the behavior of a tube may be represented by an amplification factor and an internal resistance even though it does not conform to Van der Bijl's relation.

The following treatment considers only the case of negative grid voltage *i.e.*, $E_g < 0$. In this case the grid current is zero. A more general treatment of the problem will be taken up in a future paper. It is assumed that the plate current I_p is a function only of the plate voltage E_p and grid voltage E_g . We write

$$I_p = f(E_p, E_g). \quad (1)$$

It is assumed that this function within the range considered has first, second and third derivatives, and that the function and its derivatives are finite, single-valued and continuous. It is necessary to specify accurately the conditions just stipulated because there are cases for which these conditions do not hold. For example, the recent experiments of R. Whittington² show clearly that there are resonance effects of the positive ions within the tube. For frequencies close to these resonant frequencies it is not legitimate to say that I_p is a single-valued function of E_p and E_g only, because it depends on the time rate of change of these quantities. But these frequencies are so high that at the ordinary frequencies used in radio one is not troubled by the complex effects observed by Whittington.

Denoting by Δx the change in any quantity x , the expansion of (1) is written

$$\begin{aligned} \Delta I_p &= I_p(E_{p_0} + \Delta E_p, E_{g_0} + \Delta E_g) - I_p(E_{p_0}, E_{g_0}) \\ &= \frac{\partial I_p}{\partial E_g} \Delta E_g + \frac{\partial I_p}{\partial E_p} \Delta E_p + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_g^2} (\Delta E_g)^2 + \frac{\partial^2 I_p}{\partial E_g \partial E_p} \Delta E_g \Delta E_p \\ &\quad + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_p^2} (\Delta E_p)^2 + \dots \quad (2) \end{aligned}$$

E_{p_0} and E_{g_0} are the values of the plate and grid voltages, respectively,

¹ Proc. Inst. Radio Eng., 7, 97, 1919.

² Radio Review, 1, 53, 1919.

when there is no high frequency voltage impressed on the grid. The values of the derivatives in (2) are to be evaluated for $E_p = E_p$ and $E_g = E_g$.

It is assumed that we deal with small values of ΔE_g and ΔE_p , so that the differentials in (2) of higher order than the second can be neglected. It is also assumed that the internal capacities of the tube are negligible.

Let us assume that the grid voltage varies with the time according to a sine law. So that

$$\Delta E_g = A \cos \omega t. \quad (3)$$

Then after initial conditions have been obliterated, the plate current can be expanded as a Fourier series in ωt .

$$\Delta I_p = b_0 + b_1 \cos \omega t + b_2 \cos 2\omega t + \dots + a_1 \sin \omega t + a_2 \sin 2\omega t + \dots \quad (4)$$

It is seen that the constant term b_0 means physically the rectified component of the change in the plate current.

We denote the absolute value of the plate circuit impedance for the frequency $m(\omega/2\pi)$ by Z_m , the resistance of that impedance by R_m , and the reactance by X_m . Then from (4)

$$\begin{aligned} -\Delta E_p = & Z_0 b_0 + Z_1 b_1 \cos \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + Z_2 b_2 \cos \left(2\omega t + \tan^{-1} \frac{X_2}{R_2} \right) \\ & + Z_1 a_1 \sin \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + Z_2 a_2 \sin \left(2\omega t + \tan^{-1} \frac{X_2}{R_2} \right). \end{aligned} \quad (5)$$

Setting (3) and (5) in (2) gives

$$\begin{aligned} \Delta I_p = & -\frac{\partial I_p}{\partial E_p} \left\{ Z_0 b_0 + Z_1 b_1 \cos \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + \dots \right. \\ & \left. + Z_1 a_1 \sin \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + \dots \right\} + \frac{\partial I_p}{\partial E_p} A \cos \omega t \\ & + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_p^2} \left\{ Z_0 b_0 + Z_1 b_1 \cos \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + \dots \right. \\ & \left. + Z_1 a_1 \sin \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + \dots \right\}^2 + \frac{1}{2} \frac{\partial^2 I_p}{\partial E_g^2} A^2 \cos^2 \omega t \\ & - \frac{\partial^2 I_p}{\partial E_p \partial E_g} A \cos \omega t \left\{ Z_0 b_0 + Z_1 b_1 \cos \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + \dots \right. \\ & \left. + Z_1 a_1 \sin \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + \dots \right\} \\ & + \text{etc.} \end{aligned} \quad (6)$$

The values of $a_0, a_1, a_2 \dots, b_0, b_1, b_2 \dots$, are obtained by identifying

the terms of (4) and (6). In so doing we make use of the method of successive approximations and neglect higher orders than the second.

Equating the constant terms we find

$$b_0 = \frac{\frac{Z_1^2}{4} \frac{\partial^2 I_p}{\partial E_p^2} (a_1^2 + b_1^2) + \frac{A^2}{4} \frac{\partial^2 I_p}{\partial E_g^2} - \frac{A}{2} \frac{\partial^2 I_p}{\partial E_p \partial E_g} (b_1 R_1 + a_1 X_1)}{1 + Z_0 \frac{\partial I_p}{\partial E_p}}. \quad (7)$$

To find a_1 and b_1 we identify the coefficients of $\cos \omega t$ and $\sin \omega t$ in (4) and (6) neglecting quantities of higher order than the first. This can be done conveniently by observing that the equations involved are all linear, the terms of (4) being $b_1 \cos \omega t + a_1 \sin \omega t$, and the terms of (6) being

$$-\frac{\partial I_p}{\partial E_p} \left[Z_1 b_1 \cos \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) + Z_1 a_1 \sin \left(\omega t + \tan^{-1} \frac{X_1}{R_1} \right) \right] + A \cos \omega t \cdot \left. \frac{\partial I_p}{\partial E_g} \right|.$$

We now introduce the symbol $\bar{e} = A e^{i\omega t}$, where $i = \sqrt{-1}$, and e is the base of natural logarithms. Let $(b_1 \cos \omega t + a_1 \sin \omega t)$ be the real part of $I_p' e^{i\omega t}$. It is seen that I_p' means the fundamental radio frequency component of the plate current.

Then

$$I_p' = -\frac{\partial I_p}{\partial E} \bar{Z}_1 I_p' + \frac{\partial I_p}{\partial E_g} \bar{e},$$

where

$$\bar{Z}_1 = Z_1 e^{i \tan^{-1} (X_1/R_1)}.$$

Therefore

$$I_p' = \frac{k \bar{e}}{\bar{Z}_1 + r_p},$$

where

$$\left. \begin{aligned} r_p &= \frac{1}{\frac{\partial I_p}{\partial E_g}} \\ k &= \frac{\frac{\partial I_p}{\partial E_g}}{\frac{\partial I_p}{\partial E_p}} \end{aligned} \right\} \quad (8)$$

The expression (8) shows the relation between the internal resistance of the tube r_p and $\partial I_p / \partial E_p$. It also gives a general meaning to the amplification factor k . It is to be noted that no use of Van der Bijl's relation has been made in the above derivation.

Further, expanding (8) we have

$$I_p' = \frac{Ak[(r_p + R_1) - iX_1][\cos \omega t + i \sin \omega t]}{(r_p + R_1)^2 + X_1^2}. \quad (9)$$

The real part of this is

$$\frac{Ak}{(r_p + R_1)^2 + X_1^2} [(r_p + R_1) \cos \omega t + X_1 \sin \omega t]. \quad (10)$$

Comparing (4) and (10) we see that

$$b_1 = \frac{Ak(r_p + R_1)}{(r_p + R_1)^2 + X_1^2} \quad \text{and} \quad a_1 = \frac{AkX_1}{(r_p + R_1)^2 + X_1^2}.$$

Substituting these values in (7) gives finally the expression for b_0/A^2 , which is the detecting efficiency for an amplitude A at frequency $\omega/2\pi$. It results that

$$\frac{b_0}{A^2} = \frac{(R_1^2 + X_1^2)k^2 \frac{\partial^2 I_p}{\partial E_g^2} + [(R_1 + r_p)^2 + X_1^2] \frac{\partial^2 I_p}{\partial E_g^2} - 2k[R_1(r_p + R_1) + X_1^2] \frac{\partial^2 I_p}{\partial E_p \partial E_g}}{4 \left(1 + \frac{Z_0}{r_p}\right) [(r_p + R_1)^2 + X_1^2]}. \quad (11)$$

If the tube obeys Van der Bijl's relation

$$k^2 \frac{\partial^2 I_p}{\partial E_g^2} = k \frac{\partial^2 I_p}{\partial E_p \partial E_g} = \frac{\partial^2 I_p}{\partial E_g^2}.$$

In this case (11) becomes

$$\frac{b_0}{A^2} = \frac{r_p^2 \frac{\partial^2 I_p}{\partial E_g^2}}{4 \left(1 + \frac{Z_0}{r_p}\right) [(r_p + R_1)^2 + X_1^2]}. \quad (12)$$

At the risk of repetition let us state again the meaning of the symbols used in (12). Z_0 , R_1 , X_1 are the direct current resistance, the high frequency resistance, and the reactance of the plate circuit, respectively. r_p is the internal resistance of the tube. r_p and the derivative $\partial^2 I_p / \partial E_g^2$ are obtained from the static characteristics of the tube.

Formula (12) depicts the manner in which the detecting efficiency depends on the constants of the circuits and of the tube. In particular it states that b_0 is proportional to A^2 for a specified frequency. This has been shown to be approximately true by direct experiment. (See the curves of the paper by the authors referred to in Section I.) It remained for further experiment to determine to what extent the assumptions underlying the derivation of formula (12) were permissible. This is taken up in the following sections.

3. APPARATUS.

The apparatus was arranged as shown schematically in Fig. 1. The condenser potential divider consisting of the three condensers C_1 , C_2 , C_3 the inductance coil L' and the thermogalvanometer T , were the same as that described in a previous paper by the authors (*loc. cit.*). By coupling L' to a suitable generating set high frequency voltage of small known amplitude and frequency was impressed on the grid of the electron tube. The device for measuring the change in the rectified component

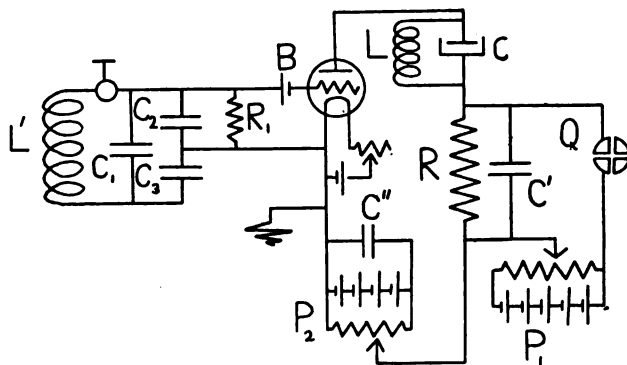


Fig. 1.

of the plate current consisted of a quadrant electrometer Q connected across a resistance R , the potential drop across R due to the plate battery P_2 being compensated by a potential divider P_1 . This has also been described in the earlier paper (*loc. cit.*). The resistance R was a non-inductive resistance made of a dilute solution of copper sulphate in water. This was shunted by a variable condenser C' . C'' was a large fixed paper condenser having a low frequency capacity of about $2\mu F$.

In the plate circuit was inserted an inductance coil L shunted by a variable condenser C . The inductance of L was $1738\mu h$, its distributed capacity about $16\mu\mu F$, and its high frequency resistance at 1150 meters was 6 ohms. The condenser C was a General Radio Company Air Variable Condenser, Type 182 A. This had a shield which was always connected to the electrometer as shown in Fig. 1. The electron tube was a Western Electric Company tube, Type V T I. The tube was used always with a filament current of 1.10 amp. and a plate voltage of 22.0 volts. The negative terminal of a standard cell B , Fig. 1, was always connected to the grid. A high resistance leak R_1 (about 2 megohms) kept the grid potential at a definite value.

4. EXPERIMENTAL TESTS.

It is seen that when R_1 and X_1 in (12) become sufficiently large b_0/A^2 becomes very small. This was verified in the following manner: Keeping the current through T constant, *i.e.*, A constant, the condenser C was varied and the electrometer deflections were observed. Curves 1 and 2, Fig. 2, show the electrometer deflection in millimeters plotted

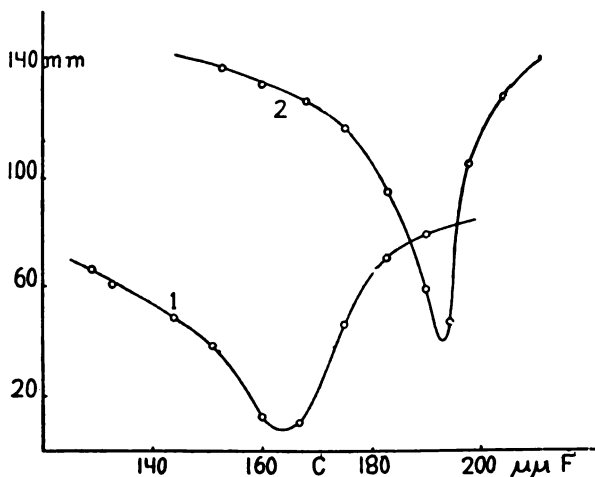


Fig. 2.

against the capacity of C in $\mu\mu F$. Curve I was taken for smaller deflections than Curve 2. Also in curve I the resistance R was about 21,000 ohms, while in Curve 2 R consisted of 421,000 ohms in series with a pair of telephones. The condenser C' was disconnected in the case of Curve 2. It is seen that in both cases the deflection dropped down for a proper value of capacity.

The explanation of these curves lies in the fact that R_1 and X_1 become very large for certain values of C . This can be seen from the expressions for the effective resistance and effective reactance of the parallel combination of an inductance coil having an inductance L and resistance R with a condenser of capacity C . If R_e and X_e are the effective resistance and reactance, respectively, it may be shown that

$$R_e = \frac{R}{R^2 C^2 \omega^2 + (LC\omega^2 - 1)^2}, \quad (13)$$

$$X_e = -\frac{R^2 C \omega + L\omega(LC\omega^2 - 1)^2}{R^2 C^2 \omega^2 + (LC\omega^2 - 1)^2}.$$

The maximum value of R_e (when C is varied) occurs very nearly at

$C = 1/L\omega^2$. The value of R_e for this value of C is $1/RC^2\omega^2 = L^2\omega^2/R$. Using the values obtained by measurement in the case of the present experiment, namely, $R = 6$ ohms, $L = 1.738 \times 10^3 \mu h$, $\omega = 1.63 \times 10^6$,

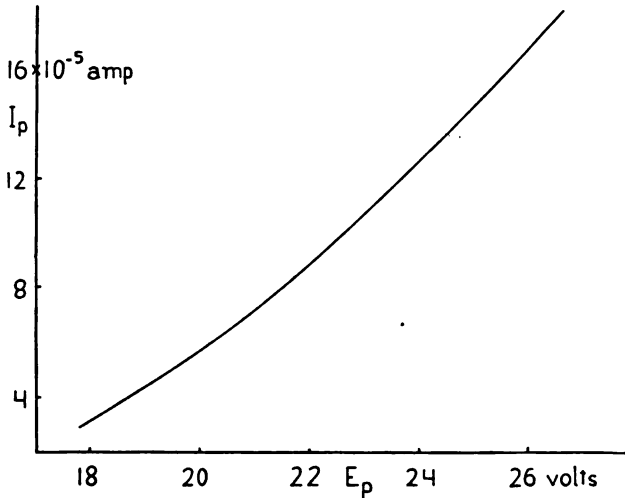


Fig. 3.

the effective resistance is found to be 1.34×10^6 ohms. This is R_1 in the notation of formula (12).

In order to determine r_p , the static characteristic curves of the electron tube were drawn. This was done by measuring the plate current I_p ,

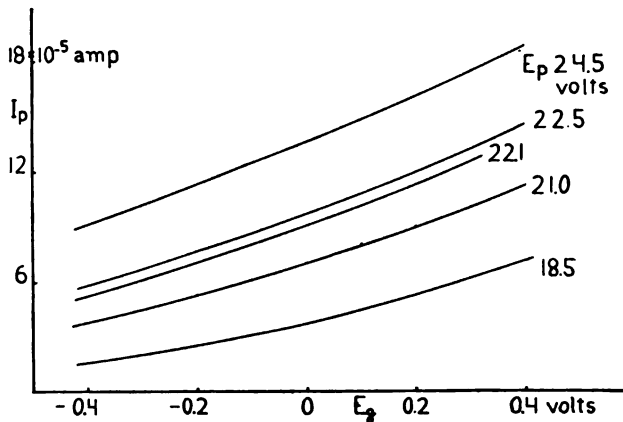


Fig. 4.

for various values of the plate voltage and grid voltage. The curves are shown in Fig. 3 and 4.

From the slope of the curve of Fig. 3 at the operating point $\partial I_p / \partial E_p$ was obtained. In obtaining the operating voltage E_p , account was taken of the potential drop in the plate circuit resistance. The reciprocal of $\partial I_p / \partial E_p$ gives r_p , which was found to be 6.5×10^4 ohms. The high value was due to the use of a small filament current (see section 3).

Returning to the experiment under discussion, R was 2.1×10^4 ohms and when C was set so that the combination of L and C were at resonance for the impressed frequency, we have

$$R_1 = 1.34 \times 10^6 + 2 \times 10^4 = 1.36 \times 10^6 \text{ ohms}$$

and

$$(R_1 + r_p)^2 = 2.01 \times 10^{12}.$$

If now C were to differ from its value at resonance by $20\mu\mu F$ the values of R_s and X_s (as computed from (13)) are 714 ohms and 3.12×10^4 ohms, respectively. Then

$$R_1 = 2.1 \times 10^4 + 0.7 \times 10^4 = 2.2 \times 10^4 \text{ ohms}$$

$$R_1 + r_p = 8.7 \times 10^4 \text{ ohms; } X_1 = X_s = 3.12 \times 10^4 \text{ ohms;}$$

and

$$(R_1 + r_p)^2 + X_1^2 = 8.55 \times 10^9.$$

Hence the electrometer deflection must have decreased in the ratio of at least

$$\frac{2.01 \times 10^{12}}{8.55 \times 10^9} = 2.35.$$

This calculation applies to Curve I, Fig. 2. An inspection of the curve shows at once that quantitatively the simple theory presented above is not borne out by experiment, for the electrometer deflection at the minimum point is about one-eighth of the deflection at $20\mu\mu F$ from the minimum. The discrepancy is to be attributed to the neglect of the effect of the internal capacities of the tube. If these capacities are taken into account there results a formula which gives better agreement with the observations. This will be taken up in a future paper. It was possible to demonstrate experimentally that the larger the capacity between the grid and plate of the tube became the less marked was the effect of making R_1 very large. This was done as follows: A variable air condenser (General Radio Company, Type 101 L) was connected from the grid to the plate of the tube. For a low condenser setting the electrometer deflection decreased perceptibly when R_1 became sufficiently high. When the condenser reached a value of about $500\mu\mu F$ the effect could no longer be observed.

Another cause of error to be considered arises from the assumption of Van der Bijl's relation. In the above case, however, the tube obeyed

Van der Bijl's law closely, as seen from the curves of Fig. 4, and so the discrepancies cannot be attributed to a deviation from Van der Bijl's relation.

There are various other factors which cause discrepancies between formula (12) and the measurements. An effect which is difficult to

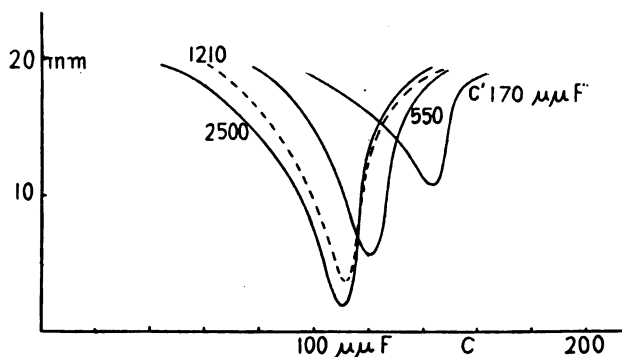


Fig. 5.

take rigorously into account is that introduced by small capacities between the various parts of the circuits. As an illustration of this effect the curves shown in Fig. 5 were drawn. In order to obtain the data for

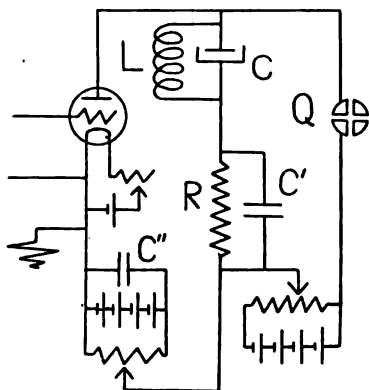


Fig. 6.

this family of curves the connections shown schematically in Fig. 6 were used in the plate circuit. It is seen that these connections differed from those used previously in that the electrometer was connected across all of the resistance in the plate circuit instead of across only a portion.

By giving C' smaller and smaller values the effect of a variation of C becomes less and less, as is seen from the curves of Fig. 5. These curves are plotted with electrometer deflections in mm. (which are proportional to the detecting efficiency) as ordinates against capacities of C in $\mu\mu F$ as abscissas for several values of C' . The grid voltage was -1.018 and the impressed wave-length 1,150 meters. It is noticed that the position of the minimum shifts. The explanation of the family of curves lies in the fact that the quadrant electrometer acted as a condenser. The position of the minimum of a curve occurs when R_1 is a maximum.

R_1 is a maximum approximately when the sum of the resistances of the parallel combination of L and C , and the parallel combination of R and C' , becomes zero. The total resistance of these parts in series depends on the setting of C' because the effective resistance of the parallel combination of R and C' is

$$\frac{R}{1 + R^2 C'^2 \omega^2}.$$

If C' is large the above quantity is small and consequently the total resistance of the plate circuit at its maximum is lessened because that resistance varies inversely with the resistance of all the parts in series.

It was of interest to study the effect with a positive voltage on the grid. With $E_g = +1.51$ volts curves similar to those of Fig. 5 were obtained and are shown in Fig. 7. It is seen that the electrometer

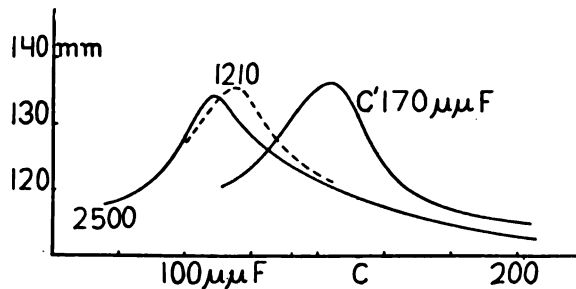


Fig. 7.

deflection, and hence the detecting efficiency, increased in absolute value, when R_1 was increased, instead of decreasing as it did in the case of negative grid voltage. It was observed, however, that the deflection in the case of a positive grid voltage and grid leak was opposite to that in the case of negative grid voltage. Thus, roughly speaking, the phenomenon may be explained by saying that there are two actions going on in the tube. One of these is due to the curvature of the grid voltage grid current curve, the other (the smaller of the two) is the same effect which has been studied with negative grid voltages. The effects on the average plate current due to these two actions are opposite. By making R_1 very large we decrease the second effect and thus increase the absolute value of the deflection. Strictly speaking this way of describing the phenomenon is not entirely correct. It gives, however, an easily memorized picture of the experimental facts.

We wish to point out the practical significance of the effects observed. We refer here to an effect frequently observed in the Armstrong regenerative circuit which to our knowledge has not been explained. In Arm-

strong's circuit the parallel combination of L and C is used just as we used it in our experiments. It is found that as C is increased, starting with small values, the intensity of the sound increases up to a certain point where it suddenly drops. Using the same circuit which we studied above and connecting the tube to an antenna it was found that this point is precisely the point where, with the same plate circuit connections, the detecting efficiency was found to be least. Further, by taking out a sufficient amount of loading inductance in the antenna and then retuning with a larger series capacity the behavior of the circuit was changed altogether. The signal no longer increased with an increase of C but remained fairly constant except in a narrow region where it became very faint. This narrow region coincided with the position of sudden decrease observed before.

JOHNS HOPKINS UNIVERSITY,
March, 1920.

VARIATION WITH PRESSURE OF THE RESIDUAL IONIZATION DUE TO THE PENETRATING RADIATION.

BY K. MELVINA DOWNEY.

SYNOPSIS.

Residual Ionization in Closed Vessels: Experimental Method.—In order to throw light upon the type of radiation responsible for the residual ionization in closed vessels, the author has measured the ionization in a sphere one foot in diameter for pressures ranging up to 21.5 atmospheres. Any corpuscular radiation not absorbed in the ionization sphere has a penetration at one atmosphere equal to or greater than the product of the dimension of the sphere and the number of atmospheres pressure. A special device was used to compensate for any fluctuations of the battery furnishing the high potential required for saturation. This consisted of two ionization spheres of the same electrical capacity which were connected to the opposite ends of a ten megohm resistance, the mid-point of which was earthed. Connections from the battery were made to the ends of the ten megohms. The final experiments were performed over water to eliminate any effect due to radio-active substances in the soil.

Nature of Ionization Curve.—The shape of the ionization-pressure curves gives information as to the process of ionization. A discussion of such curves obtained by other observers has been given in connection with those of the author. The curves obtained in the present experiment give a linear relation. This indicates that the ionization is not due to a soft radiation from the walls of the sphere, but that it is due either to the direct action of a penetrating radiation without any effect produced by secondary β rays from the gas, or to a secondary, corpuscular radiation from the walls of the vessel. If the latter is responsible for the main portion of the ionization, the experiments indicate that it has a penetration of at least six and one half meters in air at atmospheric pressure.

Diurnal Variation.—A few curves obtained during the preliminary observations have been given. Provided that the sky is not cloudy, these indicate the existence of a diurnal variation.

Ionization in Air Due to Gamma Rays of Radium.—The ionization in a sphere one foot in diameter has been measured for pressures ranging up to 40 atmospheres. From four to twenty atmospheres a linear relation existed. The change in slope at twenty atmospheres to a value about six tenths of that at lower pressures may be attributed to some of the β rays having completed their paths. At the high pressures there is a suggestion of an effect due to the secondary radiation from the air.

INTRODUCTION.

IF air or any other gas is enclosed in a metallic vessel, experiments show that ions are produced in the gas at a constant rate even in the absence of any obviously apparent ionizing agent. The number of pairs of ions produced per c.c. per second varies slightly with the material of the vessel but over land is of the general order of magnitude of 10.

A part of this ionization is certainly due to the radiation from radio-active materials in the soil and the radio-active emanation in the air. That this source accounts for only a portion of the ionization is verified by the observations over the great oceans. Although the amount of radio-active material in these oceans and in the air over them is insignificant, yet values of the order of 4 pairs of ions per c.c. per second are obtained in closed vessels over these oceans. Thus, Simpson and Wright obtained values ranging from 4 to 6 ions per c.c. per second over the Atlantic and Indian oceans;¹ and, on the fourth cruise of the yacht *Carnegie*² an average value of 3.8 ions was obtained over the Pacific and sub-antarctic oceans. Again, when the ionization vessel is shielded on all sides by water sufficient in thickness to absorb the known radiation, there is still a residual ionization as was first shown by the experiments of Rutherford and Cooke.³

There are two views prevalent as to the origin of this residual ionization in gases. One attributes it to impurities in the walls of the vessel, and the other to a penetrating radiation having its origin outside the vessel. Much work has been done on this subject, particularly by McLennan and his students. An experiment of special interest is the one in which McLennan used an ionization chamber of ice. When this was placed over Lake Ontario, the ionization value obtained was only 2.6 ions per c.c. per second. McLennan seems to favor the view that even this residual ionization was due to radio-active impurities in the ice, but no test for the radioactivity of this ice-receiver appears to have been made.

Perhaps the most convincing evidence in favor of a true penetrating radiation in the atmosphere and against any explanation founded on the assumption of radio-active contamination is given by the balloon observations of Kleinschmidt and of Kohlhörster. The latter found, for example, that at an altitude of 9,300 meters, the value obtained for the residual ionization was 80.4 ions per c.c. per second.⁴ The value at the surface of the earth would be only about 4 ions per c.c. per second if we neglect the contribution due to the gamma rays from the radio-active material in the soil, which contribution would be quite negligible at an altitude of even 2,000 meters. Kohlhörster's results thus suggest a non-terrestrial radiation, increasing in intensity with altitude.

¹ Roy. Soc. Proc., A, Vol. 85, pp. 196-198, 1911.

² Results of Atmospheric Electric Observations made aboard the *Galilee* (1907-1908) and the *Carnegie* (1909-1916), reprinted from Publication No. 175 (Vol. 3) of the Carnegie Inst. of Washington.

³ Phys. Review, Vol. 16, p. 183, 1903.

⁴ Deutsch. Phys. Gesell. Verb., 16, p. 719, 1904.

In the present work, the primary object was to measure the natural ionization in a closed vessel for pressures ranging up to 20 atmospheres, and to eliminate, as far as possible, the ionization resulting from radioactive materials in the soil by performing experiments over a body of water of sufficient thickness to cut off such radiation. The effects of pressure would then give information as to the nature of the agency responsible for the ionization. If the radiation is of an easily absorbable nature, the ionization will reach a saturation value at a comparatively low pressure; while for a range of pressures extending to even higher values the ionization would continue to increase in the case of the penetrating radiation. That is, the more penetrating the radiation, the higher the pressure for which a saturation value will be obtained. A more detailed discussion of the ionization-pressure relations, in so far as they are influenced by the properties of the radiation producing the ionization, will be given later. Several investigators have examined the variation of residual ionization for pressures below one atmosphere,¹ while Wilson,² and McLennan and Burton³ have made measurements for pressures above one atmosphere. These results will be discussed in connection with the distinctive features of the writer's experiments. It may be noted here that the pressure giving a saturation value of the ionization in any experiment depends upon the linear dimension of the ionization chamber, so that as much information may be given at a low pressure by a vessel of large dimensions as is given at a high pressure by a vessel of correspondingly smaller dimensions. In the course of the work it was desirable to observe also the variation of ionization with pressure when the ionization was caused by the rays from a sealed tube of radium bromide. These experiments, which were carried out up to 40 atmospheres, will be described in connection with the main experiments.

DESCRIPTION OF APPARATUS AND METHOD.

The form of apparatus used was suggested by Professor W. F. G. Swann. The general arrangement of the apparatus will be apparent from the diagram (Fig. 1).

The ionization chambers *A* and *B* are two cast steel spheres each having an internal diameter of one foot and walls one inch in thickness. The central rod electrode of each sphere is joined to a common wire which in turn is connected to the fiber of a string electroscope of the

¹ For example: J. Patterson, *Phil. Mag.*, Ser. 6, Vol. 6, p. 231, 1903, also Kingdom, *Phil. Mag.*, Vol. 32, p. 396, 1916.

² W. Wilson, *Phil. Mag.*, Ser. 6, Vol. 17, p. 216, 1909.

³ *Phys. Review*, Vol. 16, p. 184, 1903.

Lutz-Edelmann type.¹ The spheres are connected to the opposite ends of a ten megohm resistance. The mid-point of this resistance is connected to the earth terminal as are also the guard rings *GG*, the shielding tube *C* and the electroscope case. Connections are made at the end terminals of the ten megohm resistance to the source of high potential

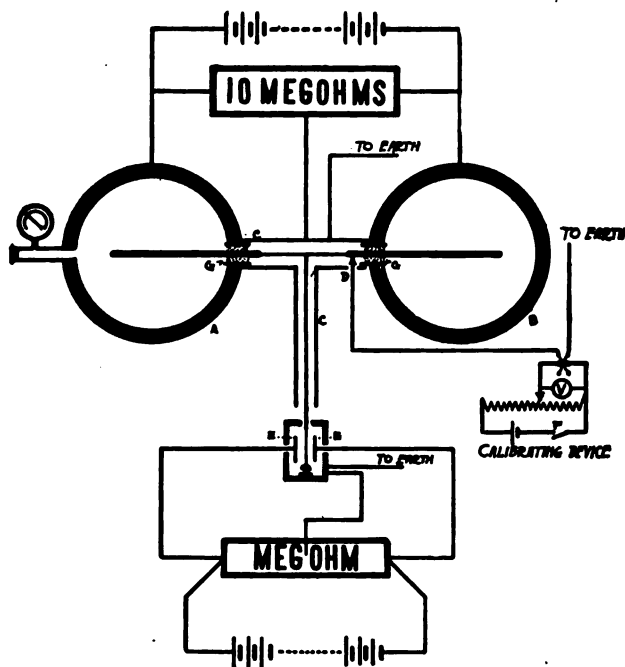


Fig. 1.

(700 volts for the residual ionization.

1500 volts for the ionization due to rays from small sample of Ra Br.)

The use of the two vessels in combination with the ten megohm resistance possesses the advantage of compensating for any effects due to fluctuations of the batteries. These effects might otherwise, be serious on account of the high potentials necessary for saturation. On the other hand, by the use of two spheres with the ten megohm resistance, the mid-point of which is earthed, any alteration of the potential of the battery results in equal and opposite alterations of the potentials of the two ends of the resistance. Then the potentials of the two spheres are altered by equal and opposite amounts and the inductive actions of the two spheres on the central insulated system annul each other. The effect is the same as if the battery were constant. It is also apparent

¹ Lutz, *Physikalische Zeitschrift*, Vol. 13, p. 954, 1912.

that there will be compensation as regards the residual ionization at atmospheric pressure if this is the same for the two spheres; consequently the rate of deflection of the electroscope fiber is determined by the excess ionization resulting from the increase of pressure in the sphere *A* above the atmospheric pressure maintained in sphere *B*.

It will be recalled that the string electroscope of the Lutz-Edelmann type consists of two parallel plates (indicated by *EE* in the diagram) with a conducting fiber stretched between them. The plates are to be maintained at potentials which are equal but of opposite signs. Since it is necessary that these potentials remain in a constant ratio, a plan similar to that given for the spheres was used. The plates, *EE*, are connected by a megohm, the mid-point of which is earthed and the ends of which are connected to the terminals of a 100-volt battery. A practically constant sensitivity of 10 divisions per volt was maintained throughout the work. For convenience a calibrating device consisting of a cell, potentiometer and voltmeter, *V*, was employed as a permanent part of the apparatus. It will be seen from the diagram that the central system was earthed through the voltmeter when the key, *F*, was open and the key, *D*, was closed.

It was obviously necessary that the potential between the spheres and the insulated, central system be sufficiently high to secure "saturation." Tests were always made to insure that there was no additional rate of movement of the fiber due to an increase of the potential beyond that assumed sufficient for saturation in any particular case.

A mixture of resin and beeswax was employed to make the vessel *A* airtight at high pressures. The bolts were tightened while the spheres were sufficiently hot to retain the mixture in a liquid form; and by this means it was possible to make the vessel stand a pressure of 65 atmospheres without appreciable leakage.

The pressure in *A* was raised by connecting a valve at its end to a cylinder which had been filled with compressed air from a liquid air machine, and the pressure was recorded by a "Bourdon" gauge. The air was thoroughly dried by phosphorous pentoxide placed inside the sphere.

For the portion of the observations where a radio-active substance was required, a sample of radium bromide of the order of two milligrams was used. This was enclosed in a lead case, the walls of which were $1\frac{1}{2}$ cm. in thickness. The lead case containing the Ra Br. was placed at a distance of eight feet from the spheres during the observations in which it was used.

Method of Calculation.—The excess rate of production of ions per c.c.

as a result of the increase in pressure in *A* above atmospheric pressure is determined as follows:

Let C = capacity of system,

ΔV = gain in potential of the fiber in e-s units,

e = electronic charge = 4.8×10^{-10} e.s. units,

v = vol. of ionization chamber in c.c.,

q = number of pairs of ions produced per c.c., per sec.

We then have $q = \frac{C\Delta V}{vet}$,

where t is the number of seconds for the fiber to acquire a potential ΔV .

The order of magnitude of the emanation effects from the table, equipment and platform would be so small that they could be disregarded in the calculations.

MEASUREMENTS.

Subsidiary Experiments.

Laboratory Conditions.—In view of the fact that a number of the experiments had to be performed in the laboratory, and since a great many radio-active measurements had been made in the building, it seemed necessary to make a thorough survey of the various laboratories so as to detect any effect due to any possible radio-active contamination of the walls. Furthermore, it was necessary to know whether there was any influence due to the radium known to be in the building. There were 2 mg. of radium in solution on the top floor and there were 3 mg. of shielded radium kept in the vault on the first floor. The laboratory, where the experiments were finally carried out, was situated in the basement.

The ionization chamber for these measurements was a cylindrical vessel having a volume of 6.8 liters. The central rod which was insulated from the vessel was connected to the string electrometer, and a connection was made to the calibrating device in a manner similar to that already given for the main apparatus. The potential (100 volts) which was applied to the vessel caused the ions of the same sign to go to the central electrode; when the connection to earth was released, the fiber would deflect. The time of the deflection of the fiber for a fixed number of divisions was recorded. The sensitivity of the electrometer was maintained at 10 divisions per volt.

The following are representative of the observations:

Electrical Laboratory (top floor), Dec. 26, 1918, values for q : 10.4, 8.75, 9.8, 9.0, 9.7, 10.0 (Mean 9.6).

Room 1 (Basement), Dec. 27, values for q : 9.8, 9.4, 9.2, 8.2, 8.7, 10.0 (Mean 9.2).

Room I (Basement), Jan. 11, q : 10.8, 11.7, 12.6, 12.6 (Mean 11.9).

Room II (Basement), Jan. 11, q : 10, 12.7, 10.8, 11.7 (Mean 11.3).

In view of the fact that the effects due to passing clouds and to the other conditions determining q are such variable quantities at different times, it is evident that there is no systematic difference between the values obtained for q in Room I. and Room II. To make a still more conclusive test, the radium in the vault was removed to the vault in the library building. The values obtained were these:

Jan. 23—Before the removal of the Ra.—12.2, 11.9, 11.9, 11.5, 11.9, 12.6 (Mean 12.0).

Jan. 23—After the removal of the Ra.—12.4, 12.8, 12.4, 12.3, 12.2, 12.9 (Mean 12.5).

Jan. 24—Before the return of the Ra. to the Physics Building—(10:30 to 11:30 A. M.)—13.8, 12.7, 12.4, 13, 12.8, 12.8 (Mean 12.9).

Jan. 24—After the return of the Ra.—(12.15 to 1:00 P. M.)—13.0, 13.7, 12.3, 12.5, 13.2, 13.8 (Mean 13.1).

There was apparently no effect due to the presence of this radium in the vault. This was also verified by experiments of an entirely different nature which were performed by Miss Herrick (a graduate student in this department).¹

Diurnal Variation.—During the course of the preceding observations, a decided variation in the ionization values was noticed at times. For quite a number of consecutive days in January, higher values were obtained in the evening than in the mid-part of the forenoon. Sometimes the difference was as much as 20 per cent.

In the interval from January 24 to February 6, several series of observations were taken, each series extending over a period of 24 hours. When the atmosphere was exceedingly clear, a maximum was obtained in the morning (7 to 9 A.M.) and a more decided maximum from 6:30 to 8:30 P.M. with the values rather high during the first part of the night. When the sky was cloudy, no very obvious diurnal variation was obtained. The curves obtained January 26, February 1 and February 4 are given in Fig. 2. January 26 was an exceptionally clear day and there was no appreciable change of the barometer. February 1 was mostly cloudy. The observations from February 3 to February 5 were begun just as a snow-fall, which had continued for some time, was ceasing. During a part of the observations on February 4 the sky was cloudy.

It would be desirable to continue these observations over a long period of time to test for the existence of the diurnal variation, particularly as there is not a unanimous agreement among investigators as to the

¹ Master's Thesis, University of Minnesota, 1919.

existence of the diurnal variation. The results obtained are in general agreement with the few observations recorded by those who have found a diurnal variation.

Saturation Voltage.—For the penetrating radiation the highest pressure used was not over 22 atmospheres. Tests for the saturation voltage

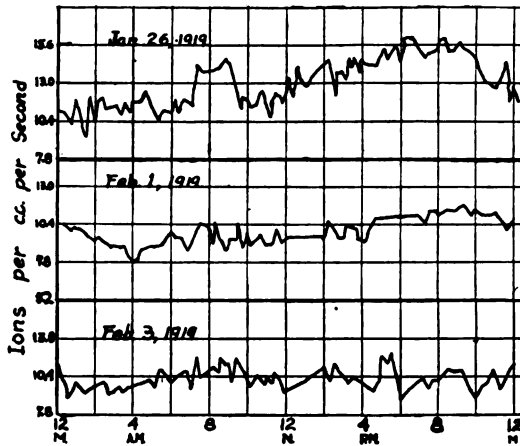


Fig. 2.

at this pressure were made. The potential of the end-terminals of the ten megohm resistance, and the corresponding values of the ionization current are given in Table I.

TABLE I.

Pressure, 22 Atmospheres

Potential.	Ionization in Arbitrary Units.
136 volts	8.28
272 "	8.37
408 "	8.35
544 "	8.33
680 "	8.39

TABLE II.

Pressure, 20 Atmospheres.

Potential.	Ionization in Arbitrary Units.
280 volts	13.69
420 "	14.90
560 "	14.78
700 "	14.74

TABLE III.

Pressure, 40 Atmospheres.

Potential.	Ionization in Arbitrary Units.
300 volts	15.87
600 "	20.00
900 "	22.22
1200 "	22.27
1500 "	22.34

It will be apparent from Table I. that for a voltage of 272 or more the current remained constant to within 1 per cent. The test for saturation voltage for the pressure of 20 atmospheres is given in Table II., which was obtained with the radium-bromide specimen in its position eight feet from the spheres. For the same position of the radium bromide the saturation voltage was also tested for a pressure of 40 atmospheres. The values obtained are given in Table III. It is evident from Tables II. and III., respectively, that 700 volts is amply sufficient for saturation

at 20 atmospheres, and 1500 volts for 40 atmospheres when the Ra Br. specimen is in the position described.

Main Measurements.

General Procedure.—The method of making observations follows the same plan as that already indicated for the preliminary work. With the source of potential connected to the spheres as indicated in the diagram (Fig. 1), and the fiber insulated from earth, the rate of deflection is proportional to the rate of production of ions. The general plan of making measurements will be apparent from the data given for a set of observations taken over the Mississippi river, May 16.

In order to avoid error due to leakage, the fiber is charged in such a way as to cross the zero during the observation and the range of scale divisions for any reading is so chosen that there is an equal number on each side

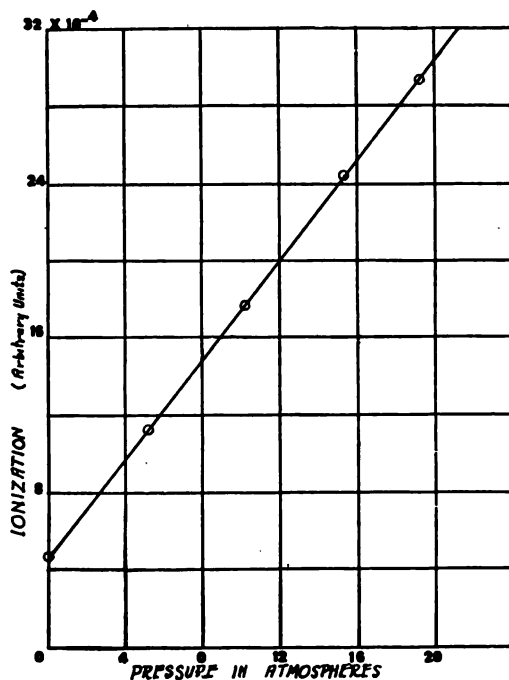


Fig. 3.

of the zero. As soon as the fiber passes the scale reading, θ_1 , the time T_1 is recorded. θ_3 is the position of the fiber at the time T_3 . ΔV is the change in the potential of the fiber in the time $T_3 - T_1$. For the set of readings above, ΔV was always 1 volt. S_1 , the sensitivity of the electrometer before the fiber was released, as well as S_2 , the sensitivity after the fiber was earthed, were recorded. P is the pressure in sphere

A in excess of one atmosphere. In order to test for constancy of conditions, the positions of the fiber were recorded both when the connection of the fiber to the calibrating device was released and when it was restored again at the end of the observation.

Curves Obtained.—The curve for the data of Table IV. is given in

TABLE IV.

θ_1 .	θ_2 .	T_1 .	T_2 .	ΔV in Volts.	S_1 (Divisions per Volt).	S_2 (Divisions per Volt).	P . (Atmos- pheres).	Time (Mid. Pt. of Reading).
			Min. Sec.					
-4	+4	0	5 40.0	1	8	8	19.35	2:10 P.M.
-4	+4	0	6 47.6	1	8	8	15.35	3:00 P.M.
-4	+4	0	9 24.3	1	8	8	10.2	3:50 P.M.
-4	+4	0	14 43.0	1	8	8	5.12	4:45 P.M.
-4	+4	0	35 33.0	1	8	8	0.	5:40 P.M.

Fig. 3. The reciprocals of the times (expressed in seconds) for the fiber to gain a potential of 1 volt. are plotted against the pressure in A in excess of one atmosphere. A linear relation was obtained. To find q ,

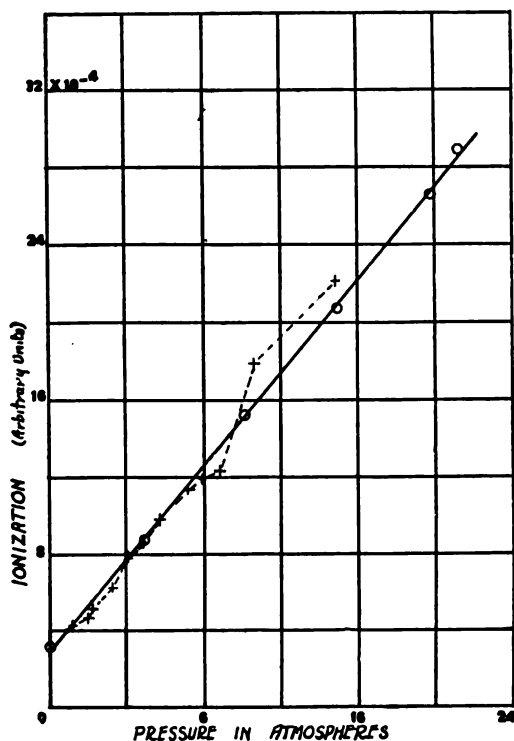


Fig. 4.

the actual numbers of pairs of ions produced per c.c. per second, these ordinates are to be multiplied by $c(1/300)/ve$

(c = capacity of the system,

$\Delta V = 1$ volt = $1/300$ e. s. unit,

v = volume of sphere A in c.c.

e = electronic charge in e. s. units).

The observations for the full line curve given in Fig. 4 were obtained over the river May 17 from 10:40 A.M. to 5:45 P.M.

The location of the apparatus over the water will be apparent from the photograph (Fig. 5). The water underneath the pier was $8\frac{1}{2}$ feet

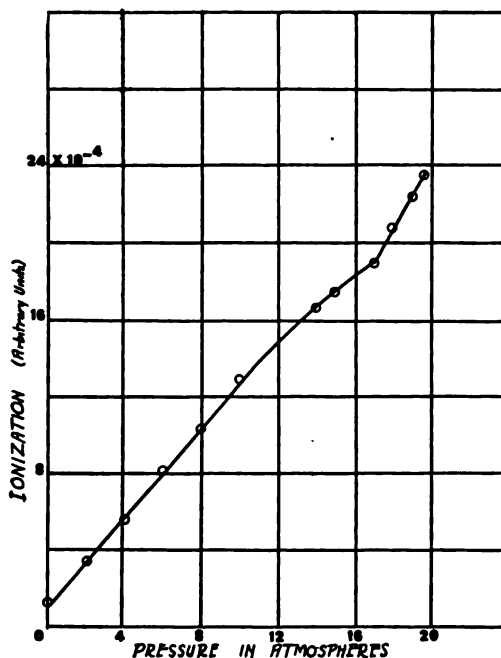


Fig. 6.

in depth. It might also be mentioned here that for the final measurements the electrometer was suspended from a rigid frame by means of springs so as to ensure freedom from the effect of any mechanical disturbances near the pier.

The curve in Fig. 6, which was obtained during the first week of May, is representative of other measurements taken in the laboratory. The

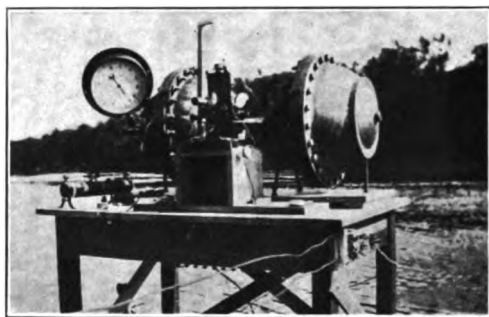


Fig. 5.

K. MELVINA DOWNEY.

measurements for Fig. 7 were made in the laboratory when radium was placed 8 feet from the spheres as previously indicated. The ordinates for Fig. 4, Fig. 6 and Fig. 7 are the same as have been given for Fig. 3.

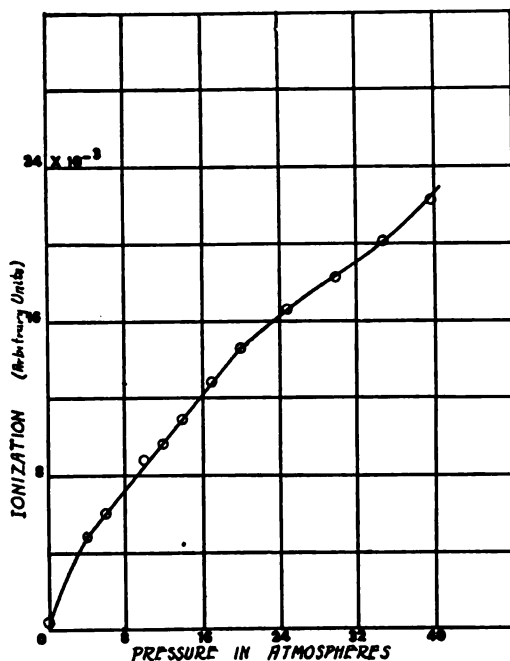


Fig. 7.

DISCUSSION.

Ionization Pressure Relations.—If the residual ionization is due for the most part to radio-active impurities in the walls of the vessel, the radiation may be of the α , β , or β and γ type, or it may be composed of all three types. The main part of this ionization would be due to the α and β rays. The α rays which are the more efficient ionizers would be absorbed by about five cm. of air at atmospheric pressure. While the β rays and the secondary corpuscular rays due to them are more penetrating, practically all the soft β rays and some of the hard rays, which have a range of several meters in air, would be absorbed at higher pressures. Thus, to the extent that the ionization is due to absorbable radiation of this kind, the ionization per gram of gas should decrease with increase of pressure. If one admits the existence of a penetrating radiation partaking of the γ ray nature, the ionization in the closed vessel may be attributed to several sources:

- (1) The direct action of the penetrating radiation in the gas;
- (2) The secondary radiation emitted from the gas by the primary penetrating radiation;
- (3) The secondary radiation emitted from the walls of the vessel by the penetrating radiation.

In addition to these agencies it is of course possible that a portion of the ionization may be due to radio-active impurities. The portion of the ionization under (1) should be proportional to the pressure. Since the number of ions produced by the source (2) varies with the number of molecules of the gas, and with the number of electrons responsible for the ionization, it is apparent that this portion of the ionization would vary proportionally with the square of the pressure. This holds only to the extent that the ionization per cm. of path of the electrons is the same at all parts of the path, and to the extent that the penetration of the electrons enables them to reach the walls before coming to rest. When the pressure is so high that the electrons complete their paths within the vessel, the increase of ionization would be less rapid than the square law would demand. But an influence in the opposite direction is to be found in the increase of ionization per cm. of path toward the end of the range of the electrons. The latter effect would be apparent at pressures below that at which the former would appear. Consequently in regard to (2) it is possible that the curve of ionization against pressure might first show a variation according to the square of the pressure, then a more rapid increase than the square law would indicate, and finally a falling down to a linear relation when the pressure became so high that the penetration of all these ionizing electrons falls within the vessel.

The portion of ionization due to (3) should obey a linear law until the pressures become so high that the more active parts of the electrons' paths fall within the vessel. At such pressures the ionization would show a more rapid increase with pressure. Finally a saturation value should be reached when the whole path is completed within the vessel.

DISCUSSION OF EXPERIMENTAL RESULTS.

A survey of Fig. 3 and Fig. 4 shows that the variation in the natural ionization with pressure obeys a linear law up to 21.5 atmospheres. In view of the preceding discussion, the linear relations given in Figs. 3, 4 and 5 indicate that the ionization within a closed vessel is not due to a soft radiation, but is determined either directly by a penetrating radiation of the γ type or by a hard corpuscular radiation emitted from the walls of the vessel by the penetrating radiation. If it is attributed to

the corpuscular radiation, this radiation itself is then so penetrating that it is not appreciably absorbed in passing through a 1-foot thickness of air at 20 atmospheres pressure. (One foot of air at 20 atmospheres pressure is equivalent in absorbing power to 20 feet of air at one atmosphere.)

It is further apparent that for the range of pressures employed, the contribution to the ionization by secondary rays emitted from the gas by the penetrating radiation is not the primary agency at work, for, if it were, the curve should become convex to the horizontal axis.

That moisture may produce an appreciable effect is shown by the curve of Fig. 6 which has a slight irregularity in the neighborhood of 17 atmospheres. This curve was obtained without any additional drying agent other than that in connection with the compressor of the liquid air machine. For the curves given in Figs. 3, 4 and 7 the drying agent was used as mentioned. For these no such irregularity was noticed. It is probable that there was some water vapor present in the case of (6). If the water vapor in the air from the compressor were as much as one seventeenth saturated, water would begin to condense at about 17 atmospheres and this would result in a change of conditions at this pressure. (Note: The sphere *A* was filled with air at the highest pressure used and the air was allowed to escape very slowly to reduce to the lower pressures.) If one considers the actual numbers of ions given by Fig. 3 and Fig. 4 and compares them with the corresponding quantities for Fig. 6, it will be observed that there is little difference—those in the laboratory were slightly lower. Since the $8\frac{1}{2}$ ft. of water was amply sufficient to cut out practically all the radiation from the soil below in the case of the measurements over the river, it seems evident that the concrete base of the building served the same function as the water. This was likewise indicated by the experiments of Miss Herrick, previously mentioned in this paper. For the work inside there was also the absorbing effect of the roof and the upper floors of the building.

It will be noted that the curves given in Figs. 3, 4 and 6 show a noticeable ionization for zero pressure. This is no doubt due to a soft radiation from the walls of the sphere *A*, in which the pressure was varied. Before the compressed air was admitted into sphere *A*, it was left in a cylinder for a time so as to allow any radium emanation present to decay partially. However since the cylinder containing the air was not allowed to stand the same length of time before each set of observations, there would be different amounts of emanation admitted into the sphere. For the observations at the higher pressures, it was found necessary to wait a time after the pressure was lowered for a constant rate of deflection of

the electrometer. This was due to the fact that it required time for the old decay products to come into equilibrium with the new amount of radium emanation. (A part of the emanation would be removed with the air while the pressure was being lowered.) It may be mentioned here that the radium emanation in air at ordinary atmospheric pressure is not responsible for more than 1.7 ions per c.c. per second.

For the purpose of comparison it was naturally of interest to test the ionization produced due to the effect of γ rays of radium when the pressure within the sphere *A* was varied. What has been given in the discussion of the ionization pressure relations under (1), (2) and (3) may be considered as applying here—the penetrating radiation for the case being the γ rays of radium and the secondary radiation from the walls and gas being rays of the β and γ type.

The curve of Fig. 7 indicates an increase of ionization with pressure. The ionization per gm. of air at pressures below 4 atmospheres is obviously somewhat larger on account of the soft radiation which is then effective. In curve 7 which has an approximately linear relation for the range of pressures from 4 to 20 atmospheres, it is noticed that the slope changes at about 20 atmospheres to a value approximately 0.6 of that for the pressures below 20 atmospheres. This departure from linearity is doubtless due to those β rays which complete their paths within the spheres at this pressure. Twenty atmospheres would be sufficient to absorb β rays having a penetration as great as 6 meters in air in one atmosphere. Those having a longer range would still be unabsorbed as would also those β rays having their paths along the shorter chords of the sphere. The results at the highest pressure indicate a tendency toward a more rapid increase of the ionization with pressure. This suggests an effect due to the secondary radiation from the air. However, it is not possible to attach much importance to this until observations have been made for still higher pressures.

It is desirable to compare the results given in this investigation with those found by other observers. Patterson, who measured the natural ionization in a closed vessel for pressures extending only to 80 cm of mercury, obtained results which indicate a marked departure from linearity over this range. From this Patterson draws the conclusion that the natural ionization is due to radio-active impurity in the vessel. The actual ionization for one atmosphere was as much as 40 ions per c.c. per second, so that without question there was an unusually large amount of radio-active impurity in the walls of the vessel used by Patterson. A large number of the ions he obtained was probably due to the α particles so that he naturally found the effect per gram to diminish

with increase of pressure (on account of the short range of the α particles). In reality the true residual ionization can have formed only a small part of the total effect measured in his experiment. In the writer's experiment results were never considered reliable unless the vessel had been so thoroughly cleaned that the ionization ranged from 10 to 13 or 14 ions per c.c. per second.

Additional results on natural ionization which were obtained by McLennan and Burton indicate an increase of ionization for the range of pressures used (4.4 to 500 c.m.), but the graph was not linear. The slope of the tangent of curve at one atmosphere is about twice that at seven atmospheres. Unfortunately McLennan and Burton have not stated any actual values for " q ." It is therefore impossible to form any opinion as to whether the departure from linearity may have been due to radio-active impurities in the walls of the vessel. However, they do give some evidence of the existence of the radio-active impurities in their experiments.

If the ionization in a closed vessel is primarily due to the corpuscles emitted from the walls of the vessel by the external radiation, departure from linearity is to be expected when the pressure attains a value so high that the linear dimensions of the vessel become comparable with the electronic penetration at that pressure. Thus in comparing the results of the different observers it is to be noted that the departure from linearity is determined not by the pressure, but by the product of the pressure and a quantity of the order of the linear dimensions of the vessel. In other words, experiments in a chamber of average dimensions, 30 cm., give at 20 atmospheres as much information in regard to this phase as experiments made up to 60 atmospheres in a vessel of only 10 cm. linear dimensions.

The results of W. Wilson are of interest in that they indicate an increase of the natural ionization with pressure. While he used pressures as high as 45 atmospheres, the results he obtained can be compared with those of the writer only at those pressures below 15 atmospheres since his ionization chamber had less than $\frac{1}{3}$ the dimensions of that used in the present investigation. The electroscope which recorded the measurements consisted of a vessel 10 cm. cube. This was also used as the ionization chambers. The small sensitivity and large leakage in Wilson's experiments made it impossible for him to obtain results with a high degree of accuracy. The electroscope leaf was maintained at a potential of 200 volts. Since in the writer's experiments the change in potential of the fiber by the natural ionization was not over 1 volt in the time necessary for a reading, it is evident that Wilson's leaf system could not

have registered this with any high degree of accuracy. The writer has plotted in Fig. 4 the values of Wilson for pressures above one atmosphere (these are indicated by means of crosses on the dotted curve). The slope of the linear part of Wilson's curve has been made the same as that of the line obtained by the writer. It will be noted that the departure from linearity of the author's results is never more than 1.30 per cent. of the highest value measured while the corresponding quantity for Wilson's results is as high as 10.50 per cent.

Wilson also obtained an ionization pressure curve for the case where radium-bromide was placed outside the ionization chamber and the pressures were varied up to 40 atmospheres. From what has been said it is clear that this would amount to a range of approximately 13 atmospheres in the writer's experiments. Wilson obtains a maximum at this pressure probably for the reason that he did not have saturation voltage. At atmospheric pressure he works with a value for q amounting to 216 ions per c.c. per second which is almost four times that obtained in the present experiment at atmospheric pressure. It was possible for the writer to work with a weak radio-active specimen on account of the compensating arrangement (the 2 spheres in combination with the ten megohm resistance). Furthermore the radium-bromide was shielded to such an extent that the γ rays reaching the ionization chamber and consequently the secondary corpuscular radiation due to the γ rays were much harder than the radiation which was responsible for the ionization in Wilson's experiments. This would naturally cause Wilson's curve to depart from linearity sooner than the writer's curve.

Additional measurements on the ionization in a closed vessel due to the effect of the γ rays of radium have been made by Kaye and Laby¹ as well as D. C. H. Florance,² but it will be seen that the range of pressures is rather limited when compared with the conditions of the present investigation. These observers worked with ionization vessels the plates of which were not over 2 cm. apart. In the work of Kaye and Laby there was a central electrode about $\frac{1}{2}$ cm. from the sides of the vessel, so that as regards ionization by electrons emitted from the walls of the vessel their curve, which was obtained for a range of pressures up to 17 atmospheres, could be compared with that of the writer only for pressure below one atmosphere.

D. C. H. Florance who used pressures up to 80 atmospheres worked with the plates 1 cm. apart, with an additional reading for a distance of 2 cm. between the plates. For the spheres used in the present experi-

¹ Kaye and Laby, *Phil. Mag.*, Ser. 6, Vol. 16, p. 879, 1908.

² D. C. H. Florance, *Phil. Mag.*, Ser. 6, Vol. 25, p. 172, 1913.

ment, the former condition would correspond to a pressure of 5 atmospheres and the latter to a pressure of 10 atmospheres. The inability of Florance to secure saturation voltage at the higher pressures is no doubt due to the unusually large amount (30 mg.) of radium-bromide used. Without saturation a large change in slope would be expected at the higher pressures.

In conclusion it is a pleasure to acknowledge my gratitude to Professor W. F. G. Swann, who has suggested the problem and directed the investigation. To him I extend my best thanks for the encouragement and advice has has given throughout this work.

I wish to thank Professor H. A. Erikson for his continued interest in the work, and also Mr. C. H. Dane, mechanician, for assistance in the solution of mechanical difficulties.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF MINNESOTA,
December, 1919.

THE KINETIC THEORY OF MAGNETISM.

BY WARREN WEAVER.

SYNOPSIS.

Magnetic Susceptibility of Gases.—The results of a previous paper by Honda and Okuba (Phys. Rev., Vol. 13, 1919, p. 6) are considered, certain important errors noted, and some aspects of the assumptions further examined and developed. The susceptibility is not the same whether or not the gyroscopic nutation of the molecules persists, and the apparent check with Langevin's result is shown to be in error. A gas is shown to be diamagnetic or paramagnetic, on this theory, according as the nutation does or does not persist. This result holds, at least without further investigation, only for smooth spherical molecules rather than as stated. A discussion is given of the effect of thermal impacts on gyroscopic motion of this type.

I. In a recent paper¹ Professors Honda and Okubo have worked out in considerable detail a kinetic theory of magnetism, which takes account of the gyroscopic precessions and nutations which the molecules of a gas will execute under the torque of an applied magnetic field. Some of their results will here be subjected to a critical examination. It will be necessary to duplicate some of their preliminary work so that matters of notation will be clear.

A given molecule is considered to be initially a magnet of moment M , which implies that the distribution of electronic orbits within it is such that the magnetic field produced by the separate electrons is such that they do not completely neutralize each other at exterior points. The change $-\delta M$ in this moment caused by the growth of the external field from a value zero to a final value H is not considered, it being supposedly very small as compared to the other effects considered. The magnetic axis of this molecule makes an initial angle α with the direction of the applied field. Suppose that this molecule is rotating about some axis, not in general coinciding with its magnetic axis. It will then have a component of magnetic moment along the direction of the axis of rotation, and one normal to this direction. Consider first the effect of the former component, assuming that the actions caused by the two components take place independently. The molecule will start to fall towards the direction of the field, the precession beginning as soon as the incipient fall develops, and the axis will perform ordinary precession about the direction of the field, and nutation about the steady precession.

¹ Phys. Rev., Vol. 13, 1919, p. 6.

What will then be the magnetic moment of the gas which is induced by the field H ? The original orientation of magnetic axes was isotropic, else the gas would have been originally magnetic, so that the number of axes which originally made with the direction of H an angle lying between α and $\alpha + d\alpha$ would be

$$dn = \frac{1}{2} \sin \alpha d\alpha. \quad (1)$$

The motion of the axis is such that, as the nutation takes place, the angle between the direction of the axis and H lies between the original value α , as an upper limit, and an angle θ_1 , as a lower limit, where θ_1 is given by the equation

$$\cos \theta_1 = -\frac{1}{a} + \sqrt{1 + \frac{2 \cos \alpha}{a} + \frac{1}{a^2}}, \quad (2)$$

where

$$\frac{a}{2} = \frac{MH}{\frac{1}{2}K\omega^2}, \quad (3)$$

K being the moment of inertia of the molecule about its axis of rotation, and ω the angular velocity of spin about this axis. For small values of a this gives

$$\begin{aligned} \cos \theta_1 = \cos \alpha + \frac{a}{2}(1 - \cos^2 \alpha) + \frac{a^2}{2}(\cos^3 \alpha - \cos \alpha) \\ + \frac{a^3}{4}\left(3 \cos^2 \alpha - \frac{5}{2} \cos^4 \alpha - \frac{1}{2}\right) + \dots \end{aligned} \quad (4)$$

II. The first important point to be noted now appears. The definition of a given by equation (3) differs from that given in the previous paper by a factor 2. The equation was given as

$$a = \frac{MH}{\frac{1}{2}K\omega^2}, \quad (3a)$$

a value which, when substituted in (2) above does not give a correct result. Comparing (3) above with Langevin's definition of his quantity a we find that they are not the same. For clearness let us write Langevin's result

$$I = MH(\coth A - 1/A), \quad (5)$$

in which

$$A = \frac{MH}{rT}, \quad (6)$$

in which r is the gas constant referred to a single molecule. Langevin was considering, in the original paper in which this result appeared,¹ a gas such as oxygen the molecules of which have, as is indicated by the

¹ Ann. de Chim. et de Phys., V., p. 70, 1905.

ratio of the specific heats, two degrees of freedom in rotation. Corresponding to each of these degrees of freedom will be an amount of energy $\frac{1}{2}rT$, or, for the total energy of rotation, rT . Since in the above work the reference axes in the molecule were so chosen that the expression $\frac{1}{2}K\omega^2$ is the whole energy of rotation it is equal to rT . Thus we have

$$2A = a. \quad (7)$$

Thus the values of a and A are not the same, as was stated.

Concerning this point Professor Honda has written me, in answer to a letter sent him before the present paper was written, the following statement. "Our value of ' a ' does not differ from that of Langevin. Your calculation is also quite correct. Our writing in p. 147 (Sci. Rep.)¹ is incorrect, but its content is correct. In considering a magnetic molecule rotating about its axis as a gyroscope acted upon by the force of gravity ($g = H$) we must take OA as the distance between the fixed point and the center of gravity of the equivalent gyroscope, but not AB . Hence in passing from the problem of the gyroscope to the molecular magnet half the pole distance must be taken in consideration. Hence

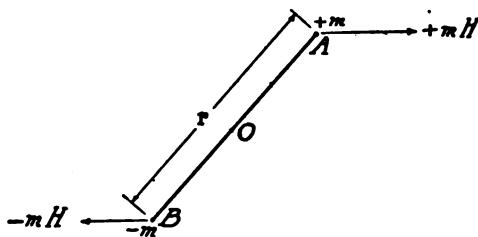


Fig. 1.

in the first equations of gyroscopic motion we must put $M/2$ for M . And consequently

$$a = \frac{4MH}{K\omega^2} \quad \text{becomes} \quad 4 \left(\frac{M}{2} \right) \frac{H}{K\omega^2} = \frac{MH}{\frac{1}{2}K\omega^2},$$

which is Langevin's ' a .'

Insofar as I understand this statement I believe it to be incorrect. The first equation of motion of the gyroscope in which the quantity M appears is an equation which expresses the fact that the sum of the kinetic and potential energies of the gyro-molecule remains constant. The potential energy of a magnetic doublet of this sort of magnetic moment $M (=mr)$ when making an angle θ with a field H is surely

¹ The text of the article in the Science Reports of the Tohoku Imperial University here referred to is the same as that which appeared in the PHYSICAL REVIEW.

— $MH \cos \theta$. The above statement seems to offer no satisfactory justification for replacing M in this expression by $M/2$.

III. In the previous article two cases are considered separately; *i.e.*, that in which the nutation persists, and that in which it does not. A discussion of which of these two descriptions is more likely to fit the actual case is curtailed because the result is there obtained that for small values of a the magnetic moment is the same in the two events. It seems clear on simple and general grounds that this can not be the case. If the precession were to take place at the larger limit of the angle which measures the nutation,—that is at α itself the magnetic moment would obviously be zero, the isotropic character of the orientations not having been changed. If the nutation is damped out in some way, or if, as suggested, the field is applied infinitely slowly so that the precession takes place steadily at the lower limit θ_1 , the magnetic moment is computed to be given by the equation

$$\sigma/\sigma_0 = a/3. \quad (8)$$

Now if the nutation actually takes place the direction of the rotation axis varies between an angle α and an angle θ_1 with the direction of the field, so that one would expect the time mean of the magnetic moment to lie between the two values which correspond to the two extreme positions, namely, between zero and $a/3$. One turns then to the analysis to see where such an error could have arisen. It is found, apparently, in the equation near the bottom of page 13, which should read:

$$k \sin^2 \theta \left(\frac{d\theta}{dt} \right)^2 = 2MH(\cos \theta - \cos \alpha)(\cos \theta - \cos \theta_1)(-\cos \theta_2 - \cos \theta), \quad (9)$$

in which there has been inserted a factor 2 on the right-hand side, and a necessary minus sign in the last term. If the calculation is carried out from this point the result obtained is

$$\sigma/\sigma_0 = a/6 + 0.24a^3 + \dots \quad (10)$$

or, for small values of a ,

$$\sigma/\sigma_0 = a/6. \quad (11)$$

This result could have been anticipated, and, indeed, obtained by means of a much simpler calculation. The angular velocities of spin with which we have to deal are large¹ of the order of 10^{16} radians/sec. while the applied couples are small, corresponding to small values of a . The precession is therefore very slow, and the nutation very rapid, being moreover between very small limits. For this case the motion may be described as a steady precession combined with a nutation in

¹ Také Soné, *Phil. Mag.*, March, 1920, p. 348.

an ellipse about the point which advances with the regular precession¹. This steady precession takes place at an angle of inclination to the direction of the field which lies half-way between α and θ_1 .² Since this elliptical motion is harmonic the time mean will be the same as if the axis precessed steadily at an angle equal to the mean of α and θ_1 .

This gives us at once for the magnetic moment per gram molecule of the gas

$$\sigma = \frac{Mn}{2} \int_0^\pi \cos\left(\frac{\theta_1 + \alpha}{2}\right) \sin \alpha d\alpha \quad (12)$$

or

$$\frac{\sigma}{\sigma_0} = \frac{1}{2} \int_0^\pi \left(\cos \frac{\theta_1}{2} \cos \frac{\alpha}{2} - \sin \frac{\theta_1}{2} \sin \frac{\alpha}{2} \right) \sin \alpha d\alpha. \quad (13)$$

Making use of (4) we find

$$\cos \frac{\theta_1}{2} = \cos \frac{\alpha}{2} + \frac{a}{2} \cos \frac{\alpha}{2} \sin^2 \frac{\alpha}{2} \dots, \quad (14)$$

$$\sin \frac{\theta_1}{2} = \sin \frac{\alpha}{2} - \frac{a}{2} \sin \frac{\alpha}{2} \cos^2 \frac{\alpha}{2} \dots$$

Which in (13) give

$$\frac{\sigma}{\sigma_0} = \frac{1}{2} \int_0^\pi \left(\cos \alpha + \frac{a}{4} \sin^2 \alpha \right) \sin \alpha d\alpha, \quad (15)$$

$$\frac{\sigma}{\sigma_0} = \frac{a}{6}. \quad (16)$$

If (5) is expanded in a power series in A the result is obtained

$$\frac{\sigma}{\sigma_0} = \frac{A}{3} - \frac{2}{90} A^3 + \dots \quad (17)$$

It is thus found that the results obtained when the nutation does and does not persist are different, as given by equations (16) and (8) respectively, and that in only one case does the result check that of Langevin, which, written in terms of a , is

$$\frac{\sigma}{\sigma_0} = \frac{a}{6} - \frac{a^3}{360} + \dots \quad (18)$$

The result of Langevin is thus seen to be the same, for small values of a as that obtained from a study of the gyroscopic motion, provided only that the nutation persists. It should be pointed out, however, that this check probably has no significance, and there is no reason why one should expect a check; for Langevin's result is a complete result, while

¹ A. G. Webster, *Dynamics of a Particle*, etc., page 286.

² Andrew Gray, *Gyrostatics and Rotational Motion*, page 101.

the result given by (16) is a partial result obtained from a study of one component only of the magnetic moment. The check would be actual, rather than merely apparent, only if the magnetic axes of all the molecules happened to coincide with their spin axes. With the ordinary picture of a molecule this is exactly what one would expect would not be the case. For example the magnetic axis of a hydrogen molecule will be along the line joining the nuclei, while the rotation as caused by thermal impact could probably have no component in this direction.

IV. If now the assumption is made that the axes of rotation of the molecules are isotropically oriented with respect to the magnetic axes, so that the number whose magnetic axis makes an angle lying between φ and $\varphi + d\varphi$ with the axis of rotation is given by an expression similar to (1) we can compute that part of the magnetic moment per gram molecule of the gas which is due to the component of magnetic moment in the direction of the axes of rotation. For if we now let M represent the total magnetic moment of a molecule the component of M along the spin axis will be $M \cos \varphi$, where $dn = \frac{1}{2} \sin \varphi d\varphi$ molecules have their magnetic axes making an angle with their axes of rotation lying between φ and $\varphi + d\varphi$. These dn molecules contribute, according to (16), a magnetic moment¹

$$d\sigma = M \cos \varphi dn \times \frac{a'}{6} = \frac{Mna}{12} \int_0^\pi \cos^2 \varphi \sin \varphi d\varphi,$$

$$\sigma = \frac{Mna}{18}, \quad (19)$$

$$\frac{\sigma}{\sigma_0} = \frac{a}{18}. \quad (20)$$

Combining this result with that obtained in the previous paper for that part of the magnetic moment per gram molecule which is due to the component of magnetic moment perpendicular to the axis of rotation we have for the complete result

$$\frac{\sigma}{\sigma_0} = \frac{a}{18} - \frac{a}{12} = -\frac{a}{36}. \quad (21)$$

It is thus seen that the magnetic susceptibility is diamagnetic if the nutation persists. If the nutation does not persist the calculation given in the previous article holds, and the result is a paramagnetic susceptibility of the same numerical magnitude.

This result will be restated, laying special emphasis upon the hypotheses under which it has been obtained. There are three chief assump-

¹ The notation of the previous article is used in the following equations.

tions involved: that the magnetic axes are isotropically oriented with respect to the spin axes, that the gyroscopic motion discussed takes place sensibly independently of and undisturbed by whatever hastening and retarding of the spin is caused by the component of magnetic moment normal to the spin axis, and that (at least on the average) thermal impacts do not disturb the complexion of the system. Under these conditions a gas is paramagnetic or diamagnetic according as the nutation of the gyroscopic motion is damped out or persists. In general it would seem theoretically possible to determine experimentally whether or not it persists by investigating whether there is a rise in temperature, otherwise unexplainable, when the gas is placed in a field: for if this motion is damped out the energy thus subtracted would have to be redistributed between the different degrees of freedom of the gas, and therefore would appear, in part at least, as ordinary heat motion. In the case of a gas satisfying the conditions above stated the matter could be settled more easily by simply observing whether the gas was paramagnetic or diamagnetic. If for any reason the nutation was partially damped out a value intermediate to the two above-stated results might be expected. The intervals of time associated with the motions of a molecule are very small indeed, and it is perhaps possible that ordinarily fields are brought into existence at a rate which could be considered infinitely slow as measured in such units, in which case one would perhaps expect to find the precession taking place steadily at the lower limit θ_1 , and the result thus paramagnetic. However for those gases which come under this theory the facts of experiment will settle definitely this question.

IV. The first of the three assumptions above stated is that the magnetic axes are distributed by chance uniformly with respect to the spin axes. This assumption would seem to be justified only for spherical molecules. In reply to a suggestion that this was probably the case Professor Honda reports that they found shortly after the publication of their paper that this assumption would indeed hold only for spherical molecules. This invalidates the latter part of their section 8, beginning with the sentence "If the form of the molecules is a body of revolution. . . ."

V. It is important to consider carefully the third assumption stated above. It is surely justified in the case of smooth, spherical molecules, for then the thermal impacts can have no effect whatsoever upon the orientations of the molecules. Translations, such as they cause, obviously do not have to be considered. It is obviously impossible, in the present state of our knowledge, to say directly whether or not a gas is composed of smooth spherical molecules, and indeed such a description

probably means little or nothing as strictly applied to a molecule, but a study of the ratio of the specific heats furnishes us with information, while indirect, of exactly the sort we require: that is, it tells us whether (whatever the shape and composition of the molecules) thermal impacts do alter the orientations of molecules. Argon, for example, would seem to satisfy the requirement. In the case of diatomic molecules, or molecules of more complicated structure, the thermal impacts surely can affect the orientations of the molecules, and it is necessary to inquire what the total average effect will be. For a rotating body whose moments of inertia about the three principal axes are of the same order of magnitude the frequency of the nutation is of the same order as the angular velocity about the axis of spin—in this case of the order of 10^{16} /sec. The frequency of molecular bombardment under standard conditions is of the order of 10^{10} /sec. For small applied moments, and high spin velocities the frequency of precession will be very small. Thus we are dealing with three intervals of time of widely different orders of magnitude. From the point of view of the molecular bombardment the precession does not exist at all, it is so slow; while the nutation is so rapid that a number of nutational oscillations of the order of 10^6 will take place, on the average, between collisions. Thus when the field is applied the orientation of magnetic axes, originally isotropic, will be slightly altered, each molecule turning so that, on the average, its magnetic axis makes a slightly smaller angle with the direction of the field than it did originally. This motion will get well under way before collisions, on the average, have a chance to alter the state of affairs. Something of the order of fifty thousand nutational oscillations will have been performed before the first impact occurs. What will be the effect of this impact? A detailed consideration would surely be a very involved matter, and perhaps one too difficult to carry through. It would require at least eighteen and probably more independent variables to specify one such impact completely. We may at least suppose, however, that after some considerable time has elapsed subsequent to the impact ("considerable" as compared to the period of the nutation) the molecule will be rotating at a new rate, and precessing about the direction of the field making a new angle with it. In other words the impact has changed it from a molecule of class *A*, which is defined as being composed of all those molecules whose spin velocity lies between ω and $\omega + d\omega$, and whose angle with the direction of the field lies between ψ and $\psi + d\psi$ over into a molecule of class *B*, in which these same quantities lie between different limits. The situation is similar to that considered by Krüger¹

¹ F. Krüger, *Annalen Physik* [iv], 50, 346.

in his theory of gyroscopic molecules, except that in our case there is an external field which is effective in producing torque. Thus on the whole, and from the point of view of one studying the nutational motion with instruments adapted for such intervals of time as are involved, the motion will consist, except for a very small fraction of the time, of the gyroscopic type first considered. Now this does not imply that for every molecule which is changed by a given impact from class *A* to class *B* there is another one which is changed from class *B* to class *A*. If this were so the total average effect of thermal impacts would be obviously null. If Langevin's analysis is to be considered completely satisfactory in the light of these ideas then we should conclude that for every molecule which is caused by an impact to leave class *A* and enter class *B* another is caused to leave class *B* and enter class *A*, the total number in a given class remaining constant: for Langevin's result is the same, for small values of α , as that demanded by the simpler theory which does not take into account the effect of impacts. The average effect of impacts would therefore have been shown null.

But if, on the other hand, a complete statistical study should reveal a different result then this different result will be absolutely independent of the initial orientation of axes which the gyroscopic theory indicates. The complexion of the system demanded by the statistical study would be established regardless of the initial orientation of axes. This initial orientation of axes, differing slightly from an isotropic one, is the sole effect of the gyroscopic action. If there is a single normal state of the gas, considered as a dynamical system, then this normal state will be attained, after a lapse of time, independent of the initial state of the gas unless indeed this initial state be such that the normal state is impossible, and provided that the collisions can effect changes in the system which make it tend towards the normal state. In illustration of this last condition we may imagine a certain normal partition of potential energy in a gas composed of smooth, spherical molecules, where the potential energy is a function of the orientation of a molecule (as it indeed is in the case of a molecule possessing a given magnetic moment), but obviously we have no reason to expect this normal partition to be effected by thermal impacts, since they can have no affect upon orientations. Thus we cannot expect the results of a statistical study, or any result which rests indirectly upon a statistical study, to furnish us information regarding the orientations of smooth spherical molecules. Langevin's result for the paramagnetic susceptibility of a gas therefore does not hold for that case, it being based on the assumption of the existence of a certain normal partition of potential energy as being

brought about by the statistical effect of collisions, when as a matter of fact the collisions could not affect the partition of potential energy at all in this case. For such a gas, therefore, the susceptibility would be given by (21) if the nutations persist, by the same quantity with the sign changed if it does not persist, and by something intermediate to them in case it is partly damped out. It is hoped to later consider, on general grounds, the statistical problem here suggested.

With respect to the problem involved in the study of the effect of thermal impacts on a gyro-molecule Professor Honda has written me as follows: "When the spinning velocity of the gyro-molecules is very large the angular velocity of precession will be slow, as you have remarked. But the effect of thermal impacts on spinning molecules is quite different from that on non-spinning molecules. (1) The thermal impact causes only momentary precession and nutation; when the impact is finished the gyro-molecules assume their original orientation before the impact. (2) Moreover the thermal impacts will take place equally often so as to rotate the axis of the gyro-molecule in the direction of the magnetic field, as in the opposite direction. Hence we cannot see the reason why the thermal impact results in preventing the molecules from directing towards the magnetic field, as Langevin admits it. In fact our investigation of the gyro-molecules gives a dynamical mechanism for the Langevin distribution of the direction of the axes of magnetic molecules."

The numbers inserted in the above quotation are to facilitate reference to a particular sentence. The sentence number (1) seems to me definitely incorrect. If a gyro-molecule is in a steady state the angular momentum about a line through O and having any direction is constant. Suppose for simplicity one chooses for the direction of this line the direction of the total resultant vector angular momentum, so that the component along the line chosen be a maximum. Then suppose there to be an impact. In general the direction of the force exerted upon the molecule under consideration will not pass through O , and the effect of the impact will be to permanently alter the value of the angular momentum. In particular the component of angular momentum along the line chosen cannot be the same after impact as it was before impact, and the gyro-molecule obviously cannot return, after the duration of the impact, to its original state.

The sentence numbered (2) can be satisfactorily substantiated only, it would seem, by the sort of detailed statistical study proposed above. It may be that Professors Honda and Okubo have successfully completed this study, and make this statement upon entirely sufficient grounds.

One's inclination is, however, that it cannot be true. The situation is so extremely complicated that it seems hardly likely that so simple a state actually exists with respect to the effect of impacts.

The remainder of the quotation is commented upon by the whole of this present section.

VI. In conclusion it should be noted that since the magnetic susceptibility appears here as a differential effect of the paramagnetic action of one component of the magnetic moment and the diamagnetic action of the other component it is perhaps not justifiable to neglect the ordinary diamagnetic effect of the growth of the field upon the elementary circuits. This effect which is a change $-\delta M$ in the magnetic moment of each molecule was spoken of in I., and although it is small it may be comparable to the difference between the two larger quantities referred to above. It is always easily included.

UNIVERSITY OF WISCONSIN.

A PHOTOGRAPHIC METHOD OF FINDING THE INSTANTANEOUS VELOCITY OF SPARK WAVES.

BY ARTHUR L. FOLEY.

SYNOPSIS.

The velocity of explosive sound waves, determined by early investigators, is briefly discussed, particularly the several experiments of Mach and his associates on the velocity of spark waves.

A new and direct photographic method for finding spark wave velocity. A shadow of a sound wave produced by an electric spark near the center of a long light-tight rectangular box was cast on a dry plate at one end of the box by the light from a properly timed spark at the other end. A notched rim steel disk was rotated at a high and measured speed just in front of and parallel with the dry plate, the shadow of the disk covering but a narrow strip along one edge of the plate. Both the sound and light sparks cast notch shadows on the plate, the distance between the two shadows of the same notch giving the time interval between the sparks, and the sound wave picture the distance traversed by the wave in that interval. Plotting distance against time, the tangent to the curve at a given point gives the instantaneous velocity of the wave at that point. The formula used in calculating time intervals is derived and discussed.

Velocity almost normal except near the spark. Measurements were made on two hundred and eighty plates, for waves from 0.32 cm. to 50 cm. in radius, for both heavy and thin sparks, and for disk edge speeds of almost two hundred meters per second. A wave velocity double the normal at 0.32 cm. from the spark axis decreased very rapidly with increasing distance and at 2 cm. from the source was only slightly above normal. At points relatively near the spark where the intensity of the expanding wave was still great, the velocity of waves from heavy sparks was greater than that from thin sparks. Reference is made to corroborative results obtained by the writer when using two other photographic methods.

Sources of error in previous investigations eliminated by the photographic method. Previous investigators have found average and not instantaneous velocities. Since the velocity very close to the source is very high, average velocities run high. However, previous results have been entirely too high due to errors inherent in the experimental methods used. Attention is directed to several, the chief one being the use of tubes or "canals." The paper gives a photograph of a sound wave, a part of which has passed through a canal of the size used by Mach, the other part expanding freely into the space about the spark, showing the former wave thirty per cent faster than the latter—due to the lower average intensity of the expanding wave.

GENERAL AND HISTORICAL.

NEWTON'S equation for the velocity of sound, according to which all sounds travel at the same rate, was derived on the assumption of infinitely small displacements. Several writers have shown that the velocity of sound waves of finite amplitude is a function of the amplitude

and therefore a function of the intensity. Numerous experimenters have verified the theory in the case of "sharp" sounds, such as are generated by electric sparks and quick acting explosives. Owing to its importance in war, sound ranging for instance, the velocity of sound produced by firing large guns has been studied extensively by military as well as by other investigators. Regnault¹ (in 1864), by electrically recording the reciprocal firing and the waves from two guns, found an average velocity of 331.37 m. per sec. for the first 1,280 m. from the gun, and a velocity of 329.9 for the next 1,165 meters. Jacques² (in 1879), using membrane receivers and a chronograph recorder, showed that the velocity of sound about a gun depends on the direction. For a cannon charged with one and one half pounds of powder the velocity immediately in the rear of the gun was below normal, and at 20 ft. the velocity was fifteen per cent. less than it was at 80 ft. behind the gun.

Wolff³ sought to determine the nature of the disturbance resulting from an explosion and concluded that the type is essentially that of a sound wave, the velocity increasing with the violence of the explosion. Vieille⁴ had previously found that he could obtain a wave velocity in air of 1,100 meters per second by sharpening the explosion of fulminate by tightly packing it. Both Earnshaw⁵ and Riemann,⁶ in a theoretical discussion of waves of finite amplitude, had shown that for such waves an increased velocity was to be expected.

In 1860 Earnshaw⁷ derived an equation for the velocity of sound in air in which he did not assume, as did Newton, that the air is a continuous medium, nor was it necessary to apply the Laplace and Poisson correction. Earnshaw's equation makes the velocity of sound in air a function of both intensity and pitch, the effect of pitch being quite small. Earnshaw¹ concluded that "there is no other limit to the velocity with which a violent sound is transmissible through the atmosphere, than that which the possibility of supplying a sufficient degree of force in its genesis may impose. Hence it is probable that there is no sound which is propagated faster than a thunder clap, the genesis of which by the electric discharge being extremely violent and almost instantaneous, and accompanied by a large development of heat. If the

¹ V. Regnault, *Mem. de l'Acad. Paris*, 37, 1, 3, 1868; *Comptes Rendus*, 66, 209, 1868; *Phil. Mag.*, S. 4, V. 35, 161, 1868.

² W. W. Jacques, *Sill. Journ.* (3), 17, 116, 1879.

³ W. W. Wolff, *Wied. Ann.*, 69, 329, 1899.

⁴ P. Vieille, *Comptes Rendus*, 127, 41, 1898.

⁵ S. Earnshaw, *Proc. Roy. Soc.*, Jan., 1859, Vol. IX., p. 590.

⁶ B. Riemann, *Göttingen Abhandlungen*, t. viii, 186, Abstract in *Fortschritte der Physik*, xv, p. 123.

⁷ S. Earnshaw, *Phil. Mag.*, June, 1860, Vol. XIX., p. 449, and Vol. XX., p. 39.

theory here advanced be true, the report of firearms should travel faster than the human voice; and the crash of thunder faster than the report of a cannon."

Earnshaw's prediction has been found to be true as far as the report of fire arms is concerned. As far as the writer knows, no trustworthy results have been obtained for the velocity of thunder. However, a large number of experimenters have worked on the problem of finding the velocity of waves from electric sparks. Although the energy involved in such sparks is not comparable to that in a lightning flash, nevertheless, the wave generation is "extremely violent and practically instantaneous," and an increased velocity of the spark wave near the source would be expected.

Most of the early work on the velocity of propagation of spark waves was done by using Antolik¹ figures, markings produced on a soot-covered or dusty plate by an electric discharge to or very near the plate. Antolik supposed the figures to be indicative of the nature of the electric discharge. They were later proven to be of acoustic origin.²

Mach and Gruss³ studied the interference strips produced on sooty glass by two sparks or by a spark and its image by reflection, noting the displacement of the soot figures when one of the sparks was retarded by connecting Leyden jars in cascade and applying Feddersen's law to determine the time interval involved. They concluded (really assumed) that the velocity of their spark waves was about 400 meters per second. Mach and Sommer⁴ continued the use of interference strips on sooty plates, but they resorted to experiment to determine the time interval between their sparks. One method was to set off the two sparks placed a known distance apart by firing a bullet between the two pairs of spark terminals, the speed of the bullet being determined by means of a ballistic pendulum. In another experiment it was found by firing a bullet horizontally through two vertical films supported on frames which were suspended some distance apart, and firing again when the two films were dropped simultaneously at the firing of the gun. The distance between the two holes in each screen gave the respective distances dropped through and by calculation the time required for the bullet to travel from the first to the second films, and finally the speed. Later they found it better to mount the two films at opposite ends of a rectangular box, to insure their simultaneous start. Still later they mounted the

¹ K. Antolik, *Pogg. Ann.*, 151, 127, 154, 14, 1875.

² E. Mach u. J. Wosyka, *Sitzungsber. der K. Gesellsch. der Wissensch. zu Wien*, 72, 1875; W. Rosicky, *l. c.*, 73, 1876.

³ E. Mach und G. Gruss, *Wien. Ber.*, 78, 14, 1878.

⁴ E. Mach und J. Sommer, *Wien. Ber.*, 75, 1877.

films at opposite ends of a shaft which was rotated at a high and optically determined speed. The results obtained by the ballistic method of determining the speed of the bullet range from 393 to 468 meters per second, for the velocity of a spark wave at distances of some 30 cm. from the source. The gravity method of obtaining the bullet speed gave even greater differences in the computed spark wave velocity. When two hammers were dropped simultaneously through unequal distances on fulminate of mercury the sound velocity ranged from 654 to 817 and 634 to 686, depending on whether the end of the tube in which the explosion occurred was closed or open. Although Mach and Sommer tried several other methods without obtaining results more consistent than those given above, nevertheless, they were able to announce that the velocity of an explosion sound wave depended upon its intensity, the method of its production, and the distance from the source. They classed spark waves as explosion waves.

A year later Mach¹ and his students published the results of an experiment in which they produced their interference figures on a soot covered revolving plate, the speed of which was determined stroboscopically. In one case the figures were obtained from the interference of the sound wave from a spark discharging directly to the rotating plate and the waves from a simultaneous (series) spark led to the plate through a tube of variable length. In another case, both sparks, connected in series, discharged directly to the rotating plate but at points some distance apart. One of the sparks produced a wave traveling in the same direction as the surface of the plate, the other a wave traveling in the opposite direction, the two waves interfering along a strip displaced toward the former spark. The former of the two methods appeared to yield the most consistent results. But as Mach's results do not agree at all with the results obtained by the writer, their discussion will be taken up in connection with the consideration of the writer's results,—after his method of determining spark wave velocities has been described.

Apparatus and Method.

The photographic part of the apparatus used in this investigation was very similar to that described by the writer in a paper on "A New Method of Photographing Sound Waves,"² to which the reader is referred for details not given in this paper.

The Leyden jars *L*, *L*, Fig. 1, charged by a Wagner static machine with two electrically driven revolving mica plates thirty inches in diam-

¹ E. Mach, O. Tumlirz und C. Kögler, Wien. Ber., 77, 1878.

² PHYS. REV., XXXV., No. 5, Nov., 1912, p. 373.

eter and yielding ten inch sparks, were discharged through the gaps *G, G*, the commutator *C*, and the two spark gaps *S* and *I*, connected in series. Partly by varying the relative lengths of the gaps *S* and *I*, but chiefly by varying the relative capacities *L* and *K*, the time of the illuminating spark *I* was made to occur a little later than the sound spark *S*, and to cast on the photographic dry plate *P*, a shadow of the sound wave produced by the spark at *S*.

A steel saw disk *D*, was mounted so as to be rotated at high speed parallel to the surface of the dry plate *P*, with the upper edge of the disk

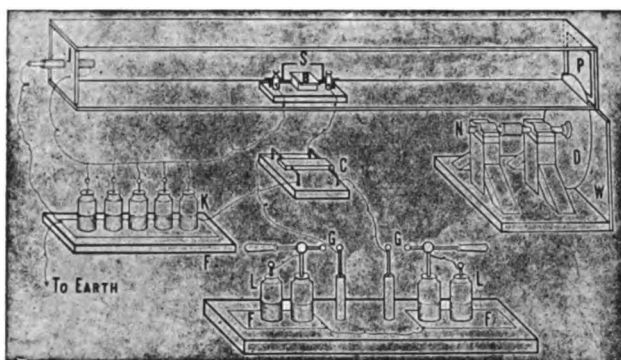


Fig. 1.

covering a narrow strip at the lower edge of the dry plate,—just enough to permit the shadow of the disk produced by the sparks at *S* and *I* to be photographed along with the sound wave shadow. The steel disk was made and mounted by a circular saw manufacturing company and was as well balanced as the manufacturer could make it. Several days were spent in testing the disk on knife edges and perfecting its balance. Notwithstanding the care taken in its making and mounting, it was thought unsafe to push the speed of rotation beyond eighty revolutions per second. As the disk was 239.39 cm. in circumference, this corresponded to a lineal speed of 191.51 meters per second, more than half the velocity of sound. At a slightly higher speed the disk began to warp and vibrate in a frightful manner. In coming up to speed the disk passed through two critical speeds at which there was some warping and vibration, but nothing compared to what a still higher speed called forth.

In partial explanation of the warping of the disk after so much work spent in balancing it, attention is called to the difference in air pressures on different parts of surface due to the fact that it did not rotate in the center of the light tight box in which it was mounted. The distance

from the surface of the dry plate to the surface of the disk was but 1.5 cm., sometimes only 1 cm. Even when the distance was 1.5 cm. the disk was rotating within 1 cm. of the edge of the plate holder and about 1.3 cm. from the slide. The unbalanced air pressure on the slide was so great at the higher speeds that it could scarcely be moved in the holder.

The edge of the disk was ground (beveled) to a chisel edge with the plane side next the dry plate and 1.5 cm. from it (only 1 cm. distant during a part of the observations). Small V-shaped notches were made in the sharp edge of the disk by means of a small three cornered file held against the edge, at right angles to the plane of the disk, and tapped lightly with a hammer. The faces of the file had been ground smooth. The burr produced when the edge of the disk was dented (notched) was carefully removed by means of a smoothing file. The notches averaged about 2 cm. apart and were purposely made of various depths (.2 to 2 mm.) and at unequal distances in order to identify their shadows on the photographic plate.

The disk *D* was driven by means of a variable speed electric motor. The shaft of the disk was connected to a revolution counter and to a Van Sicklen-Elgin chronometric tachometer.

The first plan for controlling the sparks across the gaps *G, G*, was to place a glass plate transversely across the middle of each gap, the plates being fastened at one end to a wooden shaft by means of which they could be simultaneously rotated from between the gap terminals to permit the passage of a spark and rotated back to prevent a second spark. In practice, however, it was found much better to rotate the plates from between the terminals and to lengthen the two gaps *G, G*, until sparks ceased to pass. Sparks were then produced at will by a quick motion of the plates toward and away from the gaps. The lengths of the four gaps were varied from time to time along with the capacities *L* and *K* to obtain the desired time intervals between the sound and illuminating sparks. On the average, each of gaps *G, G* was made from 6 cm. to 8 cm. and *S* and *I* from 2 cm. to 3 cm. in length.

The Leyden jars used for the capacities *L* and *K* varied in capacity from 510 cm. to 536 cm. averaging 525 cm. The much smaller jar on the electric machine together with the capacity of the machine itself, appeared to be roughly equal to the capacity of one of the 525 cm. jars, and is so considered in the data recorded in this paper.

Care in arranging and adjusting the apparatus will not secure a uniform time interval between the sound and illuminating sparks. If one takes a sound wave picture and in an hour takes another, making no

change whatever in the apparatus or its adjustment, the first time interval may be so short that the sound wave does not have time to get started, the second so long that the wave passes beyond the limits of the plate; or vice versa. However, most of this time variation can be overcome by frequently polishing the spark knobs (brass spheres about 2 cm. in diameter) of the spark gaps *G*, *G*, and by sparking the apparatus several times before making an exposure.

The procedure finally adopted was as follows: While an assistant *C* was bringing the disk up to speed, an assistant *R*, by rotating the glass plate previously referred to, produced sparks at regular time intervals, usually about three seconds. With eyes protected from the intense light of the illuminating spark by a pair of clouded glasses, and with a ground glass covering the end of the box instead of the plate *P* shown in Fig. 1, the writer viewed the shadow of the sound spark and adjusted the capacities and spark gap lengths until sound waves of the desired radius appeared on the ground glass. A long wooden bar, not shown in the figure, extending from the plate end of the box to a lever system at the illuminating gap enabled the observer to adjust the gap while viewing the waves. When once the capacities were adjusted to give waves, adjustment of the illuminating gap alone was sufficient to get them of any desired radius. This having been done, the ground glass was replaced by a dry plate and the exposure made by drawing the slide during the three second interval between the sparks. Tachometer readings of the speed of the disk were made immediately before, at the time of, and just after the exposure. Temperatures were taken both outside and inside the light tight box, the latter at a point near the sound spark gap. Finally, barometer and wet bulb hygrometer readings were taken, though not used in the calculations.

The sound and illuminating gaps, being placed longitudinally in the box, may be considered as point sources of light, as far as the plate *P* is concerned. When the sound spark occurred a shadow of the rotating disk *D* was cast on the dry plate, the V-shaped notches being clear cut, notwithstanding the high speed of the disk. Later the illuminating spark occurred and cast a shadow of the wave produced by the sound spark, together with a second shadow of the rotating disk. The distance between the two shadows of the same notch enables one to calculate the time interval between the sparks. From the radius of the wave shadow as measured on the plate, one readily calculates the actual wave radius. The quotient of the radius by the time might be supposed to give the average velocity. However, this statement involves two assumptions which the writer will call in question later.

To obtain pictures of waves greater than 15 cm. in radius, which otherwise would have been beyond the limits of the dry plate, a plane reflector was placed on each side of the sound spark gap, at such a distance from the gap that the wave, after any number of reflections, was always shadowed on the plate. In this manner waves were measured as large as 50 cm. in radius. There would have been no difficulty in extending the method to waves of much greater radius.

Figure 7 shows the vertical sound gap, the hot gases, the spark axis, the reflected wave and the notched edge of the rotating disk. Much of the detail, particularly the definition of the notches in the disk, has been lost in reduction and reproduction. In judging of the definition of the wave shown on this plate, one should remember that the sound spark is much nearer the dry plate than the illuminating spark and that nothing can be placed between the sound spark and the dry plate to prevent the light of the sound spark from fogging the plate as was done by the writer when he was using a horizontal sound gap. The light intensity of the illuminating spark must be many times that of the sound spark to obtain the definition shown on Fig. 1. The details of the illuminating gap are given in the writer's paper to which reference has already been made.

CALCULATIONS.

To determine from the distance measured on the plate the actual distance through which a disk notch turned during the time interval

between the sparks, the writer assumes that the cord of a short arc of a disk thirty inches in diameter may be taken equal to the arc.

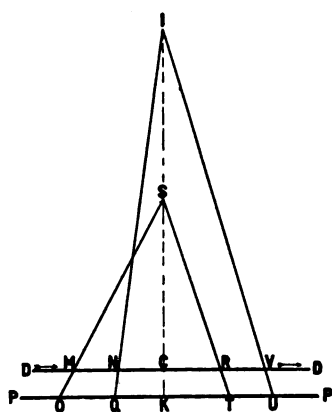


Fig. 2.

Figure 2 is a vertical projection in which *I* is the illuminating spark, *S* the sound spark, *D* the edge of the disk, *P* the edge of the dry plate. Suppose the disk moving rapidly in the direction of the arrows. When the sound spark occurs at *S* the shadows of notches *M* and *R* are cast at *O* and *T* respectively. By the time the illuminating spark occurs at *I*, the notches *M* and *R* have moved to *N* and *V*, and

their shadows are cast at *Q* and *U*, respectively. Let *C* and *K* be points on the disk and dry plate, respectively, and on a straight line through *S* and *I*.

Let

$$\begin{aligned} MN &= d, \\ RV &= d', \\ OQ &= X, \\ TU &= X', \\ QK &= D, \\ KU &= D', \\ \frac{SC}{SK} &= a, \\ \frac{IC}{IK} &= b. \end{aligned}$$

Then

$$\begin{aligned} d &= a \cdot OK - b \cdot D \\ &= a(OK - D) + (a - b)D \\ &= ax + (a - b)D, \\ d' &= bD' - a \cdot KT \\ &= a(D' - KT) - (a - b)D' \\ &= ax' - (a - b)D'. \end{aligned}$$

But

$$d = d' = \frac{1}{2}(d + d'),$$

$$(1) \quad d = \frac{1}{2}[a(x + x') + (a - b)(D - D')].$$

The following are the dimensions used during the greater part of this investigation:

$$\begin{aligned} SC &= 148 \text{ cm.}, & SK &= 149.5 \text{ cm.}, & IC &= 277.3 \text{ cm.}, \\ \text{and} & & IK &= 278.8 \text{ cm.} \end{aligned}$$

Using these dimensions to obtain the values of a and b and supplying in (1)

$$(2) \quad d = .495(x + x') - .0023(D - D').$$

The "constants" appearing in (2) were varied occasionally by changing the distance between the disk and dry plate, a ranging in value from .495 to .493 and b from .0023 to .004.

Let t = the time interval between the sound and illuminating sparks, $2\pi r = 239.39$ cm. = circumference of disk, n = number of revolutions of the disk per second:

$$(3) \quad t = \frac{d}{2\pi rn} = \frac{d}{239.39 \times n} \text{ seconds.}$$

The value of d given by equation (2) was checked as follows: With the disk at rest, a dry plate in position and the illuminating gap short circuited, a spark was sent through the sound gap. The disk was then rotated a measured distance, the sound gap short-circuited and a spark passed through the illuminating gap. The measured value of d and the value calculated from the plate by equation (2) agreed to within 0.1 per cent. A higher degree of accuracy in measuring d , and therefore in measuring the time interval t , is not necessary, because the radius of a wave near the source can not be measured to any higher degree of accuracy. The wave shadow fades out more or less gradually on each side of the point of maximum density. Consequently one can not locate exactly the wave front. Then, too, the changing form of the wave and its variation in intensity affect somewhat the distribution of light in the wave shadow.

RESULTS.

Altogether the writer made measurements and computed the velocities of the waves on two hundred and eighty plates. Some of the data and results are given in Table I., the only point considered in their selection being to show waves of widely different radii.

TABLE I.

Plate Number.	r , Cm.	D' , Cm.	D , Cm.	x , Cm.	d by Equation 1, Cm.	Wave Radius.		Disk Turns per Sec.	Time Sec. $\times 10^4$.	Wave Velocity.	
						Meas- ured on Plate, Cm.	Actual (Calcu- lated), Cm.			"Aver- age" Radius Time, M.	Instan- taneous from Curve, M.
143	0.021	2.1	7.2	0.057	0.03744	0.75	0.349	58.9	0.2687	1,295	678
111a	0.063	3.0	1.7	0.098	0.08415	1.11	0.615	34.82	0.8851	695	546
123a	0.216	1.16	6.24	0.274	0.2213	2.27	1.247	41.83	2.216	562	423
124a	0.85	4.35	3.9	0.91	0.7686	6.4	3.52	41.5	7.759	454	383
123K	1.87	3.6	3.9	1.93	1.8718	13.15	7.48	41.07	18.82	398	358
1253	5.4	0.0	0.0	4.0	9.312		33.86	41.5	93.97	360	357

Figure 3 is a plot of time and wave radius. Inasmuch as it is not practicable to show two hundred and eighty points on so small a scale, most of the points represent averages of several time and space measurements, the circles for strong sparks, the dots for weak sparks. For strong sparks each of the capacities L , L , Fig. 1, consisted of seven jars (525 cm. each) and K of eighteen jars; for weak sparks, two and six jars respectively.

The tangent to a curve drawn through these points gives the instantaneous velocity of sound at a distance from the source corresponding to the point at which the tangent is drawn. In Fig. 3 the writer has

drawn a straight line at a slant corresponding to an average sound velocity of 354 meters per second. Inasmuch as the experimental observations were made at a temperature of 24° C., at which the normal velocity of sound is 346.8 m. per sec., and the straight line appears to

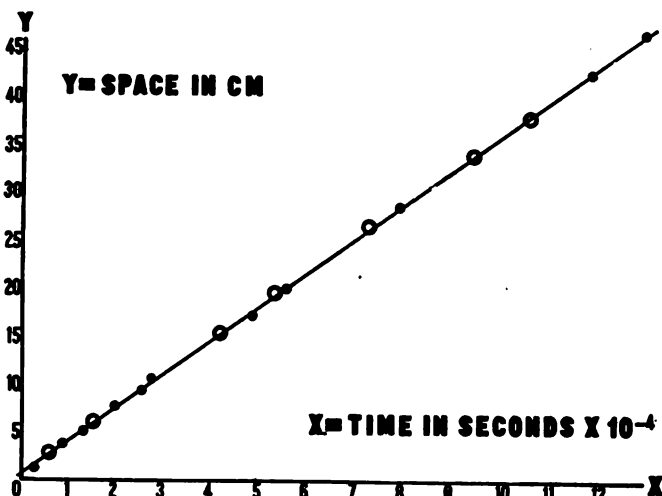


Fig. 3.

fit the data fairly well, it might be inferred that the velocity of a spark wave is constant and slightly greater than normal. But when the plot is made on a much larger scale, and for points nearer the sound source, Fig. 4, one sees that the velocity of a spark wave is not constant in this

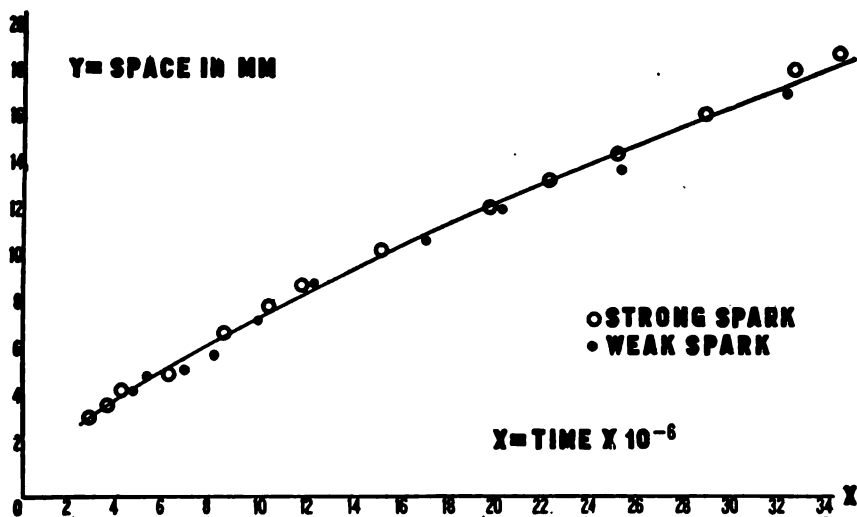


Fig. 4.

region. Note that observations have been made at points as close as 3 mm. from the spark axis. Note, too, that the curve, if continued regularly, would not pass through the origin. The writer is studying the spark itself and will have something to say later about what happens in this region. It is evident, however, that the velocity of sound at points near the source can not be determined accurately by any method which involves the ratio of total space to total time. This ratio, for the nearest point on the curve, gives an average velocity of 1295 m. per sec., while the tangent to the curve at that point gives an instantaneous velocity of 678.6 m. per sec. The last two columns of Table I. give average and instantaneous values for other and more distant points. These values approach one another as the distance from the source increases but at points near the source they are widely different.

An ordinate of Curve I of Figure 5 gives the *instantaneous velocity* of a

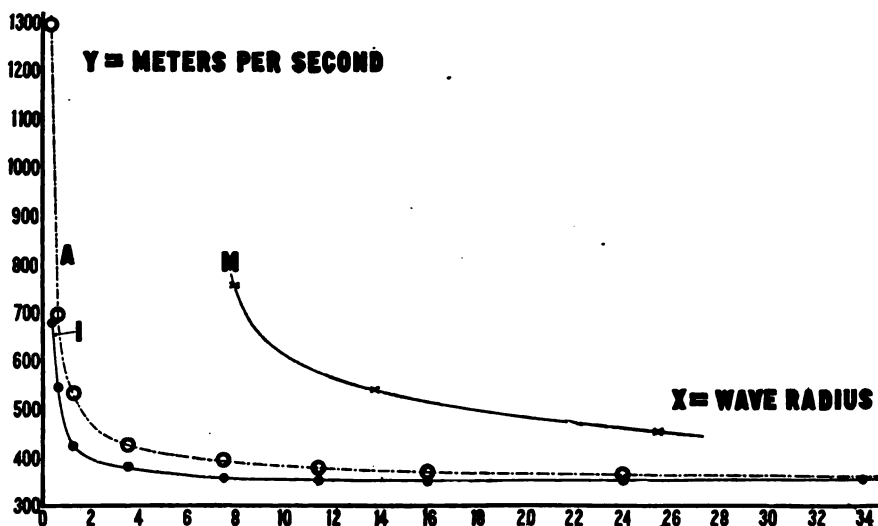


Fig. 5.

spark wave as it passes a point whose distance from the source is the abscissa. An ordinate of Curve A gives the *average velocity* for the corresponding distance from the source, both I and A being plotted from the writer's measurements. Curve M is an *average velocity* curve plotted from Mach's measurements.

Comparing curves M and A, one observes that the spark wave velocities obtained by Mach are much higher than those obtained by the writer, and that they decrease much less rapidly with increasing distance from the source. The lack of agreement is doubtless due in large measure to a

difference in the intensity of the sound waves used. Fig. 4 shows a slight difference between the velocities of waves produced by strong and weak sparks. But the writer's strongest sparks were much less intense than those used by Mach. However, the question of the intensity of the sparks would have been of much less moment had Mach experimented with spark waves in free air. The intensity of such waves near the source decreases so rapidly with increasing distance that a high initial velocity soon becomes almost normal. But if the spark wave is generated at or near the end of a tube the initial energy of the portion of the wave entering the tube remains more nearly constant, giving an increased velocity through the tube, as the writer has previously shown.¹ As a

¹ PHYS. REV., XIV., No. 2, Aug., 1919, p. 143.

matter of fact, in most of the work done by Mach and others on the velocity of spark waves near the source, the observers have passed the waves through "canals." Figure 8 shows photographically what

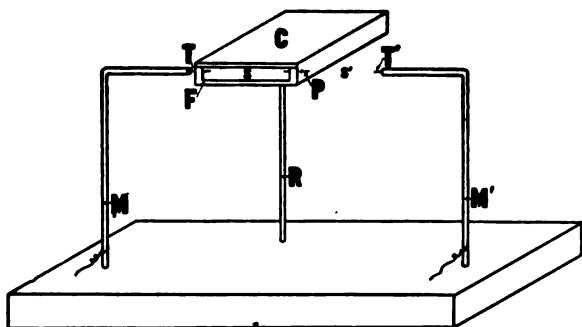


FIG. 6

Fig. 6.

happened when the writer produced a wave by a spark in the end of a rectangular canal of the size used by Mach. Fig. 6 shows the arrangement of the apparatus at the sound spark.

A rectangular tube was made by planing a groove 3 mm. deep and 1.8 cm. wide in a block of insulating fiber *F*, 5 mm. thick, 2.5 cm. wide and 6 cm. long, and covering with a piece of fiber *C*, 1 mm. thick. At *T* and *P*, 1 mm. from the end of the tube, holes were bored through the walls. One of the platinum terminals of the sound spark gap extended through one side wall of the tube. A short piece of platinum wire extended through the other wall. Thus the sound gap *TT'* was broken into two halves, *S* and *S'*. A spark wave produced at *S* was forced to travel 6 cm. through the rectangular tube before emerging into free air, while the wave produced simultaneously at *S'* was free to expand from the start.

Figure 8 shows that the wave through the tube is considerably in advance of the one in free air. Calculation shows that the wave velocity in the tube for "strong" sparks was some twenty-two per cent. greater than the velocity in free air. For weak sparks the velocity difference was somewhat less.

Much of Mach's work was done with a tube closed at the spark end. When the writer closed the end of the tube leaving everything else as shown in Fig. 6, he found a wave velocity in the tube more than thirty-five per cent. greater than the velocity in free air. For as heavy sparks as those used by Mach, doubtless the velocity differences would have been still greater. In short, we have here at least a partial explanation of the very high spark wave velocities obtained by several previous experimenters and of the lack of agreement between their results and those obtained in this investigation. It is clear that sound waves should not be confined in small tubes or "canals" in any investigation involving their velocity in free air.

The method described in this paper is the writer's third one, the first two having been published previously.¹ While the third method gave more uniform results than either of the others, their averages are in good agreement and confirm the conclusions stated below.

SUMMARY AND CONCLUSIONS.

1. The instantaneous velocity of a spark wave has been determined by a photographic method which is free from the various sources of error of previous methods.
2. Heretofore no velocity measurements have been attempted at points nearer than 8 cm. from the spark. This investigation carries the measurements to within 0.32 cm. of the source.
3. The instantaneous velocity of spark waves was found to depend on wave intensity and therefore to depend on both spark intensity and distance from the source.
4. A spark wave velocity almost double the normal velocity of sound at 0.32 cm. from the source decreased to almost normal at a distance of only 2 cm. from the source.
5. The very high velocity obtained by Mach at 8 cm. from the source is explained as due in part to the use of very heavy sparks, but chiefly as error inherent in the methods used.
6. The writer shows photographically that the velocity of spark waves through canals such as used by Mach is far greater than the velocity of the same wave in free air, and that the velocity in a canal increases with the intensity of the spark.

¹ A. L. Foley, Proc. Indiana Acad. Science, 1919, p. 221.

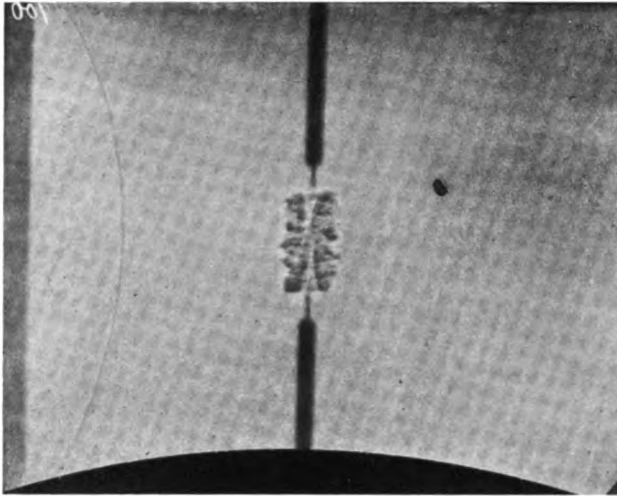


Fig. 7.

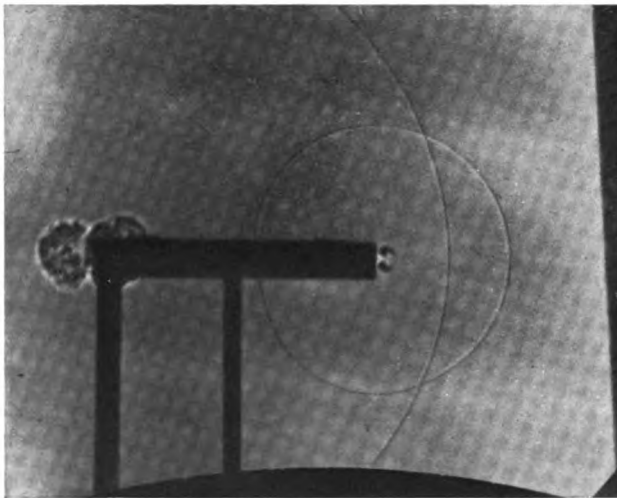


Fig. 8.

ARTHUR L. FOLEY.

7. Due to the motion of the air set up by a rotating disk measurements of sound wave velocity near the surface of the disk are wholly unreliable.

8. Owing to the practically instantaneous generation of a spark wave and its first appearance at some distance from the spark axis, the quotient of wave radius by time (for points near the source) gives an "average velocity" much greater than the instantaneous velocity at the nearest point at which the wave can be differentiated from other spark phenomena.

The writer wishes to acknowledge the aid given him in this research by a grant from the American Association for the Advancement of Science.

WATERMAN INSTITUTE FOR RESEARCH,

INDIANA UNIVERSITY,

May, 1920.

IS THE ATOM THE ULTIMATE MAGNETIC PARTICLE?

BY ARTHUR H. COMPTON AND OSWALD ROGNLEY.

SYNOPSIS.

Effect of Magnetization of Crystal on Intensity of X-ray Reflection; Theory.—The intensity of reflection of X-rays from a crystal depends upon the arrangement of the atoms within the crystal and upon the arrangement of the electrons within the atom. If the ultimate magnetic particle is a group of atoms, such as the chemical molecule, magnetization of the crystal will change the orientation of the group and hence change the positions of the individual atoms. If the ultimate magnetic particle is the atom, magnetization will change the orientation of the atom and hence alter the arrangement of the electrons. In either case magnetization of a crystal should be accompanied by a change in the intensity of a beam of X-rays reflected from its surface.

Effect of Magnetization of Crystal on Intensity of X-ray Reflection; Experiment.—Such an effect was sought for by reflecting X-rays from a crystal of magnetite and measuring the intensity of the reflected beam by a sensitive balance method. The test was made on the first four orders of reflection from the natural (111) face when the crystal was magnetized perpendicular to its reflecting surface and on the third order when magnetized parallel with this surface. On magnetizing the crystal to $1/3$ of saturation and on demagnetization, no change in intensity of the reflected beam was observed, though a variation of 1 per cent. would have been detected.

Molecule as Ultimate Magnetic Particle.—A displacement of the atoms due to magnetization by $1/3000$ of their distance apart would have produced a detectable effect. This experiment therefore affords a strong confirmation of the conclusion reached by K. T. Compton and E. A. Trousdale that the elementary magnet is not a group of atoms.

Motion of Elementary Magnet in Strong Magnetic Field.—An argument is presented which shows that when saturation occurs the elementary magnets very probably have their axes nearly parallel with the direction of magnetization.

Atom as Ultimate Magnetic Particle.—Subject to the validity of this conclusion, it is not found possible to explain the negative result of the experiment if the atom as whole acts as the elementary magnet. Certain other explanations are also discussed and found unsatisfactory.

Electron or Positive Nucleus as Ultimate Magnetic Particle.—Either of these conceptions is in accord with the result of the experiment, but auxiliary evidence favors the electron as the probable elementary magnet.

A FEW years ago K. T. Compton and E. A. Trousdale described in this journal¹ an experiment which led them to the conclusion that the ultimate magnetic particle is not any group of atoms, such as the chemical molecule, but is rather the atom or something within the atom. It has long been a favorite method of explaining the magnetic

¹ K. T. Compton and E. A. Trousdale, *Phys. Rev.*, 5, 315 (1915).

properties of matter to interpret them in terms of the magnetic moment of electrons rotating in orbits. This hypothesis has seemed to receive support by the successes of the Rutherford-Bohr theory of atomic structure, which assumes the atom to consist of electrons revolving in orbits in such a manner that the atom must possess a large magnetic moment. Furthermore, the experiments of Barnett,¹ Einstein and de Haas² and J. Q. Stewart³ have shown that the magnetization of iron is unquestionably accompanied by a change in angular momentum of the order of magnitude to be expected if the iron's magnetic properties are due to electrons revolving in orbits.

Experimental.—We have accordingly performed an experiment designed to test the hypothesis that the elementary magnet in a ferromagnetic substance is an atom consisting of rotating rings of electrons. In this experiment an attempt was made to detect a difference in the intensity of a beam of X-rays reflected from a crystal of magnetite when the crystal was magnetized and when unmagnetized.⁴ It will be assumed for the present that all the elementary magnets of which a ferromagnetic substance is composed are arranged with their axes parallel with the magnetic field when the substance is magnetically saturated. If these ultimate magnetic particles are the individual atoms, the orientation of the atoms due to magnetization of the crystal will change the positions of the electrons of which the atoms are composed. In virtue of the fact that the intensity of a beam of X-rays reflected from a crystal face depends upon the arrangement of the electrons in the atoms which make up the crystal, such a shift of the electrons should make itself known by changing the intensity of the reflected beam.

Consider for example a crystal composed of atoms of the Rutherford type, each atom having all its electrons arranged in the same plane and perpendicular to its magnetic axis. When the crystal is unmagnetized, the axes of the electronic orbits will be oriented in all possible directions, so that most of the electrons will be at an appreciable distance from the mid-planes of their atomic layers. If, however, the crystal is magnetically saturated perpendicular to the reflecting face, the electronic orbits will all lie parallel to this face. The electrons will now, therefore, be *in* the midplanes of the layers of atoms which are effective in producing the reflected beam. Such a shift of the electrons should produce a very considerable increase in the intensity of the reflected beam of

¹ S. J. Barnett, *PHYS. REV.*, 6, 240 (1915).

² Einstein and de Haas, *Verh. d. deutsch. Phys. Ges.*, 17, 152 (1915).

³ J. Q. Stewart, *PHYS. REV.*, 11, 100 (1918).

⁴ Cf. A. H. Compton and Oswald Rognley, *Science*, 46, 415 (1917); and *PHYS. REV.*, 11, 132 (1918) for preliminary accounts of this work.

X-rays, since the rays scattered by all the electrons will now be in the same phase. If the crystal is magnetized parallel to the reflecting face, the turning of the orbits will carry the electrons farther on the average from the middle of their atomic layers, the phase difference between the rays scattered by the different electrons will be greater, and a decrease in the intensity of reflection should result. If, on the other hand, the electrons are arranged isotropically in the atom, or if the atom is not rotated by a magnetic field, which would be the case if it is the individual electron or the positive nucleus that is the ultimate magnetic particle, no such change in the intensity of the reflected beam should be observed.

In our search for this effect a null method was employed. The ionization due to the beam of X-rays reflected from a crystal of magnetite was balanced against that due to a beam of the same wave-length reflected from a crystal of rock-salt, so that a very small change in the relative intensity of either beam could be detected while variations in the X-ray tube itself had but little effect. The experimental arrangement is shown diagrammatically in Fig. 1. From the target *A* X-rays fell upon

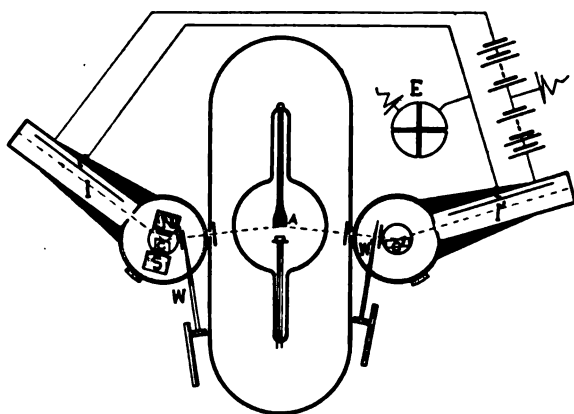


Fig. 1.

the crystals *C* and *C'*, and were reflected into the ionization chambers *I* and *I'*. One of these chambers was at a positive and the other at a negative potential, and their electrodes were both connected to the same pair of electrometer quadrants. Thus if the ionization current was the same in both chambers there was no deflection of the electrometer. The beams could be balanced accurately against each other by use of the aluminium absorption wedges *W* and *W'*, operated by means of micrometer screws. The crystal *C* was cemented to one pole of an electromagnet with a laminated core, so that by changing from direct to alter-

nating current which could be gradually reduced to zero it was possible to magnetize or demagnetize the crystal at will.

For the source of X-rays we used a Coolidge tube with a molybdenum anticathode, kindly supplied by Dr. W. D. Coolidge. As the source of high potential, a Snook-Roentgen machine was employed. The first order of the molybdenum α line ($\lambda = .721$ A. U.) was always reflected into the ionization chamber I' , while different orders of the same line were reflected by the magnetite crystal into the chamber I . It was possible with this arrangement to detect with certainty variations in the relative intensity of either reflected beam as small as 1 per cent. Unfortunately our magnet was not strong enough to saturate the magnetite crystal. The intensity of magnetization of the magnetite was about 150, which is just over $1/3$ of the saturation intensity for magnetite as determined by du Bois.¹

The effect of magnetization perpendicular to the plane of the crystal face was investigated in the first four orders. On account of mechanical difficulties, the test was made in only the third order spectrum when the crystal was magnetized parallel to the reflecting surface. In no case was a change as great as 1 per cent. observed in the intensity of the reflected beam when the crystal was magnetized or demagnetized. Our experimental results therefore may be summarized by the statement that *the intensity of the X-ray beam reflected from magnetite is not altered by as much as 1 per cent. in the first four orders when the crystal is magnetized to $1/3$ of saturation perpendicular to the reflecting surface, nor in the third order by similarly magnetizing the crystal parallel to the reflecting face.*

The Orientation of the Elementary Magnets.—The conclusions which are to be drawn from this experiment depend upon the conception that we have of what occurs to the ultimate magnetic particles when a ferromagnetic substance is magnetized. It is usually supposed that these elementary magnets have their axes so oriented by the external magnetic field that at saturation they are very nearly parallel with this field. Indeed, the manner in which this conception accounts for the phenomenon of saturation was originally perhaps the strongest argument in favor of the so-called "molecular" theory of magnetism. It should be noted, however, that the phenomenon of saturation is not in itself sufficient evidence that the ultimate magnetic particles have their axes oriented along the magnetic lines of force. This phenomenon would also occur if the elementary magnets were turned through only a small angle, but from one position of stable equilibrium to another beyond which it might be incapable of turning.

¹ Du Bois, Phil. Mag., 29, 293 (1890).

Perhaps the most convincing argument in favor of a complete rotation of the elementary magnets by a strong magnetic field is that which was brought forward originally by J. Swinburne¹ in connection with Ewing's suggestion that the orientation of the "molecular magnets" is determined by magnetic rather than by frictional forces. Swinburne points out that on this hypothesis, if the magnetization of a piece of iron is reversed by a strong rotating field instead of by a field alternating through zero, the loss in energy should be little or nothing, for *if the molecules rotate with the field* no unstable movements are possible. Experiments by F. G. Baily² on iron, by R. Beattie³ on nickel and cobalt, and especially by Weiss on pyrrhotite,⁴ show that this is actually the case. For large fields, where saturation is approached, the hysteresis loss per cycle when a rotating magnetic field is applied is found in every case to be only a small fraction of the hysteresis loss per cycle due to an alternating magnetic field. There would seem to be no reason for this great reduction in the hysteresis if the axes of the elementary magnets are unable to follow closely the rotating field. It seems difficult, therefore, to avoid the conclusion that *the elementary magnet*, whatever it may be, *is free to be oriented in any direction.*⁵ This conclusion will accordingly form the basis of our interpretation of the experiment just described.

¹ Cf. Encyclopædia Britannica, 11th ed., Vol. 17, p. 350.

The referee to whom the editors submitted this paper has offered the following comment: "It would seem that additional strong evidence is given by the experiments of Weiss and Kamerlingh Onnes in 'Researches sur L'Aimantation aux Tres Basses Temperatures,' *Journal de Physique*, vol. 9, pp. 555-584, 1910. In this research it is shown that the moment per c. c. resulting from complete orientation of the magnets differs by only 5 per cent in the case of nickel, 2 per cent in the case of iron, and 6 per cent in the case of magnetite from the experimentally measured saturation moment at ordinary temperatures."

It is indeed difficult to explain the very nearly complete saturation which this result indicates as due to anything other than an almost perfect alignment of the elementary magnets with the external field.

² Ibid.

³ Ibid.

⁴ Cf. E. H. Williams, *The Electron Theory of Magnetism*, p. 40.

⁵ In any case the axis of the ultimate magnetic particle must describe a closed conical surface under the action of a rotating magnetic field in order to account for the reduced hysteresis. This conical surface is the locus of the possible stable orientations of the axis. If the elementary magnet is free to align itself with the external rotating field, the apex angle of this cone will be π ; but if it is free to turn through only a small angle, the conical surface will have a sharp apex. The possibility suggests itself that the axis of this cone may represent some fixed axis in the elementary magnet, such as for example an electric axis, which may be slightly inclined to the magnetic axis and about which the magnetic axis may be free to turn. But any such rotation of one axis fixed in the particle about another means a rotation of the particle as a whole about the second axis. We cannot avoid in this manner, therefore, the conclusion that the elementary magnet is rotated by a rotating magnetic field.

In nature it is usual for an object to be capable of stable orientation in one direction,

The Molecule as the Ultimate Magnetic Particle.—It is obvious that if the elementary magnet consists of some group of atoms within the ferromagnetic material, the arrangement of the atoms in a crystal will be greatly altered when the groups of atoms are rotated by a magnetic field. One would therefore expect, as was pointed out by K. T. Compton and E. A. Trousdale, that magnetization would change the positions of the spots in Laue photographs taken through magnetic crystals, since these positions are determined by the arrangement of the atoms. As we have seen, the negative result obtained by these experimenters indicates that the arrangement of the atoms is not greatly changed by magnetization of ferromagnetic crystals.

The experiment which we have performed gives still more definite information with regard to the displacement of the atoms in a magnetic field. It has been shown by W. H. Bragg¹ that the minute changes in the positions of the atoms due to a relatively small rise in temperature is sufficient to affect appreciably the intensity of a reflected beam of X-rays. The negative result of our experiment shows clearly, therefore, that the atoms in a crystal are not moved as far by magnetization as they are by a change of temperature well within the melting point of the crystal. In fact direct calculation shows that a displacement of the atoms by $1/3000$ th of the distance between the successive atomic layers would have caused a detectable change in the intensity of the 4th order spectrum. Our experiment therefore affords a very sensitive confirmation of the conclusion reached by K. T. Compton and E. A. Trousdale that *the ultimate magnetic particle is not a group of atoms, such as the chemical molecule, but is the individual atom or something within the atom.*

The Atom as the Ultimate Magnetic Particle.—If the atom as a whole is the ultimate magnetic particle, it is natural to assume that its magnetic moment is due to the presence of electrons revolving in orbits. The simplest example of this type is the Rutherford form of atom, in which all the electronic orbits lie in the same plane, perpendicular to the magnetic axis. Let us therefore calculate the order of magnitude of the effect due to magnetization on the intensity of a reflected beam of X-rays if the reflecting crystal is composed of atoms of this type.

Professor W. H. Bragg has shown² that the intensity of reflection of or at most in a finite number of directions. The ability to possess stable orientation in any direction in the surface of a narrow cone would require a mechanism that is difficult to imagine. It is certainly a much more artificial hypothesis than the conception of freedom for orientation in any direction.

¹ W. H. Bragg and W. L. Bragg, *X-rays and Crystal Structure*, p. 195.

² W. H. Bragg, *Phil. Mag.*, 27, 881 (1914).

X-rays from a crystal in which the successive atomic layers are similar and similarly spaced falls off approximately according to the law

$$(1) \quad E = \frac{C(1 + \cos^2 2\theta)}{\sin^2 \theta} e^{-B \sin^2 \theta}.$$

In this expression C and B are constants, and θ is the glancing angle at which the X-rays strike the crystal face. This law represents an average for a number of different kinds of crystals. Darwin has found,¹ however, from theoretical considerations, that if all the electrons are in the mid-planes of the atomic layers to which they belong, the intensity should fall off according to the law,²

$$(2) \quad E_m = \frac{C'(1 + \cos^2 2\theta)}{\sin \theta \cos \theta} e^{-B \sin^2 \theta}.$$

This expression corresponds to reflection from a crystal composed of atoms of the Rutherford type which is magnetically saturated perpendicularly to the crystal face. Expression (1) indicates the reflection from an unmagnetized crystal, in which the magnetic axes of the atoms are oriented at random. The ratio of the intensity of reflection from the magnetized to that from the unmagnetized crystal should therefore be

$$\frac{E_m}{E} = \frac{C'}{C} \tan \theta.$$

For our approximate calculations, since θ is never large we may substitute $\sin \theta$ for $\tan \theta$. Thus in virtue of the relation $n\lambda = 2D \sin \theta$ this ratio becomes,

$$(3) \quad \frac{E_m}{E} = \frac{C'}{C} \cdot \frac{n\lambda}{2D} = kn,$$

where $k = C'\lambda/2CD$ is a coefficient independent of the order of reflection n . By extrapolation for the intensity of the X-ray spectrum line of zero order, where theoretical considerations show that E_m/E must be unity, it has been shown by one of the writers³ that for the first order reflection

¹ C. G. Darwin, *Phil. Mag.*, 27, 675 (1914).

² This expression assumes that the effective absorption coefficient of the X-rays in the crystal is the same for all orders of reflection. It is possible that the effective value is less in the higher orders. This would mean even less rapid diminution of intensity for the higher orders, and the predicted change due to magnetization would be even greater.

³ A. H. Compton, *Phys. Rev.*, 9, 52 (1917). It should be noted that Bragg's expression (1) must of necessity break down when applied to estimate the intensity of the zero order spectrum line. The extrapolation here referred to is accordingly based upon certain particular arrangements of the electrons in the atoms. Any reasonable arrangement, however, will lead to values for this ratio which do not differ greatly from those here given, so these values cannot be much in error.

from a cleavage face of rock-salt the ratio E_m/E is about 1.5 and for calcite is about 1.2. The ratio is doubtless of the same order of magnitude for other crystals. Taking 1.3 as a mean value when $n = 1$, this gives in place of equation (3),

$$(4) \quad E_m/E = 1.3n.$$

The intensity of reflection should therefore be increased by a factor of about 1.3 in the first order, 2.6 in the second order, 3.9 in the third order and 5.2 in the fourth order, if the crystal is composed wholly of atoms of the Rutherford type which are oriented with their axes parallel to the magnetic field.

It might be supposed that in the case of magnetite the iron atoms only would be oriented by the magnetic field while the oxygen atoms would remain unaffected. While this would reduce to some extent the change in intensity to be expected, it is obvious that the change would still be comparatively large. It is thus apparent that the hypothesis that atoms of the Rutherford type constitute the ultimate magnetic particles is incompatible with the negative result of our experiment.

Theoretical considerations seem to lead to the conclusion that, in order for an atom consisting of electrons revolving in orbits to be stable, the orbits must all lie in the same plane. It is therefore difficult to defend the hypothesis of a magnetic atom in which the electrons lie in different planes. It will be profitable, nevertheless, to consider the change to be expected in the intensity of the reflected X-ray beam if the atoms of the reflecting crystal have a more isotropic form.

Perhaps the most nearly isotropic form that has been suggested for the iron atom is that proposed by Hull¹ to account for the X-ray spectrum obtained from an iron crystal. This atom has two electrons near the center, the remaining 24 electrons being situated at the corners of three similarly oriented, concentric cubes whose diagonals are in the ratio $\frac{1}{4} : \frac{1}{2} : 1$. The diagonal of the outer cube, at whose corners the "valence electrons" are situated, he estimates as 1.25×10^{-8} cm. It is possible to calculate the order of magnitude of the effect due to the change in the orientation of such atoms by the help of the formula given by Darwin² and one of the writers,³ in which the energy of the different orders of the reflected X-ray beam is expressed as a function of the distances of the various electrons from the mid-planes of the layers of atoms to which they belong and the distance between the successive layers of atoms. Since the value of D for the natural (111) faces of magnetite is known, 4.10

¹ A. W. Hull, *Phys. Rev.*, 9, 85 (1917).

² C. G. Darwin, *loc. cit.*

³ A. H. Compton, *loc. cit.*

$\times 10^{-8}$ cm., the intensity of the reflected beam can thus be determined for any given orientation of the atoms and for any order of reflection.

In Table I. we have shown the results of this calculation for different orders of reflection from magnetite. The numbers given in the second and third columns represent the ratio of the intensity when the crystal

TABLE I.

Order.	E_1/E_u .	E_2/E_u .
1.....	1.000	1.004
2.....	.96	1.03
3.....	.86	1.09
4.....	.51	1.09

is magnetized to its value when unmagnetized. The intensity E_u for the unmagnetized crystal is calculated for random orientation of the atoms. In the second column the intensity E_1 is calculated for the case in which the cube face of the atom is parallel with the crystal face, *i.e.*, on the assumption that the magnetic axis of the atom is perpendicular to its cube face. The intensity E_2 , used in the third column, is estimated for the atoms with their cube diagonals perpendicular to the crystal face. In performing these calculations it has been assumed that the oxygen atoms in magnetite have their electrons arranged in the same manner as in calcite¹ and that they are unaffected by the magnetic field. It is also supposed that parallel to the (111) planes of magnetite, which were those used in the experiment, all the atomic layers are similar and are similarly spaced. Whether or not these assumptions are strictly accurate, the change to be expected if the atoms are rotated by a magnetic field should be of the order of magnitude here estimated.

It is obvious from these calculations that if the iron atoms in a magnetite crystal are of the type suggested by Hull, a rotation of the atoms due to magnetization would have easily been detected in our experiments.

The surprisingly large calculated variation in the intensity of the reflected beam for a magnetic atom as nearly isotropic as that proposed by Hull makes it appear improbable that any reasonable fixed distribution of the electrons in the iron atom would be so isotropic as to make possible a rotation of the atoms without detection. There remains the possibility that the electrons may be arranged at random as a sort of atmosphere about the atomic nucleus. While this would result in an atom which would be on the average isotropic, it is obvious that such an atom would not as a whole have any polarity and could not therefore

¹ A. H. Compton, *loc. cit.*

as a whole act as an elementary magnet. Subject to some uncertainty with regard to the extent of the orientation of the ultimate magnetic particles, the conclusion may therefore be drawn from our experiment that *the elementary magnet is very probably not the atom as a whole.*

If one admits the possibility of only a very slight change in orientation of the elementary magnets, the effect of magnetization might have escaped notice even though the atoms are not isotropic. For example, if the iron atoms are built on Bohr's model, the magnetic moment per atom will be so great that the saturation intensity of magnetization of iron would require a maximum change of orientation of only 6 degrees. This would produce an effect which if an iron crystal had been used would have just been detectable in our experiment, but with the crystal of magnetite which we employed would have been considerably smaller than the errors of measurement. However, the argument given above for the nearly complete orientation of the elementary magnets seems to eliminate the necessity of considering this possibility.

Other Possible Explanations of Ferromagnetism.—There remain four possible explanations of the magnetic properties of iron: (1) Only a small fraction of the total number of atoms may be subject to orientation by an external magnetic field, these atoms possessing a very large magnetic moment. (2) The magnetic properties of the atom may be due to a few electrons revolving in very small orbits, a change in the orientation of these orbits occurring without any change in the orientation of the remainder of the atom. (3) The positive nucleus of the atom may be magnetic and subject to orientation by the magnetic field. And (4) the electron itself may be magnetic and subject to orientation by the magnetic field.

1. A minimum limit may be placed upon the number of atoms of the Rutherford type which may be turned around by the magnetic field without being detected in our experiment. It has been shown above that a complete orientation of all the atoms should increase the intensity of reflection in the fourth order by a factor of 5. Thus the orientation of 1/500th of the atoms would be sufficient to produce a change of 1 per cent. Since in our experiment we worked at only about .4 of the saturation intensity of the magnetite crystal, for complete saturation it is possible that 1 atom out of 200 might be turned by the magnetic field.

Considerations of symmetry make it appear improbable that in a homogeneous crystal of similarly arranged atoms so small a fraction of the atoms should be subject to orientation by a magnetic field. But perhaps a more serious difficulty with this hypothesis is the large magnitude of the magnetic moment which it is necessary to assign to the

particular iron atoms which are subject to orientation. The magnetic moment per atom in magnetite is

$$\frac{\text{saturation intensity of magnetization}}{\text{number of atoms per cubic cm.}} = 5.1 \times 10^{-21} \text{ e.m.u.}$$

This value¹ is of the same order of magnitude as the moment 12×10^{-21} e.m.u. of an atom of oxygen,² for example, which can be calculated on the basis of Langevin's theory of a paramagnetic gas. If, however, only 1 in 200 of the atoms is subject to orientation by the external field, the magnetic moment of the mobile atoms must be $200 \times 5 \times 10^{-21}$ or 1×10^{-18} e.m.u. Suppose that all the electrons in this atom rotate in coplanar orbits, each with angular velocity $h/2\pi$, as assumed by Bohr. The magnetic moment per electron will then be 9.2×10^{-21} e.m.u.,³ and for all 26 electrons will be $26 \times 9.2 \times 10^{-21} = .24 \times 10^{-18}$ e.m.u. This hypothesis would thus necessitate an atom with the apparently prohibitive magnetic moment of not less than 4 times that supposed by Bohr.

2. If the magnetic properties of magnetite are to be accounted for by a single electron in each atom rotating in an orbit whose plane can be altered without changing the orientation of the atom as a whole, it can be shown that the radius of this orbit must be less than 1×10^{-9} cm. in order to account for the negative result of our experiment. A study of the relative intensities of the different orders of an X-ray spectrum line shows, however,⁴ that the outer electrons of an atom are at a distance of the order of 1×10^{-8} cm. from the center of the atom. If the mobile electrons are rotating about the nucleus, this hypothesis would therefore mean that the electrons responsible for the atom's magnetic properties are in one of the inner rings whose radius is less than 1/10 that of the atom. There are a number of effects, however, which are difficult to explain unless an atom's magnetic properties depend upon its surface electrons. Among these may be noted:⁵

1. The profound effect of chemical constitution on the magnetic properties of an atom,
2. The effect of temperature on magnetic properties,
3. The effect of mechanical jars in facilitating the orientation of the elementary magnets.

These phenomena make it appear improbable that the ultimate magnetic

¹ S. Dushman, *Theories of Magnetism*, p. 44.

² *Ibid.*, p. 24.

³ O. W. Richardson, *Electron Theory of Matter*, p. 395.

⁴ A. H. Compton, *PHYS. REV.*, 9, 52 (1917).

⁵ Cf. K. T. Compton and E. A. Trousedale, *loc. cit.*

particle consists of a ring of electrons revolving as near the center of the atom as the result of our experiment requires.

It is possible, however, that these small orbits may be those of the outer valence electrons revolving about positions of equilibrium. If this is the case, there must be intense forces acting on these electrons of a kind concerning which we have as yet no knowledge. Thus the central force required to hold an electron in so small an orbit with angular momentum sufficient to account for the magnetic moment of the iron atom in magnetite must not be less than that due to a positive charge of 10 electronic units placed at the center of the orbit. In view of our ignorance concerning the character of the infra-atomic forces, it cannot perhaps be said that forces of this magnitude do not occur near the surface of the atom. We have, however, no other reason to suspect the existence of such forces, and the assumption that they exist would mean a very radical departure from our usual ideas of electrodynamics. One would therefore wish to consider this explanation of our experiment only as a last resort.

3. The considerations just brought forward as indicating that the magnetic properties of an atom depend in large measure upon its surface electrons are of equal weight as opposed to the hypothesis that the positive nucleus is the elementary magnet. Moreover the experiments of Barnett on magnetization¹ by rotation and by Stewart on rotation by magnetization² show that at least the major part of ferromagnetism is due to the motion of *negative* electricity. This is difficult to explain if the *positive* nucleus of the atom is the ultimate magnetic particle. Our experiment, however, brings forth no new evidence on this point.

4. It is clear that the result of our experiment is in accord with the hypothesis that the electron itself is the ultimate magnetic particle, unless the electron is assumed to have dimensions so great that a change in its orientation will produce an appreciable effect. Let us suppose with Parson³ that the electron is a ring of electricity with a magnetic axis perpendicular to its plane, and in addition that on magnetization the axis of the electron is brought perpendicular to the reflecting surface of the crystal. If all the electrons are oriented by magnetization and if it is assumed that the different elements of the electron move under the action of the incident wave as if they were independent charged particles of definite mass, calculation then shows that the effect should not be noticeable in our experiment if the radius of the electron is less than 4×10^{-10} cm. Thus if the radius is 2×10^{-10} cm. as one of us has esti-

¹ S. J. Barnett, loc. cit.

² J. Q. Stewart, loc. cit.

³ A. L. Parson, Smithsonian Misc. Collections, Nov., 1915.

mated on the basis of the scattering of X-rays and gamma rays,¹ the hypothesis that the electron is the ultimate magnetic particle is in accord with our experimental results.

Conclusions.—The fact that on magnetizing the crystal of magnetite the intensity of the reflected X-ray beam did not change by as much as 1 per cent. in any of the first four orders therefore supports the conclusion of K. T. Compton and E. A. Trousdale that the elementary magnet in a ferromagnetic substance is not a group of atoms.

Subject to the validity of our argument for the nearly complete alignment of the elementary magnets with the external magnetic field, it is also apparently impossible to explain this result on the hypothesis that the atom as a whole acts as the ultimate magnet, since an orientation of the atoms by the applied magnetic field would have made a noticeable change in the intensity of the reflected X-ray beam.

The hypotheses that only a small fraction of the atoms are turned by the magnetic field or that the magnetic effects are due to certain electrons whose orbits may change without affecting the remainder of the atom, have not appeared to be plausible explanations of our result.

Our experiment is in accord with the suggestion that *the magnetic properties of matter are due either to the nucleus of the atom or to the individual electrons*. Auxiliary evidence, however, indicates that *the electron is the more probable elementary magnet*.

This experiment was performed in the Physics Laboratory of the University of Minnesota during the winter of 1916-17. We take pleasure in thanking Professor K. T. Compton, of Princeton University, for his helpful suggestions in interpreting the results of this work.

A. H. C.,
Cavendish Laboratory, Cambridge.
O. R.,
Bureau of Standards, Washington.

May 22, 1920.

¹ A. H. Compton, *PHYS. REV.*, 14, 31 (1919).

ON THE FREE OSCILLATIONS OF SPHEROIDS.

BY RAJENDRA NATH GHOSH.

SYNOPSIS.

Oscillations of liquid drops, assuming the potential energy for any momentary form to be that due to capillarity only. Rayleigh's method of energy is applied so as to yield a formula for the frequency of oscillation for cases where the momentary surface differs from that of equilibrium by a spherical harmonic deviation of arbitrary order and rank.

THE theory of oscillations due to capillarity of spherical drops and cylinders of liquids has been discussed in detail by the late Lord Rayleigh,¹ when the displacements of each particle are infinitesimal. In practice the theory requires certain modifications, as has been shown by Pederson² and Bohr,³ of whom the latter has particularly found that the discrepancy between theory and practice is due mostly to the finite displacements of the particles, and has calculated the effect in the simple case of two-dimensional oscillations of circular laminæ of fluids. The case of spheroidal drops of fluids does not seem to have been touched, and the present note contains an attempt to that effect. We shall follow Rayleigh's method of energy, and make use of Spheroidal Harmonics in the Calculation. Professor G. H. Bryan⁴ has discussed the forced oscillations and waves on a rotating spheroid. He has shown that the frequency of free waves depends upon the eccentricity of the spheroid, the rank and degree of the Harmonics. In the present case of free oscillations due to capillarity, the frequency depends entirely upon the dimensions of the spheroid, the capillary force and the rank and degree of the Harmonics.

In order to effect the calculation, we begin by putting down the volume and surface of a spheroid having a and c for axes, $a < c$.

The volume V is given by

$$V = \frac{4}{3} \pi a c^2 \quad (1)$$

and the surface S is approximately given by

$$S = 4 \pi c^2 (1 - \frac{1}{2} e^2). \quad (2)$$

¹ Theory of Sound, Vol. II, Arts. 364.

² Phil. Trans., Vol. 207 (1907).

³ Phil. Trans., Vol. 209 (1909).

⁴ Phil. Trans., Vol. 150 (1889).

Let us now assume that the radius vector bounding the liquid surface at time t at any point, θ, ϕ , is given by¹ where $P_n^s(\mu)$ is the associated

$$r = a_0 + \sum_1^n \alpha_n P_n^s(\mu) \cos s\phi \quad (3)$$

function.

For the deformed volume V' included within the liquid surface we have

$$V' = \frac{4}{3}\pi a_0^3 + \sum_1^n \frac{2\pi}{2n+1} \cdot \alpha_n^2 a_0 \frac{n+s!}{n-s!}. \quad (4)$$

Since the density of the fluid remains the same we have after equating the volumes V and V'

$$a_0 \left[1 + \frac{3}{2} \sum_1^n \frac{\alpha_n^2/a_0^2}{2n+1} \frac{n+s!}{n-s!} \right]^{1/3} = c(1 - e^2)^{1/6}. \quad (5)$$

The deformed surface S' is determined by the integral

$$S' = \int_0^{2\pi} \int_{-1}^{+1} d\mu d\phi \left\{ r^2 + \frac{1}{2} \left(\frac{dr}{d\theta} \right)^2 + \frac{1}{2} \frac{1}{(1-\mu^2)} \left(\frac{dr}{d\phi} \right)^2 \right\}.$$

On substituting the value of r from (3) we obtain

$$\int_0^{2\pi} \int_{-1}^{+1} r^2 d\mu d\phi = 4\pi a_0^2 + \sum_1^n \frac{2\pi \alpha_n^2}{2n+1} \frac{n+s!}{n-s!}$$

and

$$\begin{aligned} \frac{1}{2} \int_0^{2\pi} \int_{-1}^{+1} \left(\frac{dr}{d\theta} \right)^2 d\theta d\mu \\ = \sum_1^n \frac{1}{2} \alpha_n^2 \int_0^{2\pi} \int_{-1}^{+1} (1-\mu^2) \left[\frac{d}{d\mu} P_n^s(\mu) \right]^2 \cos^2 s\phi d\mu d\phi. \end{aligned}$$

In order to effect the integration we observe that

$$\begin{aligned} \int_{-1}^{+1} P_n^s(\mu) \frac{d}{d\mu} \left\{ (1-\mu^2) \frac{d}{d\mu} P_n^s(\mu) \right\} d\mu \\ = \int_{-1}^{+1} \frac{s^2}{1-\mu^2} [P_n^s(\mu)]^2 d\mu - n(n+1) \int_{-1}^{+1} [P_n^s(\mu)]^2 d\mu \end{aligned}$$

or

$$\int_{-1}^{+1} (1-\mu^2) \left[\frac{d}{d\mu} P_n^s(\mu) \right]^2 d\mu = \frac{2n(n+1)}{2n+1} \frac{n+s!}{n-s!} - \frac{s}{1} \frac{n+s!}{n-s!}.$$

Hence

$$\begin{aligned} \sum_1^n \frac{1}{2} \alpha_n^2 \int_0^{2\pi} \int_{-1}^{+1} (1-\mu^2) \left[\frac{d}{d\mu} P_n^s(\mu) \right]^2 d\mu \cos^2 s\phi d\phi \\ = \sum_1^n \frac{\pi \alpha_n^2 n(n+1)}{2n+1} \frac{n+s!}{n-s!} - \frac{\pi s \alpha_n^2}{2} \frac{n+s!}{n-s!} \end{aligned}$$

¹ This has been shown in the case of spherical drops. Todhunter's Functions of Laplace Lamé and Bessel, Art. 138. See also Byerly's Spherical Harmonics, page 197.

and lastly

$$\frac{1}{2} \int_0^{2\pi} \int_{-1}^{+1} \frac{1}{1-\mu^2} \left(\frac{dr}{d\phi} \right)^2 d\mu d\phi = \sum_1 \frac{\pi \alpha_n^2 s}{2} \cdot \frac{n+s!}{n-s!}.$$

Therefore the whole deformed surface S' is equal to

$$S' = 4\pi a_0^2 + \sum_1^n \frac{\pi \alpha_n^2}{2n+1} (n^2 + n + 2) \frac{n+s!}{n-s!}. \quad (6)$$

The velocity potential ψ of the fluid motion must satisfy the equation

$$\frac{\partial}{\partial \mu} \left\{ (1-\mu^2) \frac{\partial \psi}{\partial \mu} \right\} + \frac{1}{1-\mu^2} \frac{\partial^2 \psi}{\partial \phi^2} = -\frac{\partial}{\partial \xi} \left\{ (1+\xi^2) \frac{\partial \psi}{\partial \xi} \right\} + \frac{1}{1+\xi^2} \frac{\partial^2 \psi}{\partial \phi^2},$$

where μ , ϕ and ξ form an orthogonal system of coördinates. The solution of this equation is given in tesseral Harmonics

$$\psi = \sum_1^n B_n P_n^s(\mu) p_n^s(\xi) \cos s\phi, \quad (7)$$

$$p_n^s(\xi) = \frac{2n!}{2^n} \frac{(1+\xi^2)^{s/2}}{(n-s)! n!} \left\{ \xi^{n-s} + \frac{(n-s)(n-s-1)}{2(2n-1)} \xi^{n-s-2} + \dots \right\},$$

B_n being determined from the boundary condition where we must have

$$\frac{\partial \psi}{\partial s_\xi} = -\frac{\partial s_\xi}{\partial t} \quad \text{when} \quad \xi = \xi_0. \quad (8)$$

The kinetic energy T of fluid motion is given by the volume integral

$$T = \frac{1}{2} \rho \iiint \left(\frac{\partial \psi}{\partial x} \right)^2 + \left(\frac{\partial \psi}{\partial y} \right)^2 + \left(\frac{\partial \psi}{\partial z} \right)^2 dx dy dz,$$

or by Green's theorem since $\Delta^2 \psi = 0$ we obtain

$$T = \frac{1}{2} \rho \int_0^{2\pi} \int_{-1}^{+1} \psi \frac{\partial \psi}{\partial s_\xi} ds_\mu ds_\phi.$$

Evaluating the integral we obtain

$$T = \frac{\pi \rho B_n^2 k}{e^2 (2n+1)} \frac{n+s!}{n-s!} p_n^s(\xi_0) \frac{\partial (p_n^s \xi)}{\partial \xi_0}, \quad (9)$$

where $k = a/\xi_0$.

From the condition expressed in (8) we obtain the value of B_n

$$B_n = \frac{k}{e} \left(\frac{d\alpha_n}{dt} \right) \frac{1}{\frac{\partial p_n^s(\xi_0)}{\partial \xi_0}};$$

hence the kinetic energy is finally found to be given by

$$T = \frac{\pi \rho}{e^2 (2n+1)} \frac{n+s!}{n-s!} \frac{k^3 p_n^s(\xi_0)}{\frac{\partial p_n^s(\xi)}{\partial \xi_0}} \cdot \left(\frac{d\alpha_n}{dt} \right)^2. \quad (10)$$

¹ G. H. Darwin, Phil. Trans., 1901, page 461.

The potential energy V of the system in any configuration, due to the capillary force, is simply proportional to the increment of the surface, viz., $S' - S$. Hence we get the potential energy

$$V = \sum_1^n T \pi \alpha_n^2 \frac{(n^2 + n - 2)}{2n + 1} \cdot \frac{n + s!}{n - s!}, \quad (11)$$

where T is the capillary force.

The sum of the potential and kinetic energies must always be constant, from which we readily obtain the relation

$$4\pi^2 N^2 = \frac{(n+2)(n-1)T}{\rho c^3} \cdot e \cdot \left[\frac{\frac{dp_n^*(\xi_0)}{d\xi_0}}{p_n^*(\xi_0)} \right], \text{ where } N \text{ is the frequency.} \\ \text{(approx)} \quad \left[\frac{dp_n^*(\xi_0)}{p_n^*(\xi_0)} \right]$$

The above formula is proposed tentatively, though we have no means at present to test it by experiment.

EWING CHRISTIAN COLLEGE,
ALLAHABAD, INDIA.

SOFT X-RAYS; A NOTE OF INTERPRETATION.

BY H. M. DADOURIAN.

SYNOPSIS.

Discussion of Earlier Paper.—Some of the results described in an earlier paper are discussed and are shown to be capable of interpretations different from those given by Miss Laird.

Additional Facts.—Certain experimental details and results which did not appear in the original paper are given here as further evidence for the soundness of the conclusions therein derived.

A NOTE¹ by Miss Elizabeth Laird, appearing in the April number of this journal raises several questions concerning the results described in my paper on soft X-rays.²

Miss Laird perhaps did not read my paper with the utmost care since she says, "No statement is made of the vacuum obtained," notwithstanding my paragraph on page 237 in which occurs the following sentence: "The three-forked brass tube, *T*, connects each of the three compartments of the main apparatus directly to a charcoal-liquid-air bulb through a glass stopcock with a bore one cm. in diameter." This statement, I thought, would be sufficient to convey to the ordinary reader the fact that a liquid-air vacuum was maintained during the experiments in question and did not consider it necessary to put in my original paper the following reassuring details: Before taking each set of readings the large stopcock *L*, Fig. 1, was closed and the cocoanut charcoal in the liquid-air-charcoal bulb, *B*, was re-vitalized by pumping the bulb (while the bulb was heated by means of a Bunsen burner) with an oil pump capable of giving a pressure of 0.001 mm. After pumping for about an hour the valve, *M*, was closed to reduce the distillation of mercury vapor from the McLeod gauge into the charcoal bulb. Then the small stopcock, *s*, was closed and a two-liter flask with liquid-air was applied to the charcoal bulb. The tubing connecting the charcoal bulb to the main apparatus was about 12 mm. in diameter and only about 30 cm. long.

Again, Miss Laird says, "My apparatus was more favorable as the airtight window kept out ions and positive rays." By this and

¹ PHYSICAL REVIEW, 15, p. 293, 1920.

² PHYSICAL REVIEW, 14, p. 234, 1919.

by repeated emphasis upon her " airtight window " she gives the impression that the window of my apparatus was not airtight and that airtightness was necessary. Had Miss Laird read my paper with more care she would have found in it the following facts: (a) that my apparatus was so constructed that the window could be made successively airtight

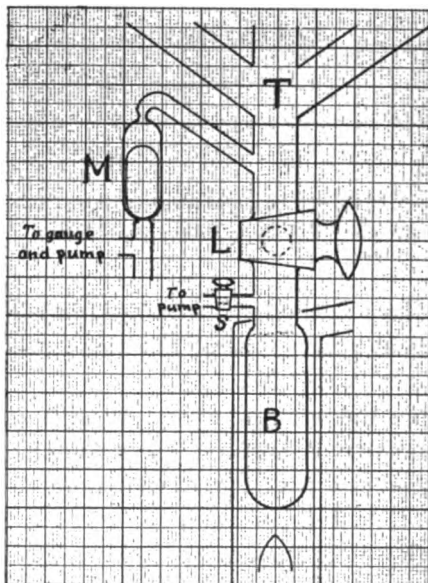


Fig. 1.

yet open to radiation, open to everything, or closed to everything; (b) that it was used in these three ways; and (c) that airtightness of the window was shown to be unnecessary.

In questioning the effectiveness of my electrostatic trap Miss Laird says, " Nothing is said as to whether the field was reversed or not," and brings in the evidence that the field in the electrostatic trap was not radial. As a matter of fact, in some of my preliminary experiments the field was reversed; furthermore the plates of the trap were connected to the terminals of a battery the middle point of which was to earth; this could be done because both of the plates were insulated. It was found that so long as the field was sufficiently strong the trap was effective irrespective of the way it was obtained. Consequently it was not considered necessary to insulate both plates in the final apparatus. Had not this result been obvious from elementary theoretical considerations and had I not developed a strong reaction against padding scientific papers with unnecessary facts and formulas, I certainly would have

incorporated the foregoing information in my paper. As to the distribution of the field, a little consideration will show that the effectiveness of the electrostatic trap depends not upon whether the field is radial or not but upon whether it is at right-angles to the velocity of the charged particles or not. This condition was fulfilled by my apparatus. As a further criticism Miss Laird says that "light is not thus trapped" by my electrostatic trap. She seems to forget that the only kind of "light" which could produce the radioelectric effects measured was the radiation due to bombardment by cathode-electrons and that it was the function of the electrostatic trap to permit the free passage of that "light" and not to trap it even if it could.

On page 294 Miss Laird says, "Dadourian's second argument rests on his attempts to measure the velocity of his photoelectrons. In Fig. 4 he shows a curve obtained by the retarded field method at 390 volts on the discharge tube. At this potential the presence of an X-radiation is not doubted, though it may be accompanied by light and other effects. I have previously shown a curve (Fig. 6, Curve II.) at 550 volts not intersecting the axis until 20 volts retarding potential is reached, as compared with his 5 volts." Here Miss Laird gives the impression that I had obtained doubtful evidence for the production of radioelectrons of only 5-volt velocity while she had obtained electrons having velocities corresponding to 20 volts. But here it must be remembered that the curve (I. of Fig. 4) which she selects for comparison represents the results of a preliminary experiment, that while it crosses the potential axis at -5 volts there is another curve in the same Fig. 4 which intersects the potential axis at -27 volts, and that in Fig. 5 there is a curve which represents results obtained under more favorable experimental conditions and which does not intersect the potential axis at all.

Referring to the effect of the field of the outer gauze upon the field of the inner gauze of my radioelectric chamber Miss Laird says, "Since in Dadourian's experiments $+5$ volts sufficed to hold back the great majority of photoelectrons, the use of $+200$ volts on the second gauze makes it highly probable that the effect of this was to counteract the assumed retarding field." There is no need to wander in the realm of probability in this connection. A glance at curve III. of Fig. 4 shows that the effect which Miss Laird would explain away by means of the second gauze is just as prominent in results obtained without the second gauze. Furthermore a comparison of the two curves of Fig. 5 shows that the effect in question is increased considerably by lamp-blackening the gauzes in spite of the $+200$ on the second gauze. Miss Laird's appeal in this connection to Von der Bijl's work with amplifiers does

not alter the situation. In amplifiers the grid is made of fine parallel wires whose distance apart is about equal to their distance from the plate and the source of electrons is very near the grid wires, while in my experiments gauzes with 1 mm. meshes were placed more than 3 mm. apart; the effective distance of the source of electrons from the inner cage being over 20 mm. Considering the relative dimensions involved and applying the inverse square law I find that the infiltration effect of the field of the "plate" into the "grid" should be in my apparatus less than one tenth of one per cent. of the effect in the amplifiers used in wireless work.

Miss Laird also finds one contradiction in my paper. She says, "It may be noted that the curves of Fig. 3 show apparently a greater proportion of the effect measured transmitted by celloidin at 80 volts than at 380. Contrary to what is given in Fig. 8." The part of the curves corresponding to 80 volts from which Miss Laird computes the proportion transmitted by the celloidin could not be used for that purpose because the corresponding readings were very small and consequently involved relatively large errors of observation. In the figure these readings appear measurable because curve I. was plotted on a very large scale in order to make its initial point coincide with that of the corresponding curve for 380 volts. It is stated in my paper that the curves were plotted on different scales. With her experience with cathode voltages below 200 she of course knows that the curves of Fig. 3 could not be used for comparing the transparency of the celloidin window for different cathode voltages.

The main object of Miss Laird's note is to question that soft X-rays are produced by cathode potentials below 200 volts. By different methods H. Dember,¹ R. Whiddington,² and Sir J. J. Thomson³ obtained evidence for the production of soft X-rays with as low cathode potentials as 18.7, 128, and 10 volts, respectively. In a previous paper⁴ Miss Laird disposed of these men and their evidence with the statement: "Various experiments make the results of previous observations of a Röntgen radiation produced by cathode rays of less than 200-volt velocity appear doubtful." Miss Laird now admits that some kind of radiation must be produced by the impact of electrons moving with less than 200 volt velocity but denies that this radiation can be called soft X-rays. She says, "It is not questioned that electrons of less than 200-volt velocity produce radiation by impact, but there appears as yet no proof that

¹ H. Dember, *Verh. Deutch. Phys. Ges.*, 15, p. 560, 1913.

² R. Whiddington, *Camb. Phil. Soc.*, 17, p. 144, 1913.

³ J. J. Thomson, *Phil. Mag.*, 28, p. 620, 1914.

⁴ E. R. Laird, *Ann. der Phys.*, 46, 605, 1915.

such radiation lies in the region designated as belonging to soft X-rays." Here the discussion is brought to a question of definition of the term soft X-rays. Miss Laird does not state the limits of "the region designated as belonging to soft X-rays," for no such limits had been designated until for reasons of expediency I defined the term "soft X-rays" in my paper as radiations, produced by the impact of electrons, which fall between the shortest known ultra-violet rays and the longest waves of characteristic X-radiation measured at the time my paper was written.

VELOCITY OF SOUND FROM A MOVING SOURCE.

BY R. B. ABBOTT AND J. W. COOK.

SYNOPSIS.

Poynting's Theoretical Velocity Equation and a discussion of it are quoted from his original article. The equation is $X = (E - r_0 U^2)(dy/dx)$, where X = external pressure, U = resultant relative velocity of propagation, E = elasticity, dy/dx = volume change and r_0 = density. The *corroborations* obtained by Major Alter are stated to be: first, hearing the whistle of high velocity shells before they strike a target near the observer; second, calculations, based upon Poynting's equation, of the velocities measured by Mach, Berthelot and Wolf at various distances from the points of explosions. The calculated values agree with the observed values.

Observations taken in the *laboratory* did not show any change in the velocity of sound due to a spark source moving 8,550 cm. *per second*. The source consisted of a spark gap mounted on the end of a vane rotating 30 R. P. S. The spark was made to occur at the same point once in a revolution. Opposite points on the spherical wave front were located by two telephone transmitters and one receiver connected to a differential transformer. A minimum sound in the receiver indicated that the wave front arrived at the two transmitters simultaneously. The radii of the spherical waves measured varied from 190 to 260 centimeters. Normal room temperature and pressure prevailed.

POYNTING'S THEORY.

IN certain recent observations and calculations, there has arisen in connection with wave propagation, an important problem with regard to the relative velocity of the waves emitted from a moving source. This problem appears to have been first considered by Poynting¹ in the form of an equation which he has developed for the propagation of longitudinal waves in an elastic medium.

In the words of the article:

"The velocity with which waves of longitudinal disturbance travel in the air or in any other fluid can be calculated from the resistance to compression and extension and the density of the fluid. . . .

"Whatever the form of the wave, we could always force it to travel on with that form unchanged, and with any velocity we chose, if we could apply any 'external' force we liked to each particle, in addition to the 'internal' force called into play by the compressions and extensions. . . .

¹ Encyclopædia Britannica, Vol. 25, 11th ed., p. 440. Poynting's Equation is:

$$X = (E - r_0 U^2)(dy/dx). \quad (5)$$

X , external pressure; U , resultant relative velocity of propagation; E , elasticity; dy/dx volume change; r_0 , density.

"We shall investigate the external force needed to make a train of plane waves travel on unchanged in form with velocity U

"We shall regard the external force as applied in the form of a pressure X per square centimeter parallel to the line of propagation and varied from point to point as required in order to make the disturbance travel on unchanged in form with the specified velocity U . In addition there will be the internal force due to the change in volume, and consequent change in pressure, from point to point. . . .

"Suppose that the whole of the medium is moved backward in space along the line of propagation so that the undisturbed portions travel with the velocity U . The disturbance, or the train of waves, is then fixed in space, the fresh matter continually enters the disturbed region at one end, undergoes the disturbance, and then leaves it at the other end. . . .

"If then we apply a pressure X given by (5)¹ at every point, and move the medium with any uniform velocity U , the disturbance remains fixed in space. Or if we now keep the undisturbed parts of the medium fixed, the disturbance travels on with velocity U if we apply the pressure X at every point of the disturbance."

CORROBORATIONS.

Major Dinsmore Alter¹ reports very interesting results noted while aboard a target-pulling tug. In listening for the whistling of shells coming directly towards the tug at about twice the normal velocity of sound, those aboard were able to hear, before the splash occurred, four out of seven shells fired. The whistling, in these cases, preceding the splash by about two seconds. He also states that more than half a dozen coast artillery officers have told him of hearing the whistling before the shell struck.

Major Alter determined the excess velocities, over the normal, from data obtained by Mach, Berthelot and Wolf² on the velocity of explosive sounds at various distances from the point of explosion and compares these with the excess velocities which would be expected by Poynting's equation. He finds these to agree very closely, thus seeming to corroborate the dependency of the velocity of the longitudinal sound waves upon the velocity of the source emitting them.

LABORATORY TESTS.

The following laboratory methods can be employed to make experimental tests of the theory.

¹ Lick Observatory Bulletin No. 310, Vol. IX., June 13, 1918.

² Handbuch der Physik, Winkelmann. Band II, Seite 523-528.

1. Telephonic method employing similar transmitters to vary normally equal currents of electricity through two independent opposing primary windings of a telephone transformer as used by Hebb¹ in his determination of the velocity of sound.

2. Change of resistance method, in essence similar to the first, but employing, instead of transmitters, heated wires which change resistance upon the impingement of sound compressional waves. This change of resistance then causes an electric current to actuate two elements of a three-element oscillograph, the third element being used for timing.

3. Photographic method in which the compressional wave is actually photographed (as already accomplished by Foley and others for stationary sources) in several positions as it expands, together with the moving source. In this way by direct measurement the relationship of velocity of source to velocity of waves can be determined absolutely.

The first method; viz., by the use of the Hebb telephone transformer, was that adopted for the present determinations, and will be discussed at length.

In order to illustrate the principle of the Hebb telephone method, Fig. 1 is presented. In this figure, let us assume ²S to be a source emitting

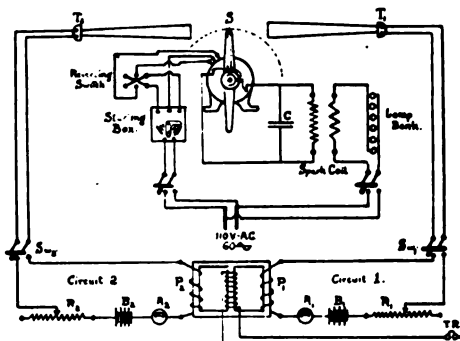


Fig. 1.

sound waves at a definite, known frequency such as would be produced by:

1. Whistle of definite pitch.
2. Buzzer of definite pitch.
3. Tuning fork.
4. Spark discharge, timed.

With such a source each compressional wave reaches the two points

¹ (a) T. C. Hebb, *PHYSICAL REVIEW*, Vol. XX., 1905, p. 89, "The Velocity of Sound."
 (b) T. C. Hebb, *PHYSICAL REVIEW*, Vol. XIV., 2d Series, 1919, p. 74, "The Velocity of Sound, and the Ratio of the Specific Heats of Air."
 (c) Barton, *Text Book of Sound*, p. 536.

of a pair such as A and A' , or B and B' , etc., simultaneously. Consequently simultaneous similar variations in the currents of Circuit 1 and Circuit 2 are set up through the two ordinary carbon grain telephone transmitters, T_1 and T_2 , placed equidistant from S at any two points of a pair. These currents pass through the equal primaries, P_1 and P_2 of the transformer, wound to oppose one another, and are neutralized if they are equal and simultaneous so that a minimum sound is heard in the telephones. In a similar manner a minimum of sound also is observed if T_1 is at any one of the points such as F , and T_2 is at any other point on either side of S , the points being at any whole multiple of the wave-length from S , or placed anywhere in the field of the sound with differences from S of exact multiples of the wave-length. But if one of the transmitters be on one of the above mentioned points and the other be not, the compressional waves will vary the currents in the two circuits at different times thus inducing secondary currents in the transformer which cause a sound in the telephones. Therefore, it becomes a very simple matter to determine the velocity of sound from the relationship

$$v = nL,$$

v , velocity of sound; n , vibration frequency of source, S ; L , wave-length. The frequency, n , has been determined previously, and the wave-length, L , is measured directly by keeping one of the transmitters at a constant distance from S and by moving the other through several points producing minimum sound. The average distance between such points is L .

EXPERIMENTAL WORK.

A moving sound source was obtained by mounting a spark gap on the end of a wooden vane which was revolved by a motor running at a constant speed, the position of the vane upon the passage of a spark being controlled by a commutator attached to the vane. This vane was constructed of $\frac{1}{2}$ inch pine 4 inches wide at the hub, tapering to 2 inches at the ends, being 91 centimeters long, and carefully stream-lined. A disc 6 inches in diameter was cut from fibre and accurately bolted to the vane at the center. Two brass segments were dovetailed into the periphery of this disc 90 degrees apart, and two wires connecting them with the gap at the end were inlaid in the wood. The radius of the gap about the shaft was 45.5 centimeters, and the vane attached to an 1800 R.P.M. synchronous motor gave a linear velocity to the source of sound of 8550 cm./sec. This is approximately one fourth of the velocity of sound. Fig. 1 illustrates the apparatus and the wiring. The spark coil, condenser, and lamp bank were adjusted to give but one spark

discharge per half cycle with the vane stationary. Then with the motor running, a single discharge was certain to be obtained at S at every revolution.

In order that the minimum might be the more easily read two similar transmitters were selected and adjusted by means of the rheostats R_1 and R_2 to give as nearly as possible the same pitch and the same loudness of tone. This adjustment was accomplished by listening in at the telephone receivers to the stationary spark as an assistant closed and opened circuits 1 and 2 alternately, varying the resistance as signaled by the observer. The telephone receivers were placed in a room in another part of the building together with a signaling system both to eliminate outside disturbances and also to remove the tendency of the observer to anticipate the minima. In order to eliminate the noise of the motor as much as possible, two large trumpet-like tubes of wrapping paper with 6-inch-diameter mouths were fitted to the transmitters. These tubes were about 8 feet in length, and could be adjusted so that hardly any outside disturbance was noticeable above the spark sound.

The method of observing with spark gap stationary or moving was essentially the same and was as follows: The observer was stationed at the telephone receivers in the distant room while the assistant operated the other apparatus. Transmitter T_2 was placed at a measured distance from spark gap S , and was left fixed while transmitter T_1 was slowly moved towards and then away from the spark gap. Each time a minimum sound was detected in the phones by the observer he signaled the assistant who immediately marked the position of the transmitter T_1 . Ten such readings were obtained from each of three observers for each setting of T_2 with the gap stationary, and a similar set was taken with the gap moving towards T_2 , and a third similar set with the motor reversed and the gap moving towards T_1 . Average readings were then computed for each set. In this way the personal equation of any one observer was eliminated in the average. The data obtained in this way are presented in the Tables I. to III. inclusive.

If the variable velocity theory were true, in this case, we should have the average readings of T_1 very much nearer T_2 with the gap moving towards the latter than with the gap stationary. Similarly, with the gap S moving away from T_2 and towards T_1 the minimum setting for T_1 should be further from T_2 than when the gap is stationary. With the particular settings employed of T_1 and T_2 if we assume the theory true, the minimum point for the moving gap should be advanced from the minimum point of the stationary gap by approximately 160 centimeters. However, no such separation of the three minimum points occurs, and

TABLE I.

*Distance of T_1 from Spark-gap to Give a Minimum Sound in Phones,
while T_1 is Fixed at 261 Cm. from Gap.*

Gap Moving Toward T_1 Veloc. = $8550 \frac{\text{Cm.}}{\text{Sec.}}$	Gap Stationary Veloc. = $0 \frac{\text{Cm.}}{\text{Sec.}}$	Gap Moving Towards T_1 Veloc. = $-8550 \frac{\text{Cm.}}{\text{Sec.}}$
OBSERVER A.		
258. cm.	259. cm.	259. cm.
258.5 "	259. "	259.5 "
258.5 "	259.5 "	260. "
259. "	260. "	260.5 "
260. "	260. "	261.5 "
260.5 "	260. "	261.5 "
260.5 "	260.5 "	262. "
261. "	260.5 "	263. "
261.5 "	261. "	263. "
262. "	261. "	263. "
Means, 260. cm.	260. cm.	261.3 cm.
OBSERVER B.		
258.5 cm.	259. cm.	260. cm.
259. "	259.5 "	260. "
259.5 "	260. "	260. "
260. "	260. "	261. "
260. "	260.5 "	261.5 "
261. "	261. "	261.5 "
261. "	261.5 "	262. "
261.5 "	261.5 "	262. "
262. "	262. "	262.5 "
262. "	262.5 "	263. "
Means, 260.5 cm.	260.8 cm.	261.4 cm.
OBSERVER C.		
259. cm.	260. cm.	260. cm.
259.5 "	260. "	260.5 "
260. "	260.5 "	261. "
260. "	261. "	261. "
260.5 "	262. "	261. "
260.5 "	262.5 "	261.5 "
261. "	262.5 "	262. "
261.5 "	263. "	262. "
262. "	263.5 "	262.5 "
262.5 "	264. "	263. "
Means, 260.7 "	261.9 cm.	261.5 cm.

instead, within the limits of experimental error, they fall precisely together.

TABLE II.

*Distance of T_1 from Spark-gap to Give a Minimum Sound in Phones,
while T_1 is Fixed at 219 Cm. from Gap.*

Gap Moving Towards T_1 Veloc. -8550 $\frac{\text{Cm.}}{\text{Sec.}}$	Gap Stationary Veloc. = 0 $\frac{\text{Cm.}}{\text{Sec.}}$	Gap Moving Towards T_1 Veloc. -8550 $\frac{\text{Cm.}}{\text{Sec.}}$
OBSERVER A.		
Means, 218.2 cm.	222.2 cm.	219.9 cm.
OBSERVER B.		
Means, 218.2 cm.	222. cm.	220.4 cm.
OBSERVER E.		
Means, 217.8 cm.	221.7 cm.	219.2 cm.

TABLE III.

*Distance of T_1 from Spark-gap to Give a Minimum Sound in Phones,
while T_1 is Fixed at 186 Cm. from Gap.*

Gap Moving Towards T_1 Veloc. -8550 $\frac{\text{Cm.}}{\text{Sec.}}$	Gap Stationary Veloc. = 0 $\frac{\text{Cm.}}{\text{Sec.}}$	Gap Moving Towards T_1 Veloc. -8550 $\frac{\text{Cm.}}{\text{Sec.}}$
OBSERVER A.		
Means, 187. cm.	187.7 cm.	189.8 cm.
OBSERVER B.		
Means, 190.5 cm.	187.3 cm.	187.9 cm.
OBSERVER C.		
Means, 192. cm.	186. cm.	190.5 cm.

The fact that a minimum occurs with T_1 at the same point whether the vane is stationary or rotating in either direction could not be due to the bringing up of the next wave, for the wave-length used was 1,104 centimeters, and the increment required by the theory with the velocity used and with the distances of T_1 and T_2 from S was approximately 160 centimeters.

CONCLUSIONS.

The conclusions are that the velocities of sound disturbances produced by a moving spark are not appreciably affected by the velocity of the gap.

Whatever increment of velocity may have been given to the sound by the motion of the gap was damped out too quickly to affect the measurements, beyond the errors involved.

UNIVERSITY OF CALIFORNIA,
May 24, 1920.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

THE SPECTRA OF COMPOUND GASES FLOWING IN VACUUM TUBES.¹

By W. H. BAIR.

THE spectra of compound gases were obtained in the ordinary discharge tube, both with the gases flowing and at rest. The gases studied were ammonia, nitrous oxide, nitrogen peroxide, and carbon dioxide.

As is well known, two bands, one in the visible and one in the ultra-violet region of the spectrum, are usually attributed to ammonia. Of these, the former lasts for only a few seconds after the fresh supply of gas is cut off, while the latter continues indefinitely. The ultra-violet band was found to be four or five times as intense with the gas at rest as with flowing gas. Thus, the present work seems to agree with E. P. Lewis's conclusion that the two bands are due to different compounds. The spectrograms show that the two bands are alike, in that each consists of two heads of unequal intensity; the more intense being on the side of shorter wave-lengths. No further structure was observable in the visible band with the dispersion used.

The two oxides of nitrogen give the same general appearance in the discharge tube, except that the phosphorescence is more intense with flowing nitrogen peroxide than with flowing nitrous oxide. Similarly, the third positive group of nitrogen bands is more intense with the flowing peroxide. As the second group is at the same time reduced in intensity, the flowing gas furnishes the best conditions for studying the third group. This group was found to contain a number of bands not formerly reported. Thus, the limits of the group, as observed, are from λ 3458 to λ 1935 instead of λ 3007 to λ 2154 as observed by Deslandres and by Schneiderjost, when working with air. Variations in the intensities of the different heads in subgroups were observed. Thus, in some of the new groups, the second head is more intense than the first. Relative intensities depended also on whether the gases were flowing or at rest. Some bands were found, which do not fit into this group. An attempt to apply Deslandres' Law showed that the wave numbers, as calculated from experi-

¹ Abstract of a paper presented at the Seattle meeting of the American Physical Society, June 18, 1920. See the preceding issue of the *Review* for abstracts of all other papers presented at this meeting.

mental data, do not increase so rapidly in the extreme ultra-violet as the law requires. That is, the groups in this region are crowded together more than the law suggests. These facts suggest that we may have a new group of bands, but against this conclusion is the fact that, in structure, the new bands are very similar to the old ones.

Spectrograms of the cathode glow with flowing carbon dioxide show the second negative group of Deslandres. A number of new bands have been observed, but no measurements have been made as yet.

UNIVERSITY OF NORTH DAKOTA.

THE PHYSICAL REVIEW.

ON K. S. MAGNET STEEL.

BY KÔTARÔ HONDA AND SHÔZÔ SAITÔ.

SYNOPSIS.

K. S. Magnet Steel.—The composition of this steel is given as C 0.4–0.8 per cent.; Co 30–40 per cent.; W 5–9 per cent.; Cr 1.5–3 per cent. *Tempering* is best effected by heating to 950° C. and quenching in heavy oil. Measurements of the *residual magnetism* for specimens of different composition gave values from 920 to 620 C.G.S. units; the *coercive force* for the same specimens ranged from 226 to 257 gauss. *Artificial aging* by heating in boiling water and by repeated mechanical shock reduced the residual magnetism by only 6 per cent. The *hysteresis curves* for a magnetizing force of $\pm 1,300$ gauss were taken for annealed and tempered specimens; for the annealed specimen the coercive force was 30 gauss and for the hardened steel the coercive force 238 gauss and the energy loss per cycle 909,000 ergs. The *hardness* of annealed and tempered specimens was found to be 444 and 652 respectively on the Brinnell scale and 38 and 55 on the Shore scale. The *microstructure* of the hardened steel showed a finer grain than for the annealed.

ON June, 1917, a new remarkable alloy steel possessing an extremely high coercive force and a strong residual magnetism was discovered by Mr. H. Takagi and one of the present writers (K. Honda). This steel is prominent as a magnet steel among those hitherto known, *i.e.*, tungsten magnet steel, and is named the "K. S. Magnet Steel," after Baron K. Sumitomo, who offered a sum of 21,000 yen to our university for the investigation of alloy steels. During the last two years, several important improvements have been made in the steel, and also numerous measurements of its characteristic constants made; but for reasons connected with the patent, the publication of these data has been suspended up to the present. In the following pages, a short account of the result of our investigation is given.

The alloy is a special steel containing cobalt tungsten and chromium. A favorable range of the percentages of these metals is given below:

C	Co	W	Cr
0.4–0.8%	30–40%	5–9%	1.5–3%

The alloy being somewhat brittle, great care is required in forging the ingot; but with a good deal of practice, one can forge it into any desired shape. For K. S. magnet steel, the best quenching temperature is 950°C. , and the best quenching bath a heavy oil. As shown below, K. S. magnet steel requires almost no heat treatment in order to be used as a permanent magnet in electrical instruments.

The following sets of measurements for K. S. magnet steel are described below:

- (a) Values of residual magnetism and coercive force.
- (b) Hardness and microstructure.
- (c) Hysteresis curves for annealed and quenched steels.
- (d) Effect of artificial aging,
 - (i) Heating in boiling water,
 - (ii) Mechanical shock.
- (e) Effect of the dimension of the specimen on residual magnetism.

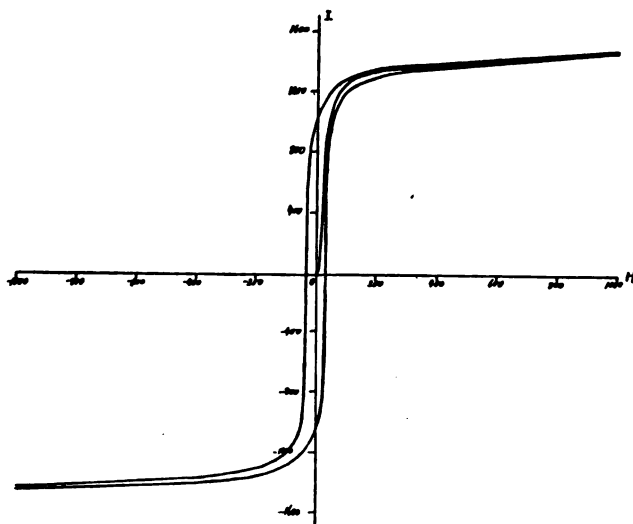


Fig. 3.

Annealed specimen.

The specimens were tested in the form of a cylindrical rod, 20 cm. long and 5 mm. thick. The intensity of magnetization was measured by the ballistic method in the usual way. The magnetizing coil was 40 cm. long and wound in thirteen layers with an insulated copper wire about 1.5 mm. thick, the coil constant being $4\pi n = 483.4$, where n is a number of turns in unit length of the coil. The effective field H was calculated by the well-known relation

$$H = H' - IN,$$

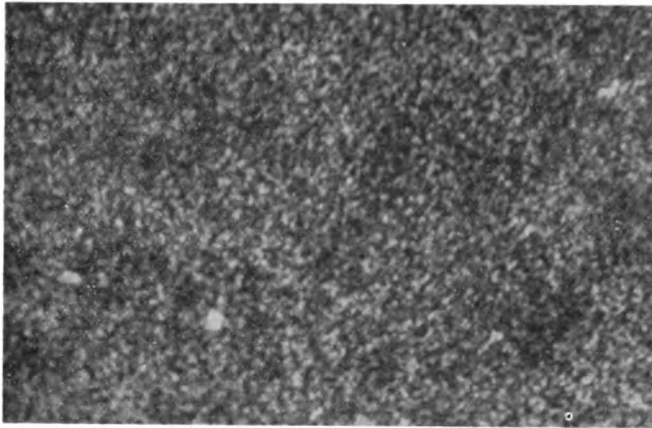


Fig. 1.

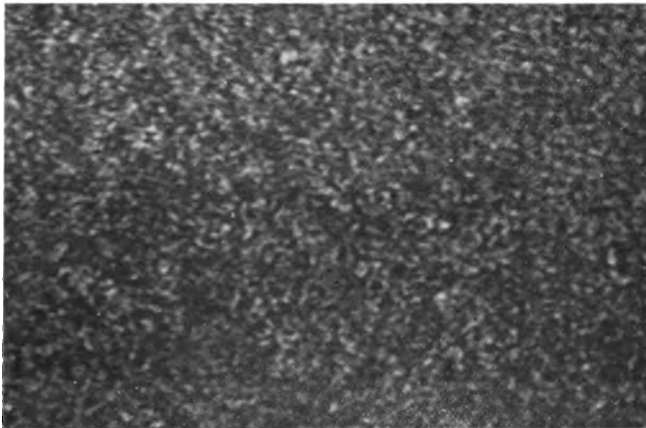


Fig. 2.

K. HONDA AND S. SAITO.

where H' is the applied field, I the intensity of magnetization and N the demagnetizing factor.

Residual Magnetism and Coercive Force.—Two important quantities for a permanent magnet are the intensity of residual magnetism I_r and coercive force H_c , of which the latter is more important than the former. Since these quantities increase with the magnetizing field, a strong field of about 1,500 gauss was applied in our experiment. By varying the percentages of carbon, tungsten, cobalt and chromium within certain limits, alloys having different values of I_r and H_c can be obtained. The following table contains some of the values hitherto obtained:

Specimen.	I_r (C.G.S.).	H_c (gauss).
No. 1	920	226
2	841	221
3	828	245
4	620	257

Thus the coercive force is about three times greater than that of the best tungsten steel hitherto known. The residual magnetism is also greater than that of the tungsten steel.

Hardness and Microstructure.—The hardness of a steel is closely related to its microstructure. Generally speaking, the harder the steel, the finer the structure. The Brinell and the Shore hardness of an annealed and quenched K. S. steel are given in the table below:

	Brinell.	Shore.
Hardness (annealed)	444	38
" (quenched)	652	55

In Figs. 1 and 2, the microphotography of annealed and quenched K. S. steel is given; the microstructure of these specimens consists of an aggregate of very fine grains.

The hardness is also intimately related to magnetism; *i.e.*, mechanically hard steel is also magnetically hard. Thus, since K. S. steel is mechanically very hard and has a very fine structure, it cannot be easily magnetized; but when it is once strongly magnetized, its residual magnetism is very large, and cannot easily be lessened.

Hysteresis Curves.—The following table contains the results of observation for the intensity of magnetization in different fields, both of the annealed and quenched states of the same specimen.

These numbers are also plotted in Figs. 3 and 4, together with the hysteresis loops for the cyclic change of magnetizing field between

Annealed.		Quenched.	
H (Gauss).	I (C.G.S.).	H (Gauss).	I (C.G.S.).
19	281	53	54
30	805	91	107
78	1,227	200	365
247	1,372	295	909
377	1,410	344	998
675	1,450	581	1,171
776	1,465	870	1,266
960	1,479	982	1,293
1,322	1,499	1,281	1,346

+ 1300 and - 1300. In the annealed state, the coercive force is only 30 gauss; but in the quenched state, it amounts to 238 gauss and the area of the loop is enormously large, the loss being about 909,000 ergs. In the best tungsten steel, the same quantity is found to be 290,000 ergs.

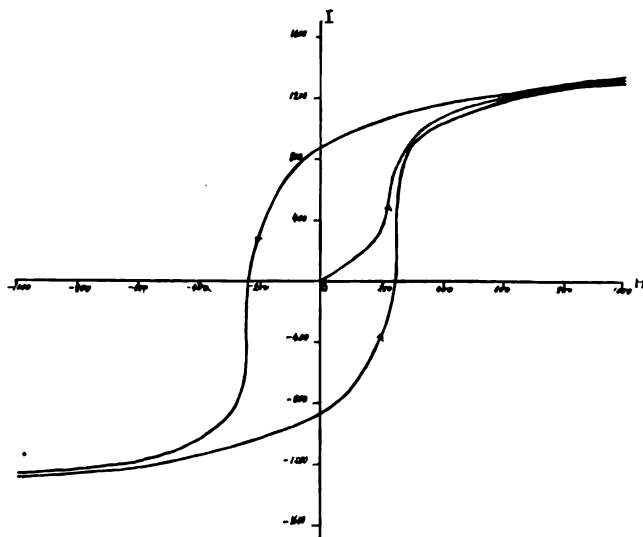


Fig. 4.

Quenched specimen.

Effect of Artificial Aging.—A permanent magnet is always acted on by a demagnetizing force due to the end distribution of magnetism; this force rapidly increases, as the dimension ratio of the specimen decreases. The heating process of the specimen is to expose it to molecular vibrations under the action of the demagnetizing force, and hence residual magnetism is usually reduced by a considerable amount during a prolonged heating, especially in a short specimen, for which the demagnetizing force is large. Since K. S. steel has a large coercive

force, it may be expected strongly to resist the weakening effect of prolonged heating.

The effect of heating the specimen in boiling water is graphically given in Fig. 5, *a*. A rod of K. S. steel having the same dimensions as before was quenched, and strongly magnetized in a coil. The specimen was afterward constantly heated in boiling water, and its residual magnetism measured from time to time for 40 hours. From the curve, we notice that during the first heating for three hours, the residual magnetism slightly increases and afterwards remains perfectly constant. In other K. S. magnet steels, it is usual to observe a slight decrease of magnetization for the first one or two hours of heating, and then an increase to its former value at about three hours, the magnetization afterwards remain-

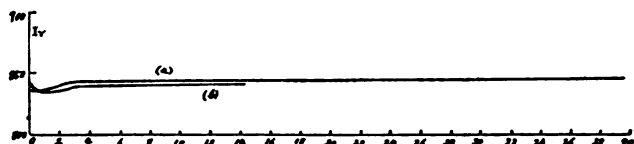


Fig. 5.
Decay curve in boiling water.

ing constant (curve *b*). This decrease and increase of magnetization are due to the simultaneous action of two opposite causes. The first decreasing cause is the molecular vibration due to heating, and the second increasing cause the tempering; that is, during heating, a small fraction of carbides, which are present as a solid solution, is set free, and hence the magnetism increases. The effects of these two causes soon reach their asymptotic values as the heating continues. Hence the superposition of these two effects results in the decay curve (*b*), as actually observed. When the first effect is very small, the curve takes the form as shown in curve (*a*).

The effect of a repeated shock was also observed; a specimen of the same dimensions as before was quenched and magnetized by a strong field; it had initially a residual magnetism of 854 C.G.S. units. Two series of experiments were made; in one, the specimen was repeatedly allowed to fall on a wooden floor from a height of one meter, and in the other, on a concrete floor from the same height. The result of experiments is graphically given in two curves in Fig. 6. The ordinate is the residual magnetism and the abscissa the number of falls. The first diminution of magnetization is comparatively rapid up to 20 falls, and afterwards very slow. The fall on the concrete floor has a greater demagnetizing effect by about 6 C.G.S. units than the fall on the wooden floor. After 850 repeated falls on the concrete floor, the intensity of residual magnetism diminished from 854 to 800 C.G.S. units.

Effect of Dimensions on Residual Magnetism.—As the dimension ratio of a permanent magnet decreases, the demagnetizing force acting in the

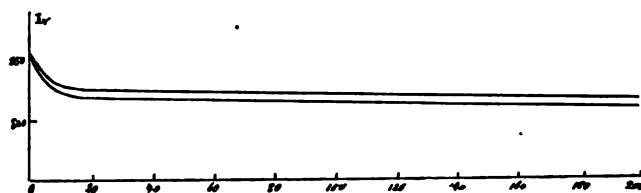


Fig. 6.

I_r —fall number curve.

interior of the magnet becomes greater, and hence the residual magnetism rapidly decreases with the dimension ratio. The resistance to this

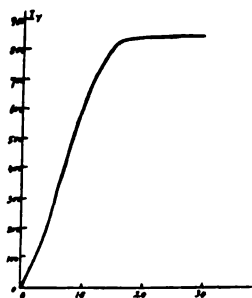


Fig. 7.

I_r —dimension ratio curve.

demagnetizing action is measured by the coercive force. Suppose the length of a bar magnet be gradually diminished, then the material having a smaller coercive force undergoes a greater diminution of magnetism than one having a greater coercive force. Hence, for a bar magnet, K. S. magnet steel has a very great advantage over the ordinary tungsten steels. For a dimension ratio of 15, the permanent magnetism of K. S. magnet steel is about 1.8 times stronger than that of the tungsten steel.

In Fig. 7, the I_r -dimension ratio curve for K. S. magnet steel is given. It shows how rapidly the residual magnetism increases with the increasing ratio of the dimensions, and that above the ratio of 20, the residual magnetism is not affected by the ratio.

SUMMARY.

1. K. S. magnet steel has an extremely large coercive force; its intensity of residual magnetism is also considerably larger than that of ordinary tungsten steels.
2. The area of the hysteresis loop of K. S. magnet steel is very large.
3. K. S. magnet steel, when quenched, is mechanically very hard, and has a very fine microstructure.
4. The residual magnetism of K. S. magnet steel does not appreciably diminish by a prolonged heating at 100° C. over many hours.
5. 850 repeated falls of the steel bar from a height of one meter on a concrete floor causes only a diminution of magnetization by 6 per cent. of its initial value.
6. K. S. magnet steel is specially suited for short bar magnets.

THE EFFECT OF FLUORESCENCE AND DISSOCIATION ON THE IONIZING POTENTIAL OF IODINE VAPOR.

BY H. D. SMYTH AND K. T. COMPTON.

SYNOPSIS.

Fluorescence and ionization.—When fluorescing under the influence of the mercury green line, *iodine vapor* shows weak ionization at a potential about 2.6 volts less than the normal ionizing potential. This is attributed to ionization by impact against molecules whose electrons have been displaced by absorption of the exciting radiant energy.

Dissociation and ionization.—Another weak ionization always observed at about 1.5 volts less than the normal ionizing potential was attributed to ionization of atomic iodine, present as result of the dissociating action of the hot filament. This was verified by experiments in a pyrex glass apparatus at such high temperatures as to cause a high percentage of dissociation, when it was found that this new type of ionization was very intense.

The observed ionizing potentials were: 6.8 volts, due to fluorescing molecules; 8 volts, due to iodine atoms; 9.4 volts due to iodine molecules. The difference between the first and last values corresponds roughly to the *quantum* of absorbed radiant energy, while the difference between the two latter values agrees accurately with the *heat of dissociation* of iodine molecules.

PART I. FLUORESCENCE.

Introduction.

ACCORDING to present theories of atomic structure an electron in an atom is in stable equilibrium only in certain definite positions or orbits and the transfer of an electron from one position to another is accompanied by the absorption or emission of energy. In an atom of gas in the normal state the electrons are grouped as closely as possible about the center leaving the outer, less stable, orbits unoccupied. On the other hand, during the emission of light, as has been very clearly set forth in a recent paper by Drs. Foote and Mohler,¹ an electron is supposed to be falling back toward the normal orbit after having been knocked wholly (ionization) or partly out of the atom. Careful researches by Whiddington² and by Henry³ have failed to detect any ionization in fluorescing iodine vapor. Apparently, therefore, since fluorescence is not accompanied by ionization, it must result from the temporary dis-

¹ Phil. Mag., 40, p. 80, 1920.

² Whiddington, Proc. Camb. Phil. Soc., 15, p. 189, 1909.

³ Henry, Proc. Camb. Phil. Soc., 9, p. 319, 1898.

placement of an electron to one of the outer orbits. On this hypothesis it is obvious that the work required to ionize a fluorescing atom or molecule should be less than that necessary to ionize an atom or molecule in the normal state. As suggested by Hughes,¹ this is probably the explanation of the results of Franck and Westphal,² who found the current in a glow discharge through iodine increased by fluorescence. The present research was undertaken in the hope of observing this predicted reduction in the ionizing potential of iodine vapor, the fluorescence of which has been so thoroughly investigated by Wood.

Apparatus and Procedure.

The ionization tube used was of the ordinary three-electrode type with plate, *P*, gauze, *G*, and filament, *F*, of platinum and a ground-glass joint to allow replacement of the filament. As will be seen from the diagram in Fig. 1, its only unusual features are its dimensions. The

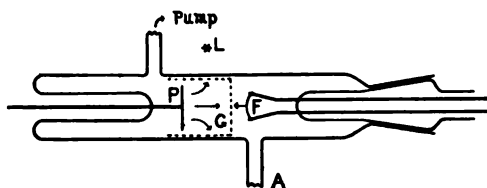


Fig. 1.

diameter of the tube was made small, about 1.5 cm., in order that the exciting light might not be too greatly cut down by the absorption of the vapor before getting into the region where impacting electrons were most numerous. The direction of the retarding field resulting from the shape of the gauze tended to spread the electrons toward the walls of the tube and was thus a second factor favoring impact of electrons with fluorescing molecules. An accelerating field was applied between *F* and *G* and a retarding field larger by a constant amount, between *P* and *G*. The gauze *G* was connected to a Leeds & Northrup high sensitivity galvanometer which measured the electronic current between *F* and *G*. The disc *P* was connected to Dolazelek electrometer of sensitivity about 3,000 millimeters per volt, making the capacity of the electrometer system of the order of magnitude of 50 cm. The potentials were read on a Robt. W. Paul voltmeter.

In order to get increased emission the filament was coated with barium oxide. It was found, however, that a very large emission in vacuum was tremendously reduced by the presence of iodine and after a few hours use became so small as to make the detection of minor effects difficult.

¹ A. L. Hughes, *Photo-Electricity*, p. 21, 1914.

² Franck & Westphal, *Verh. d. D. Phys. Ges.*, 14, p. 159, 1912.

To introduce the iodine, the distillation apparatus shown in Fig. 2 was sealed on to the side tube, *A*, of the ionization tube. The whole system was first heated and pumped out with a Gaede mercury pump for several hours, and a liquid air trap between the pump and the apparatus was given time to take out all mercury vapor. Then the distillation tube at *C* was opened, Kahlbaum re-sublimed iodine introduced into *M*, and the glass again sealed off. Keeping liquid air on *M*, the pump was started again and after further heating of *N* and evacuation the iodine was sublimed over into *N*. Now, by raising *N* above liquid-air temperature iodine vapor was made to flow through the apparatus to the trap on the other side, thus giving a pressure in the apparatus somewhere between the vapor pressures at the temperature of *N* and at the temperature of liquid air. By the above procedure and by keeping the pump running continuously, disturbances due to gases evolved by the glass or by the filament were reduced to a minimum.

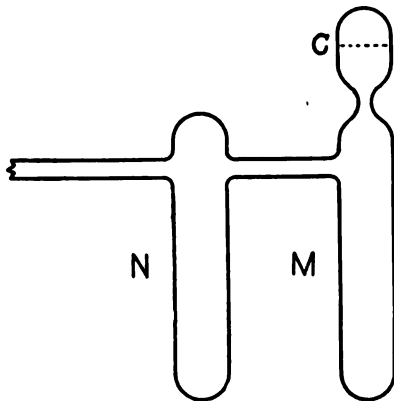


Fig. 2.

To excite fluorescence, a Cooper Hewitt quartz mercury arc was used. It was mounted about 2 in. above the ionization chamber and across it, so that the maximum amount of light fell on the part of the tube between *P* and *G*. A sheet copper shutter was arranged so as to completely cut off the light, making possible alternate readings with and without fluorescence.

Since the electrometer system had a considerable zero leak, readings were started with the accelerating field V_A sufficiently low to establish a good zero line. Then V_A was increased in small steps and two readings taken at each value, one with the shutter open and one with it closed. In several runs this was further checked by reading with the shutter closed, then open, then closed again at each voltage. Ionization was, as usual, detected by a sharp change in the rate of deflection of the electrometer. Velocity distribution measurements were made frequently though, as will appear later, no great importance was attached to them.

Experimental Results.

The results of two typical runs are plotted in Figs. 3 and 4 below. In each case there are two curves for each run, one drawn through points

taken with the shutter open (circles) and one through the points with the shutter closed (dots).

On the first of these runs, No. 4, Fig. 3, in the fluorescence curve there are three distinct break points at about 5.5, 6.6 and 8.4 volts respectively,

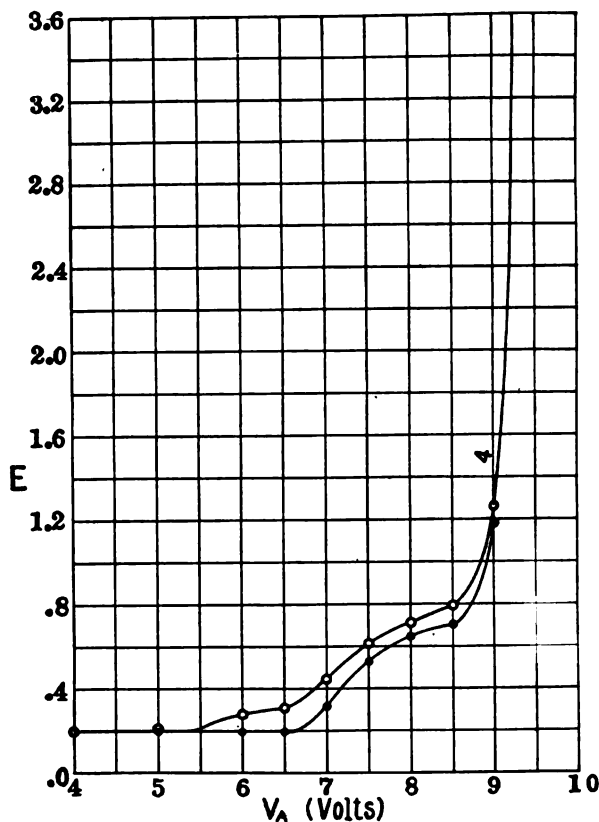


Fig. 3.

but only the two higher of these appear on the normal curve. Run No. 5, Fig. 4, shows exactly the same effects coming at 6, 7.3, 8.5 and 7.4 and 8.5 volts, but the curves rise much less steeply, as would be expected from the fact of smaller emission. Apparently the two lower breaks are due to very improbable types of impact since they both seem to approach saturation rapidly as the voltage is raised. The lowest break appearing only on the fluorescence curves, is naturally attributed to ionization of fluorescing molecules. To explain the middle break which was totally unexpected, the authors were forced to the conclusion that it is due to the ionization of atomic iodine produced by the dissociation of molecules by the hot filament. The correctness of this view was

demonstrated by further experiments which are described in the second part of the present paper. Since atomic iodine has no absorption bands in the visible spectrum¹ it would not be affected by the presence of the mercury arc. All discussion of this second effect will therefore be post-

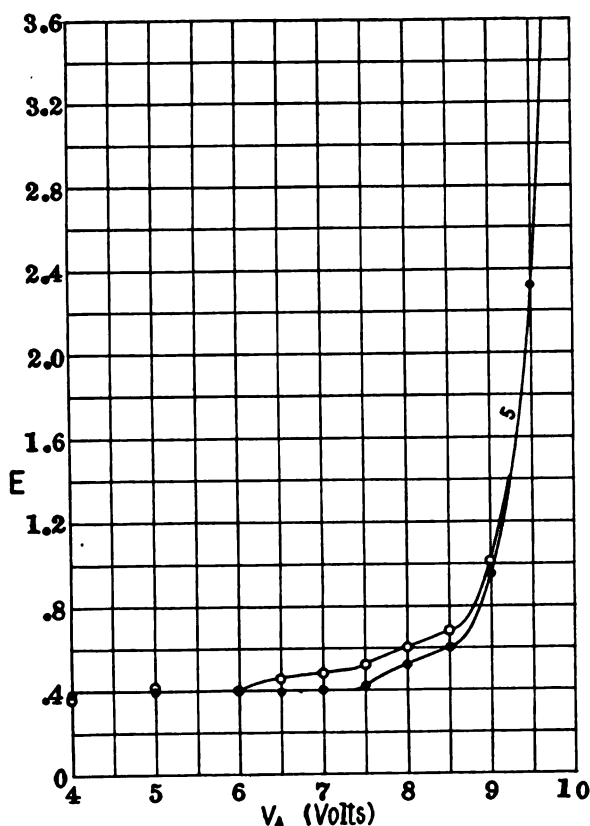


Fig. 4.

poned to the second part of the paper. The third break point which appeared very strongly even with very small emission is presumably due to ionization of normal, unexcited, diatomic iodine vapor.² Before further discussion it will be well to present a complete summary of our experimental results.

where

p = pressure in mm. Hg. (approximate),

G = current to gauze in amperes $\times 10^{-9}$.

¹ E. J. Evans, *Astro. Phys. J.*, 32, p. 1, 1910.

² F. L. Mohler & P. D. Foote, *PHYS. REV.*, 15, p. 321, 1920.

Discussion of Results.

The values given in this table have been corrected for velocity distribution as nearly as possible according to the formula derived in a previous paper¹ but on account of the shape of the gauze used and the difficulty of estimating the probability of ionization by a collision in iodine, no great degree of accuracy can be claimed, probably considerably less than indicated by the calculated probable error.

However, it is not so much the absolute values of the points *a* and *c* which concern us here as it is the difference between them, as given by the last column of Table I., and this is independent of the velocity

TABLE I.

Run.	β .	<i>G</i> .	Corrected Breaks.					
			Fluorescence.			Normal.		<i>c</i> - <i>a</i> .
			<i>a</i> .	<i>b</i> .	<i>c</i> .	<i>b'</i> .	<i>c'</i> .	Volts.
1	.005	2.6	6.4		9.		9	2.6
2	.15	14.2	6.9	8.2?	9.5	8.2	9.7	2.6
3	.15	4.9	6.7	8.2	9.7	8.2	9.7	3.
4	1.	155.	6.2	7.3	9.0	7.3	9.2	2.8
5	.25	9.5	6.	7.3	8.5	7.4	8.5	2.5
6	.15	2.5	5.5?	6.7?	8.5		8.5	3. ?
7	1.	700.		8.7?	10.2?	8.7	10.2	
8	1.	150.	6.9	7.8	9.7	7.8	9.7	2.8
9	.13	11.4	7.	8.6	9.5	7.7	9. ?	2.5
10	.13	21.	6.	7.7	9.5?	7.2	9.5?	3.5?
11	1.	11.5	5.7	6.8	8.2	6.8	8.4	2.5
Average omitting doubtful (?) values			6.42 ±0.11	7.67	9.07 ±.42	7.7	9.21	2.66 ±.04

distribution corrections. The mean value of *c* - *a*, $2.66 \pm .04$ volts, should be the difference in the work necessary to remove an electron from a fluorescent molecule and from a normal molecule. Now the shortest lines observed by Wood² in the fluorescent spectrum of iodine excited by a quartz mercury arc are of the group of - 1 order having a wave-length about 5370 Å.U. while the wave-length of the exciting light is 5460.7. If we apply the quantum relation to these wave-lengths we get 2.3 volts and 2.26 volts respectively. We would expect, then, 2.3 volts to be the maximum gain in potential energy of a molecule due to fluorescence and the greatest possible shift in its ionizing potential. The value of the resonance potential for iodine found by Mohler and Foote³ is $2.34 \pm .2$ volts

¹ Smyth, *PHYS. REV.*, 14, p. 409, 1919.

² R. W. Wood, *Researches in Physical Optics*, Part II., p. 29.

³ Loc. cit.

which would indicate that this is a natural quantum for an iodine molecule to absorb and thus agrees approximately with our last statement.

The discrepancy between this theoretical value of about 2.3 volts and the observed values of $2.66 \pm .04$ may very likely be due to some constant experimental error but this seems sufficiently improbable to make a brief consideration of other more deep-seated possibilities worth while. The first of these is the fact that there is a strong absorption band¹ for iodine in the blue green region corresponding to 2.66 volts and there are lines in the mercury arc at $\lambda = 4358$ and 4916.4 corresponding to 2.83 and 2.51 volts. The iodine molecule might absorb energy in this region but radiate it in two or more steps so that it would not give lines of such short wave-length in the fluorescent spectrum. In this case, however, we would still expect a stronger effect from the much more intense green line.

A second possibility is that, due to the increased radius and instability of a fluorescent molecule, the probability both of collision and of ionization at collision is greatly increased. This ionization effect is therefore much more probable per molecule than an ordinary ionization and therefore tends to be detected at a relatively earlier point on the curve. The much greater number of the normal molecules would counteract this tendency to a greater or less degree.

A third possible contributing factor is the absorption of heat waves from the filament after the molecule has already absorbed the quantum corresponding to 2.26 from the mercury arc. The difference between 2.26 and 2.66, or 0.4 volt, gives a wave-length of $2.09 \mu\mu$ which would certainly be present in the radiation from the filament.

Finally, the discrepancy between the observed and expected values may be due to the effect of a negative surface layer on the platinum gauze *G*, tending to repel electrons and consequently tending to make the observed values of V_A greater than the actual values. Such layers are known to occur in the electronegative gases. If present, such a layer would reduce the effective accelerating field to the greatest extent where the electron current is most intense, *i.e.*, near the center of the gauze. The electrons whose impacts contribute most to the ionization of fluorescent molecules pass through the gauze near the outside of the tube, while the intensity of the ordinary ionization is due principally to electrons which pass near the center of the gauze. Consequently the observed value of the normal ionizing potential would be shifted more than the fluorescent ionizing potential, and in a direction to account for the discrepancy.

¹ Wood, *op. cit.*, p. 23.

In view of these considerations, the agreement of observed and expected values is probably as good as could be expected. In any case, it is important to have definitely established the effect of fluorescence in decreasing the ionizing potential.

PART II. DISSOCIATION.

Introduction.

According to the results of Starck and Bodenstein,¹ iodine vapor at a pressure of .01 mm. and temperature of 450° C. would be nearly 50 per cent. dissociated or at a pressure of .05 mm. it would be 25 per cent. dissociated. At 500° C. and .03 mm. the degree of dissociation would be about 40 per cent. From this it is evident that any effect due to atomic iodine should be greatly accentuated by running at a temperature between 450° C. and 500° C. if the pressure is kept low. This suggested the following method for investigating the ionization of atomic as distinguished from molecular iodine.

Apparatus and Procedure.

As has already been stated the effect *b* in the experiments on fluorescence was attributed to the ionization of atomic iodine. In order to test this theory a new apparatus, made of pyrex glass, was mounted in an electric heater so that its temperature could be raised to more than 500° C. The electrodes were the same as before but the tube was about twice as long so that each end projected outside the heater. This made it possible to keep the ends of the tube cool and so helped to prevent electrical leaks. Platinum guard rings wrapped around the outside of the tube served the same purpose. The temperature was measured by a platinum resistance thermometer.

The iodine distilling apparatus in this case was connected with a T-joint between the ionization tube and the pump. As before, mercury vapor was kept out by a liquid air trap but now the pressure was regulated by the temperature of a U-tube between the T-joint and the main apparatus, a large amount of iodine being first condensed in it by using liquid air. In this way, using the equation given by Baxter and Grose² for the vapor pressure of iodine, and knowing the exact temperature of the U-tube a much more accurate value for the pressure could be obtained than in the previous case, where there was a flow through the tube.

In spite of all precautions the electrical leak at high temperature was serious. It was found, however, that it increased uniformly with voltage

¹ G. Starck & M. Bodenstein, Zs. Elch., 16, p. 966, 1910.

² G. P. Baxter and M. R. Grose, J. Am. Ch. S., 37, p. 1061, 1915.

so that a good zero line could be determined for each run by a series of measurements with the filament turned off. Otherwise the procedure was similar to that of the previous experiment.

Experimental Results.

In Figs. 5 and 6 below are given the graphs of three runs which illustrate the confirmation of our theory by this high temperature apparatus. Thus, in run 15, Fig. 5, taken at 515° C., we have both break points appearing at 8.3 and 9.7 volts, respectively, but by comparing with Figs. 3 and 4 we see that the first effect is very much greater than in any

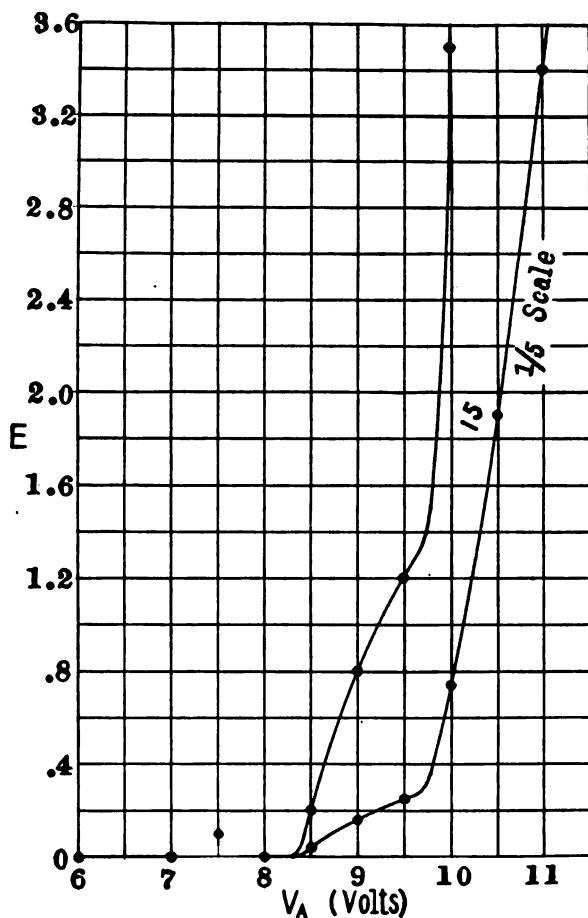


Fig. 5.

of the runs on fluorescence. Again, in Fig. 6, are plotted two runs taken one immediately after the other under conditions identical except for the temperature of the furnace. In the case of run 17 at 475° , the first

type of ionization is so intense as to completely mask the second whereas in run 18 at 25° it is barely detectable.

The other curves obtained were similar to those shown. A summary

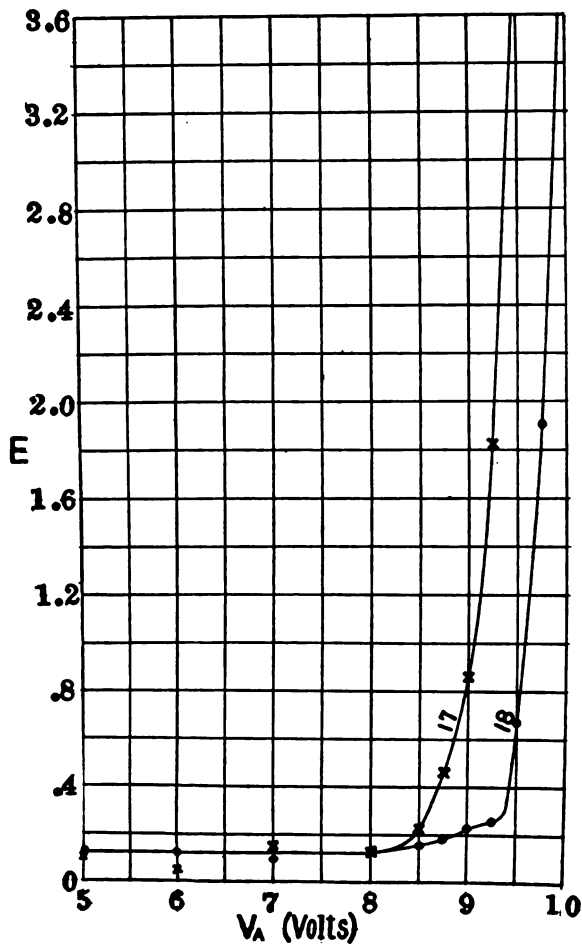


Fig. 6.

of results is given in Table II. The same uncertainty applies to the velocity distribution correction as in the former runs.

where

p = pressure in mm. Hg.;

T = centigrade temperature of apparatus;

G = current to gauze in amperes $\times 10^{-9}$.

TABLE II.

Run.	p .	T .	G .	Corrected Breaks.		
				b .	c .	$c-b$.
12	.01	430°	38-15	8.6	10.2	1.6
13	.01	475°	9-2	8.5	10.2	1.7
14	.032	476°	1.8-2.6	8.7?	—	—
15	.032	515°	3.7±	8.9	10.3	1.4
16	.032	475°	5.5	9.3	10.5	1.2
17	.032	475°	3.5	8.5	—	—
18	.032	25°	4.5-4	8.4	9.6	1.2
Average omitting doubtful values				8.7	10.16	1.42
Average including b , c , and b' , c' from runs 1-11				8.0 ± 0.1	9.4 ± 0.1	$1.47 \pm .045$

Discussion of Results.

The final values at which we arrive, therefore, are 8.0 ± 0.1 volts for ionization of an iodine atom and 9.4 ± 0.1 for ionization of the molecule where uncertainties not taken care of by the calculated probable error may probably reduce the accuracy of these values considerably. The fact that the values in runs 12 to 18 are consistently larger than in runs 1 to 11 emphasizes the possibility of some constant error. Due consideration of the various doubtful factors leads to the conclusion that 8.0 ± 0.5 and 9.4 ± 0.5 would roughly represent the accuracy of our results. The value 9.4 ± 0.5 for molecular iodine agrees within the limits of experimental error with 10.1 ± 0.5 found by Mohler and Foote.¹ The ionizing potential of atomic iodine has not been measured before.

However uncertain may be the determination of absolute values of the above potentials, the difference between them is definitely given. By averaging the values of $c - b$ and $c' - b'$ for runs 1 to 11 and $c - b$ for runs 12 to 18 we get the value $1.47 \pm .045$. Now the heat of dissociation of iodine vapor is about 35,000 calories per gram molecule² which corresponds to 1.52 volts per molecule. The natural inference drawn from this close agreement is that ionization of molecular iodine is accompanied by dissociation. This is predicted by Bohr's theory for hydrogen and has apparently been found experimentally by Mohler and Foote³ and by Franck, Knipping and Krüger.⁴ The latter authors, however, also attribute one of their effects to ionization without dissociation, the occurrence of which in iodine was not observed. Due to the difficulty

¹ Loc. cit.² Starck & Bodenstein, loc. cit.

Bjerrum, Zs. Physik. Ch., 81, p. 281, 1912.

³ F. L. Mohler and P. D. Foote, J. Optical Soc. Am., IV, p. 49, 1920.⁴ Franck, Knipping and Krüger, Ber. d. D. Phys. Ges., 21, p. 728, 1919.

in getting thermal dissociation of hydrogen, the methods of distinguishing effects of atomic and molecular ionization are necessarily indirect and there has as yet been no work done on this question for other diatomic gases.

. PART III. GENERAL DISCUSSION.

The results of the foregoing investigations are of interest in that they afford evidence regarding possible modes of ionization and of photochemical action.

It has been suspected that ionization may, under suitable conditions, be accomplished in two stages, the first being the removal of an electron to a less stable position by the absorption of the appropriate amount of radiant energy of proper frequency, and the second being the further complete removal of this electron by a collision. This method of ionization would account for the ionization which has been observed at the resonance potential in low voltage arcs, and has been indirectly shown to be effective in the ionization of helium.¹ It appears to be proven directly in the present experiments.

It is generally assumed that the primary effect of light in photochemical reactions is to displace or "loosen" an electron of the activated molecule so as to render it physically less stable, or chemically more active. There is some doubt regarding the nature of this "loosening." Experimental evidence, and also the known conditions of ionization, render it unlikely that this loosening consists in complete ionization in the great majority of cases. On the other hand, partial ionization or displacement seems capable of accounting for the so-called "primary" photochemical actions; and such displacement with reëmission of radiant energy following the chemical reaction seems to account for the "secondary" actions.² The present investigation of fluorescing iodine may be thought of as a measurement of the degree of loosening of the electrons by light, and such loosening has been found to be given, at least approximately, by the quantum relation. Apparently the effect of the light is not, in this case at least, the dissociation of some of the molecules—an explanation of photochemical action recently suggested by Nernst.

The results of the present investigation are evidently in accord with the viewpoint recently suggested by Perrin,³ whereby quanta of radiant energy are emitted or absorbed in all chemical reactions.

PALMER PHYSICAL LABORATORY,
PRINCETON, NEW JERSEY.

¹ K. T. Compton, *Phil. Mag.* in print.

K. T. Compton, E. G. Lilly, and P. S. Olmstead, *Phys. Rev.* in print.

² Bodenstein, *Zeitsch. Phys. Chem.*, 85, p. 329, 1913.

³ *Ann. de Phys.*, 11, p. 5, 1919.

Note added with the proof.—Clifton G. Found has published data on the ionizing potential of iodine, among other gasses, in a recent number of the PHYSICAL REVIEW (Vol. 16, p. 41, 1920). He interpreted his results as due to a *single* type of ionization, drawing an average smooth curve through his plotted points, and concluding that the ionizing potential is $(25)^{2/3}$ or 8.5 volts. A closer examination of his results shows, however, that there are *two* break points, a pronounced one at $(31)^{2/3}$ or 9.8 volts and another, less sharply defined, at about $(24)^{2/3}$ or 8.3 volts. It is significant that these agree, within the limits of probable error, with the ionization potentials of molecular and atomic iodine found by us.

THE ABSORPTION OF SOUND BY RIGID WALLS.

BY PAUL E. SABINE.

SYNOPSIS.

Absorption assumed to be due to porosity, the absorbing walls being so massive that flexural yielding is presumably negligible. Upon this assumption, *Rayleigh's theory* indicates that the absorption by such a wall is very nearly proportional to the square root of the frequency of the sound.

Absorption is approximately a linear function of the square root of the frequency, as shown by experiments for a range of frequencies between 64 and 4096 periods per second, but the absorption frequency curve does not pass through the origin, and at higher frequencies the absorption is greater than can be accounted for by porosity alone.

Selective transmission of a frequency whose wave-length in the wall is twice the thickness gives a possible explanation. Absorption experiments at still higher frequencies are desirable.

PROF. WALLACE C. SABINE has shown¹ that the rate of decay of sound in a room may be expressed by the equation

$$(1) \quad \frac{dI}{dt} = -AI,$$

A being a function of the dimensions and shape of the room, and of the absorbing properties of the walls, and the various exposed surface in the room. If we consider simply the absorption of the bounding surface of an empty room then the value of A is given by the equation

$$(2) \quad A = \frac{va}{sp},$$

in which v is the velocity of sound, a is the total absorbing power of the room, s the total surface and p the mean free path of the sound between reflections. The absorbing power a is the sum $a_1s_1 + a_2s_2 + \text{etc.}$, a_1 , a_2 , etc., being the ordinary absorption coefficients of the bounding surfaces s_1 , s_2 , etc., respectively. The values of these absorption coefficients for materials ordinarily used in wall and ceiling construction are of fundamental importance practically in architectural acoustics. They have been determined with great care for a large number of materials over the entire range of frequencies used in speech and music, and are given in various papers published by Professor Sabine.²

¹ Architectural Acoustics, Part 1.

² Proc. Am. Acad. Sci., Vol. LXII., No. 2, 1906; The Brickbuilder (Boston), Jan., 1914.

Qualitatively, the theory of sound absorption is simple. The dissipative forces through which the sound energy is converted into heat are introduced in two ways. The yielding of reflecting surface as a whole, or in large units of area under the alternating pressures of the sound wave calls into play damping forces, which dissipate the sound energy. In the case of porous surfaces, the viscosity of the air and the transfer of the heat of compression from the air to the walls of the minute channels formed by the pores also become effective. It is probable that in most cases both of these effects are present, so that while qualitatively simple, the problem in general does not admit of quantitative theoretical treatment. For example, it would seem obvious that in heavy felt and fabrics, porosity is the predominating factor, yet the curves for the absorption coefficient of such materials plotted as a function of the frequency of the sound show maxima and minima, indicating that resonant yielding of the material as a whole plays a part. The separation of the two factors in any particular case is difficult. Making certain simplifying assumptions, Lord Rayleigh,¹ following the method of Kirchoff,² developed an expression for the absorption of a porous non-yielding surface. He assumed that the depth of the pore channels is small compared with the wavelength, but large relative to the diameter, and that the flow into the channels, is sensibly equal to that across them at a certain short distance from the surface. The value for a , defined as above, is given by the equation

$$(3) \quad a = \frac{4M}{2M^2 + 2M + 1},$$

where M is defined as equal to

$$\frac{2(1 + g) \sqrt{\mu' \gamma}}{r \sqrt{n}},$$

in which g is the ratio of the perforated to the unperforated area of the porous surface, μ' the kinematic viscosity of the gas, μ/ρ^0 , γ the ratio of the specific heats of the gas, r the radius of the pores considered as cylindrical, and n is $2\pi \times$ the frequency of vibration. It should be noted that the absorption coefficient, as defined, includes the entire fraction of the incident energy that is not reflected.

Putting in reasonable values for g and r , M^2 is found to have values much greater than unity for the frequencies of the ordinary musical tones, so that, neglecting unity in the denominator (3) may be written

$$(4) \quad a = \frac{2}{M + 1} = \frac{2r \sqrt{n}}{(1 + g) \sqrt{\mu' \gamma} + r \sqrt{n}}.$$

¹ Theory of Sound, Vol. II., p. 328.

² Pogg. Ann., CXXXIV., p. 177, 1868.

Lord Rayleigh points out that equation (4) holds good only in case the radius of the pores is sufficiently small to allow free exchange of heat between the air and the sides of the channels and estimates that if r is much greater than .01 cm. the equation does not hold.

The flexural yielding of massive masonry is clearly a negligible factor in the absorption of sound. Such walls ought to satisfy approximately the assumptions involved in the derivation of equation (4). The following table gives the absorption coefficients for 18" walls of hard brick set

	Frequency.		
	1	2	3
64.....	.021		
128.....	.023	.0092	.0079
256.....	.026	.0097	.0084
512.....	.032	.0120	.0104
1024.....	.040	.0150	.0144
2048.....	.052	.0190	.0174
4096.....	.070	.028	.025

in mortar. In column 1 are the values given by Prof. Sabine,¹ for unpainted brick surface, column 3 is for a surface of gypsum plaster with a so-called "putty finish" taken about three months after being put directly on to an 18 in. brick wall and column 2 is for the same surface a year afterwards. The values for columns 2 and 3 were obtained in this Laboratory. These results are plotted as ordinates in the accompanying figure, with the square roots of the frequencies as abscissæ. The increase in absorption of the plastered wall with time is probably due to an increase in the porosity of the surface, as the walls become more thoroughly dry. It has been frequently noted that the reverberation in empty rooms with newly plastered walls is decidedly greater than in the same rooms later on. Obviously the curves may be represented by an equation of the form

$$a = K_0 + K_1(f)^{1/2} + K_2(f).$$

For curve 3, K_0 , K_1 and K_2 have the values .007, .00013, and .0000026 respectively with corresponding values for curves 1 and 2.

Turning now to equation (4) and expanding in a power series, the value of a is given by the expression

$$a = K_1'(f)^{1/2} - K_2'n + K_3'(f)^{3/2} - \text{etc.}$$

Since for frequencies not greater than 4096 p.p.s. M is large in equation (4) in comparison with 1, the sum of the terms after the second will be small in comparison with the second term, so that the sum of terms after

¹ Proc. Am. Acad. Sci., Vol. XLII., No. 2, 1906.

the first will be intrinsically negative. Plainly the theoretical curve will be concave toward the axis of abscissæ.

A comparison may now be made between the results of experiment and Rayleigh's theory.

(a) The fact that the experimental curves do not pass through the origin as does the theoretical curve indicates that not all the loss of energy in reflection is accounted for by viscosity and heat conduction.

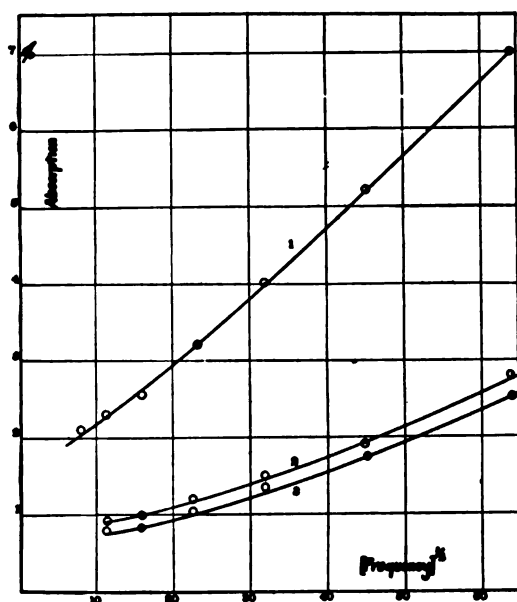


Fig. 1.

The refraction of sound into the wall considered as a second medium of different density and compressibility, may be shown to be negligible in comparison with the absorption at zero frequency as determined by extrapolation of the experimental curves. Computation based on the relative velocities of sound in the air and in the wall shows that the transmitted energy is of the order of .00025 times the incident energy, if the wall be considered simply as a continuous medium, transmitting a portion of the incident energy, and reflecting the rest.

(b) The experimental and the theoretical curves have curvatures of opposite signs. If the whole absorption of the wall is to be accounted for by the porosity the value of the absorption should approach a limit, as the frequency is increased whereas the experimental curves do not show this tendency over the range of frequencies employed.

It appears then that the dissipation of sound energy in a room with

massive walls can not be wholly accounted for by the porosity of the walls. In the analogous case in light, the shape of the experimental curves would be explained by the assumption of an absorption band in the region of the higher frequencies due to the natural free period of the electrons in the absorbing medium.

In the case under consideration, such a natural frequency may be supposed to be present because of the limited thickness of the wall. Under the alternating pressure of the incident sound wave, a compressional wave is excited in the wall, which will travel with a velocity determined only by the density and the compressibility of the wall material. When the frequency of the sound is such that the thickness of the wall is half the wave-length of the transmitted sound, the inner and the outer wall surfaces become the loops of the stationary wave set up by internal reflection from these surfaces, and the wall will exhibit selective transmission for this frequency. Chladni gives the velocity of sound in masonry as approximately 12,000 ft. per sec. The frequency natural to a wall 18 in. thick is approximately 4,000 p.p.'s, which is a little less than the highest frequency in the experiments. Some experiments conducted in this Laboratory in connection with another problem indicate that the response of an extremely massive body to its own natural vibration frequency is probably sufficiently large to show an appreciable effect in the absorption of sound by the body. It would consequently be of great interest to carry the absorption measurements to still higher frequencies but unfortunately sources of sound of determinable rates of emission are not yet available for such an extension. Moreover, the results by the reverberation method would be in considerable doubt since the response of the ear to very high frequencies is far from possessing the uniformity which it possesses for lower tones.

In conclusion, it may be said that although the Rayleigh formula does not completely account for the experimental results, yet the absorption of walls so thick that flexural yielding does not occur, may be approximately represented as a linear function of the square root of the frequency, over a considerable range of frequencies.

WALLACE CLEMENT SABINE LABORATORY,
RIVERBANK, GENEVA, ILL.

ON THE THEORY OF POWELL'S BANDS AND THE GROUP-VELOCITY IN DISPERSIVE MEDIA.

BY NIHAL KARAN SETHI.

SYNOPSIS.

Powell's Bands: Observations with liquid mixtures of variable refractive index.—A description is given of the changes in the position, width and degree of visibility of the bands seen in the spectrum when a flat-sided cell containing liquid of which the refractive index is gradually varied is placed behind the prism on the table of a spectroscope, a transparent plate of glass being immersed in the cell so as to cover half the aperture. The liquid chosen for the experiment was a mixture of CS_2 and benzene of which the proportions could be altered in any desired manner. Special attention is paid to the stage at which the position of the plate in the cell has to be reversed in order that the bands might continue to be visible.

Powell's Bands: Condition of visibility.—Mascart's and Schuster's statements on this point in their treatises are criticized, and it is shown that the side of the cell on which the plate should be immersed is determined not by the relation between the refractive indices of the liquid and the plate but by the group-velocities in the two media. Owing to the highly dispersive nature of the liquid employed in the experiments, the position of the plate has on this account to be changed from one side to the other at a stage at which the refractive index of the liquid is still lower than that of the plate for the whole of the visible spectrum.

Powell's Bands: Relation to Laminar Diffraction.—Spectroscopic analysis of the laminar diffraction either of the Fraunhofer or the Fresnel type formed by the plate immersed in the liquid gives oblique interference lines, photographs of which are given. Their number and the positions at which they cross the spectrum are identical with those of Powell's bands and their inclination to the direction of the spectrum indicates at once the side on which the plate should be immersed for observation of Powell's bands and the degree of visibility of these bands. The visibility is maximum when the interference lines are inclined at 45° to the length of the spectrum.

Group-velocity of Light: Direct determination of the position of its identity in two media.—This is rendered possible by the fact that the oblique interference lines mentioned above curve round and become inclined in the opposite direction at this position as shown in the photographs given.

Determination of Refractive Indices and Dispersive Powers.—As the oblique interference-lines in the spectrum of laminar diffraction are seen clearly and sharply-defined irrespective of the thickness of the plate or the aperture of the telescope within wide limits, it is suggested that they may be used in the preference to Talbot's and Powell's bands for such determinations.

I. INTRODUCTION.

POWELL¹ observed a remarkable system of interference bands when a hollow prism containing liquid was used to form a spectrum and

¹ Baden Powell, Phil. Trans., 1848, Pt. II., page 213.

a transparent plate with its edge parallel to the refracting edge of the prism was immersed inside the liquid so as to cover half the effective aperture of the beam passing through it. A convenient method of studying these bands was suggested by Stokes.¹ An ordinary spectro-scope with glass prism may be used and a flat-sided cell containing the liquid chosen is put on the table of the instrument. The plate is then immersed inside this cell so as to cover half the aperture, leaving the other half covered by the liquid only. Whether the bands are visible or not in any case depends on the side of the beam which is covered by the plate. Mascart and Schuster state in their treatises² that it is necessary, if the bands are to be seen, to place the plate on the thicker side of the prism if the refractive index of the liquid is greater than that of the plate, and to put it on the thinner side if the refractive index is less. The present author had occasion recently to make some observations of Powell's bands with plates immersed in liquid mixtures of varying composition, from which it would appear that the foregoing statement is not correct and that the actual condition for the visibility of the bands should be stated somewhat differently. The side on which the plate should be placed is determined by the relation between the *group-velocities* of light in the two media and not by their refractive indices as stated by Mascart and Schuster. When the group-velocity is less in the liquid than in the plate, the plate should be placed on the thicker side of the prism and when greater the plate should be placed on the thinner side. In the case of a plate immersed in a highly dispersive liquid of nearly the same refractive index, *e.g.*, a plate of glass immersed in a mixture of carbon disulphide and benzene it is possible to have the group-velocity in the liquid to be less than that in the plate, though the refractive index of the latter may be greater throughout the visible spectrum. In such a case, the position of the plate for visibility of the bands would actually be the reverse of that necessary if the differences of refractive indices had been the criterion. It is proposed in the present paper to describe some experiments illustrative of this aspect of the theory of Powell's bands and to explain a simple method by which the identity or difference between the group-velocities in the two media may be ascertained by direct observation without requiring a knowledge of their dispersive powers.

¹ Stokes, Phil. Trans., 1848, Pt. II., page 227, and Math. and Phys. Papers, Vol. II., page 14.

² Mascart, *Traite D'Optique*, Tome premier, page 474.
Schuster, *Theory of Optics*, second edition, page 119.

2. THEORY OF POWELL'S BANDS.

The condition for the visibility of Powell's bands may be found by applying in their case a train of argument similar to that which has been used by Professor Schuster himself¹ to explain the formation of Talbot's bands. When impulses of white light pass through a prism, they are dispersed and the impulses passing through the thicker portion of the prism are retarded more than those passing through the thinner. Accordingly, at the focus of a lens placed to receive the beam, the impulses from the thin end of the prism will be the first to arrive and will be followed in succession by those that have passed through increasingly thicker portions of the prism. These impulses as such cannot interfere because they do not arrive at the *same time* but if it is so arranged that the impulses coming through the two halves of the prism arrive in pairs *simultaneously*, it is possible to produce interference effects. This can be done either by a relative retardation of those impulses which have travelled through the thinner portion of the prism or by a relative acceleration of those that have passed through the thicker. If on the contrary the reverse of this is done, the interval between the arrival of the impulses will be further augmented and no interference will be possible. In the case of Powell's bands the relative retardation or acceleration is produced by means of covering half the aperture by a plate immersed in a liquid. It is clear that the plate will retard the impulses passing through it when the velocity with which they travel through it is less than that with which they travel through the liquid. The impulses travel through the liquid and plate with the respective group-velocities in the media which depend not only on their refractive indices but also on their dispersive powers. If the dispersive power of the liquid is greater than that of the plate, it is possible to have the group-velocity in the liquid less than that in the plate even when the refractive index of the latter is higher. In such a case, in spite of its higher refractive index, the plate will not act as a retarding plate but will result in a relative acceleration of the impulses passing through it. Consequently it will have to be put on the thicker side of the prism. Thus it is clear that the correct condition for the visibility of the bands is that stated above, namely that so long as the group-velocity of light in the plate is less than in the liquid, the plate must be introduced on the thinner side of the prism. When it is greater, the plate must be introduced on the thicker side. The transition from one case to the other takes place when the group-velocities in the two media are equal.

¹ Schuster, Phil. Mag., Vol. VII., Jan., 1904, page 1. See also R. W. Wood, *Physical Optics*, second edition, pp. 259 and 657.

3. OBSERVATIONS OF POWELL'S BANDS WITH LIQUID MIXTURES.

The foregoing remarks are most conveniently illustrated by using a mixture of carbon disulphide and benzene as the liquid in the cell. The mixture has a high dispersive power and by altering the proportions of the constituents the refractive index may be adjusted as desired. The state of the mixture as regards its refractive index can be very simply ascertained by looking at the incandescent filament of an electric bulb obliquely through the edge of the plate while immersed in the cell. The position of the spectrum formed by the edge of the plate with reference to the filament gives the necessary indications.

In some observations made by the author, the cell containing the liquid was placed between the prism and the observing telescope of a spectrometer. The experiment was begun with carbon disulphide alone and very sharp and narrow bands were observed throughout the spectrum when a thin glass plate was immersed in the cell on the thicker side of the prism. As the refractive index of the liquid was lowered, the bands increased in width and a thicker plate (about 5 mm.) had to be used in order to keep them of a convenient size. This plate was cut from an ordinary good quality mirror glass. When the refractive indices of the plate and the mixture became more nearly equal, even the thickness of this plate proved to be less than the "best." As it was not convenient to use thicker plates, the device of cutting down the aperture of the beam passing through the cell was adopted. Nothing special happened when the refractive indices for the red, yellow, green, blue and even the violet were equalized in succession, except that very narrow apertures down to a slit about 1 mm. in width had to be employed to keep the bands at maximum visibility. Although the refractive index of the liquid had now become less than that for the plate almost throughout the visible spectrum, the bands remained as clear and distinct as ever with the plate still on the thicker side of the prism. The width of the bands, however, did not remain uniform, those towards the red end being wider apart. With further diminution of the refractive index, the bands in the red became very broad indeed and gradually decreased in visibility and ultimately disappeared altogether, and this was followed by the disappearance of the bands in the yellow and the green. At this stage, on introducing the plate on the other side, broad bands could be seen in the yellow and red regions, becoming narrower and more distinct towards the red side, thus showing that the same combination of the liquid and the plate gives bands in one part of the spectrum with the plate on the thinner side of the prism (Fig. 1) and in another part with the plate on the thicker side (Fig. 2). By further addition of benzene, the

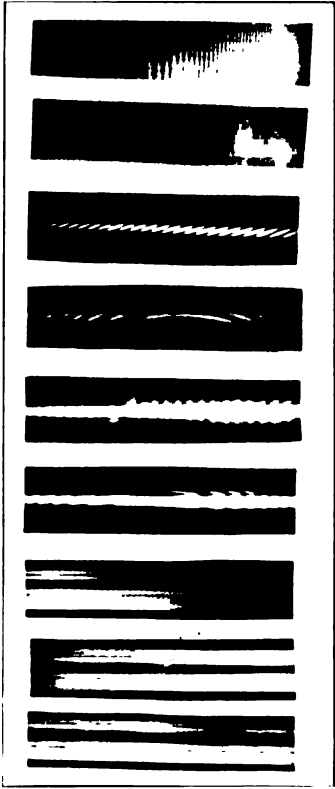


Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

N. K. SETHI.

point of zero visibility of the bands separating these two parts of the spectrum could be shifted further and further towards the violet end until finally the bands in the visible spectrum could be seen only with the plate on the thinner side of the prism.

4. RELATION BETWEEN LAMINAR DIFFRACTION AND POWELL'S BANDS.

A study (by the method of spectroscopic analysis) of the laminary diffraction pattern produced by the plate while immersed in the liquid is very instructive and serves clearly to demonstrate that the group-velocities in the plate and the liquid are identical at the point of zero visibility of Powell's bands in the spectrum. For this purpose, the laminary diffraction pattern either of the Fraunhofer or the Fresnel type may be used. White light from a narrow slit is allowed to fall on the cell containing the plate and the liquid, the slit being parallel to the edge of the plate. The plate should be adjusted so as to be quite normal to the light by tilting it about its edge till the shadow of this appears as a fine sharp line. If the Fresnel pattern is to be observed, all that is necessary is that the slit of a direct-vision spectroscope be placed across the shadow of the edge at a distance of about 15 or 20 cm. from the plate. If the Fraunhofer pattern is to be observed, a lens has to be added to the above arrangement in order to focus the image of the first slit on the slit of the spectroscope. The aperture of the lens should also be limited so as to make the central band sufficiently wide. The edge of the plate should bisect this aperture.

Figs. 3, 4, 5 and 6 show photographs of the spectrum of the Fraunhofer diffraction pattern in different cases. Figs. 3 and 5 are respectively dense and light prints from the same negative taken when the refractive indices of the plate and the liquid were equal for the blue-violet region of the spectrum. It will be noticed that in the deeper print (Fig. 3) a series of interference bands are seen running obliquely through the spectrum but there is nothing which specially distinguishes the position of equality of refractive indices,¹ the bands running in the same direction throughout. The lighter print (Fig. 5) shows the fainter bands above and below the central band in which a certain asymmetry of illumination can be seen. This is due to a slight prismatic action of the edge of the plate and it can be seen that these bands become reversed at the wavelength in the spectrum at which the refractive indices are equal. Figs. 4 and 6 were taken when the refractive index of the liquid was lower than that of the plate throughout the visible spectrum. It will be seen that

¹ Except a certain increase in the intensity of the band cutting obliquely through the spectrum at this point which is however not very strikingly shown in the print.

in this case the bands are much wider apart and in proceeding along the spectrum they gradually change their inclination, curve round and become inclined in the opposite direction.

Figs. 7, 8 and 9 show the spectrum of the Fresnel pattern of which Figs. 7 and 8 refer to the same condition of the liquid as with Figs. 3 and 5. In Fig. 8, the plate was purposely kept a little tilted about its edge and it shows the asymmetry of illumination on the two sides more clearly. Fig. 9 was obtained in circumstances similar to those of Fig. 4 and it will be seen that as in that case the dark interference bands appearing in the center change their inclination and curve round to the opposite direction.

It was found experimentally that the position of the curved band in Figs. 4 and 9 corresponded exactly with the point in the spectrum at which the visibility of Powell's bands is zero. The fact that the direction of the spectrum is tangential to the interference lines in the pattern at this point shows clearly that the phase change produced by the plate measured in radians for the various wave-lengths in this neighborhood is constant. If μ_g and μ_l are the refractive indices of the plate and the liquid respectively, λ the wave-length of light and e the thickness of the plate, this condition gives

$$\frac{2\pi e}{\lambda} (\mu_g - \mu_l) = \text{constant},$$

i.e.,

$$\frac{d}{d\lambda} \left(\frac{\mu_g - \mu_l}{\lambda} \right) = 0$$

or

$$\mu_g - \lambda \frac{d\mu_g}{d\lambda} = \mu_l - \lambda \frac{d\mu_l}{d\lambda}.$$

which on transformation in terms of the wave-velocities readily gives

$$V_g - \lambda \frac{dN_g}{d\lambda} = V_l - \lambda \frac{dN_l}{d\lambda}.$$

The expressions on the two sides of this equation represent the group-velocities in the plate and the liquid respectively. Thus the curved band marks the position in the spectrum at which the group-velocities in the two media are equal to each other.

That the interference lines appearing in the spectrum of the diffraction pattern curve round and reverse their inclination beyond a certain point in the spectrum obviously stands in intimate relation with the fact that the position of the plate has to be reversed in passing from one side of this point in the spectrum to the other side in order that Powell's

bands might continue to be visible. There is in fact a definite quantitative relation between the obliquity of these interference lines and the degree of visibility of Powell's bands. It may be readily shown that the maximum visibility corresponds to an inclination of 45° to the length of the spectrum. $\tan^{-1} 0$ and $\tan^{-1} 2$ correspond to zero visibility of Powell's bands and mark the extreme limits between which they are visible.¹

It may be remarked in conclusion that the determination of refractive indices and dispersive powers for which Talbot's and Powell's bands have been used² could no doubt be made more conveniently by using the spectrum of laminar diffraction pattern preferably of the Fraunhofer type. The oblique bands found in this way cut the spectrum exactly at the same points as the corresponding Talbot's or Powell's bands and they are generally easier to see. There is no question of having the "best thickness" of the plate and no necessity to limit the aperture of the observing telescope in order to see the bands clearly.

The experimental work was carried out in the Laboratory of the Indian Association for the Cultivation of Science and the writer desires to express his gratefulness to Prof. C. V. Raman for his very helpful interest in the work.

210, BOWBAZAR STREET,
CALCUTTA, INDIA.

¹ The distance of the diffraction pattern from the lens forming it is taken as equal to the focal length of the spectrometer telescope.

² A very useful bibliography on this point is given in a paper by T. E. Doubt in *Phys. Rev.*, Vol. X., series II., page 332, Oct. 1917.

ON THE X-RAY SPECTRA OF TUNGSTEN.¹

BY WILLIAM DUANE AND R. A. PATTERSON.

SYNOPSIS.

Critical Absorption and Emission Wave-lengths in K and L Series.—This paper describes accurate measurements of the critical absorption wave-lengths associated with the K and the L series, and also of the wave-lengths of most of the emission lines in the L series. The measurements were made by means of a new ionization spectrometer, similar to the one described in the *PHYSICAL REVIEW* for Dec., 1917, at page 624. A high tension storage battery supplied the current through the X-ray tube, the voltage and current being kept constant by the procedure indicated in the reference. To eliminate errors due to the penetration of the rays into the crystal and to lack of exact adjustment between the axis of rotation of the crystal and the *effective* reflecting plane, etc., the authors employed the third method mentioned in the article referred to, above. In this method two slits between the X-ray tube and the spectrometer define the width of the beam of X-rays that strikes the crystal, and the slit in front of the ionization chamber has sufficient breadth to allow the entire reflected beam to enter the chamber. With this arrangement of slits the angle through which the crystal turns (not the angle made by the *reflected* beam of rays with the zero of the instrument) measures the glancing angle that is used in the formula to calculate the wave-length.

Mean Values of the Wave-lengths Measured in Spectra of the 1st, 2d and 3d Orders.—The following table contains the weighted mean values of the wave-lengths obtained from measurements in spectra of the 1st, 2d and 3d orders, together with an estimate of the precision of the measurements. To estimate the *absolute* accuracy of the data we must take into account the errors in the value of the grating constant of the calcite crystal used. These add up to about 0.07 per cent. In the text the values of the emission wave-lengths obtained by this ionization method are compared with those measurements of the same lines by photographic methods in which special procedures have been employed to minimize the effects of the above mentioned errors.

*X-Ray Spectra of Tungsten.*Grating Constant for Calcite $2a = (6.056 \pm .004) \times 10^{-8}$ cm.Critical Absorption Wave-Lengths, $\lambda \times 10^4$ cm.

K α	L α_1	L α_2	L α_3	
.17806 $\pm$ 7	1.2136 $\pm$ 1	1.0726 $\pm$ 5	1.024 $\pm$ 3	
Emission Wave-Lengths, $\lambda \times 10^4$ cm.				
K α_2	K α_1	K β	K γ	
.21341 $\pm$ 3	.20860 $\pm$ 4	.18420 $\pm$ 3	.17901 $\pm$ 6	
L η	L α_3	L α_1	L η	
1.6756 $\pm$ 10	1.4839 $\pm$ 3	1.47306 $\pm$ 11	1.4176 $\pm$ 7	
L β_4	L β_1	L β_3	L β_2	L β_5
1.2985 $\pm$ 4	1.27892 $\pm$ 9	1.2601 $\pm$ 3	1.24193 $\pm$ 12	1.2040 $\pm$ 7
L γ_1	L γ_3	L γ_2	L γ_4	
1.09608 $\pm$ 7	1.0655 $\pm$ 4	1.0596 $\pm$ 3	1.0261 $\pm$ 6	

¹ A paper presented to the American Physical Society at its New York meeting, Feb. 28, 1920.

The K emission wave-lengths have been taken from a paper by Dr. Stenström and one of the authors.

The Relative Intensity of the Lines.—The ionization spectrometer furnishes the best method of estimating the relative intensity, the relative breadth, etc., of emission and absorption lines, provided that they are so near together that the effect of absorption by the tube, etc., may be neglected or corrected for. The numerous curves in the text give a good idea of the relative intensity, etc., of the lines.

The Differences between the K and L Absorption Frequencies Equal the $K\alpha$ Emission Frequencies in Each Case to Within the Errors of Measurement.—The object of the research has been to obtain accurate values of both the absorption and the emission wave-lengths measured under the same experimental conditions. This becomes particularly important in testing theoretical laws and relations. According to the theory of the mechanism of radiation based on the Rutherford-Bohr atomic model the K critical absorption frequency should be complex, if, as Sommerfeld assumes, some atoms contain elliptic orbits and other atoms of the same chemical element contain circular orbits. Further the difference between the *average* K critical absorption frequency and one of the L critical absorption frequencies should not exactly equal the frequency of one of the $K\alpha$ emission lines. As a matter of fact the authors have not observed a doubling of the K absorption frequency, and their new measurements verify the results obtained last year by Duane and Shimizu.¹ The difference between the K absorption frequency and one of the L absorption frequencies equals the frequency of one of the $K\alpha$ emission lines in each case to within the errors of measurement. Hence, if the effect due to different kinds of tungsten atoms exists at all, it is too small to be detected, at least for chemical elements of high atomic numbers.

Test of Sommerfeld's Formula for the Frequency Interval of the L Doublet.—A Sommerfeld² has deduced from Bohr's theory an expression for the frequency interval of the L series doublet by making an interesting application of one of the equations in the quantum theory, which enables him to calculate the eccentricity of an elliptic orbit. The expression contains one undetermined constant, to which he gives the value 3.63. This value he obtains by using the published wave-lengths of the lines in the L series of a great many chemical elements. Although there is some difficulty in giving a physical interpretation to the theory, yet, when one substitutes the constant in the formula, one obtains the frequency difference of the L series doublet in the tungsten spectrum to within less than one per cent. of the value we found by experiment.

Object.—The object of the research reported in this paper has been to measure both the critical absorption and the emission wave-lengths associated with the K and the L series of tungsten. Values of these absorption and emission wave-lengths measured under the same experimental conditions are not available at present, and, evidently, for the purpose of testing certain relations deduced from theories of the structure of atoms and the mechanism of radiation, such a series of comparable wave-lengths should be obtained.

Apparatus and Method.—In order that the values of the wave-lengths might be as free as possible from errors due to the penetration of the X-rays into the reflecting crystal, and to lack of exact adjustment

¹ PHYSICAL REVIEW, July, 1919, p. 67.

² Atombau und Spektrallinien, Chapter 5.

between the axis of rotation of the crystal, the incident beam of rays, the *effective* reflecting plane in the crystal, etc., we have employed a new ionization spectrometer, similar to the one described in detail in the PHYSICAL REVIEW for December, 1917, at page 624. The new instrument does not differ essentially from the old one, but has circular scales that appear to be somewhat more exact. We were able to detect no difference between the readings of the two verniers attached to the crystal table that would give us a perceptible correction for excentricity in the parts of the scale used.

We have adopted the third method of arranging the spectrometer slits mentioned in the article referred to above. A narrow beam of X-rays, defined either by two slits in lead blocks between the X-ray tube and the spectrometer or by one slit and the focal spot on the target, falls on the reflecting crystal. The third slit, that in front of the ionization chamber, has sufficient breadth to allow the entire reflected beam to enter the chamber. The chief advantage of this arrangement lies in the fact that no correction is necessary for the penetration of the rays into the crystal and for the lack of exact adjustments in certain parts of the instrument. By taking readings corresponding to the same spectrum line on both sides of the zero, twice the glancing angle of incidence can be calculated by simply subtracting the angle through which the crystal has been turned from 180° . The angle through which the ionization table has been turned does not enter into the calculation. In those methods of using a spectrometer in which the glancing angle has to be calculated from positions of the reflected beam of rays (photographic methods, for instance) corrections for the above mentioned sources of error must be applied, or else special devices must be employed to eliminate them.

In making a series of measurements the crystal and ionization chamber are rotated through successive angular increments, the chamber always being moved twice as far as the crystal. In each position of the crystal the ionization current is measured. The graphs representing ionization currents as functions of the crystal table angles (see the figures) indicate peaks, corresponding to characteristic emission lines, and sharp drops, corresponding to characteristic absorption lines. To get the double glancing angle of incidence, 2θ , for substitution in the wave-length formula,

$$\lambda = 2a \times 10^{-8} \sin \theta \text{ cm.},$$

we measure from the tips of the peaks or the centers of the drops, as the case may be.

To sum up, the above described arrangement of slits and general

procedure eliminates errors due to the following causes: (a) penetration of the rays into the crystal; (b) the axis of rotation of the crystal not lying exactly in the effective reflecting plane; (c) the center of the incident beam not passing exactly through the axis of rotation and (d) certain of the defects in crystal structure.

A further advantage of the ionization method lies in the fact that very good estimates of the relative intensity of spectral lines may be

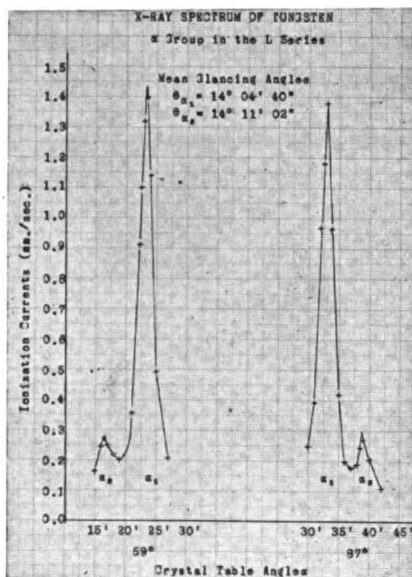


Fig. 1.

obtained, provided that the lines lie so close together that we may neglect, or correct for errors due to changes in the coefficients of absorption and of reflection with wave-length.

For the tungsten emission spectrum we used an X-ray tube of the Coolidge type equipped with a tungsten target. The rays emerged from the tube through a thin glass window blown in a side arm attached to it. This materially reduced the absorption of the relatively soft L series lines. The rays emerging through this window left the target at almost grazing angles. This gave so narrow a source of rays that only one slit was needed to make the incident pencil of rays as narrow as desired.

As in earlier researches a high tension storage battery supplied the current through the X-ray tube, the voltage being kept constant during an experiment by gradually changing a resistance in series with the tube.

The usual method of control of the current through the tube enabled

us to maintain the apparatus in a steady state, and to keep the temperature of the target, etc., constant.

We used a lead ionization chamber filled with methyl iodide or ethyl bromide and provided with a very thin mica window to reduce the absorption of the reflected beam of X-rays as it entered the chamber. A quadrant electrometer measured the ionization currents.

Tungsten L emission Lines.—Thirteen lines in the L series of tungsten have been measured. Figs. 1, 2 and 3 show typical curves platted from actual measurements on the α , β and γ groups of lines respectively. The position of a peak can always be located as between two crystal

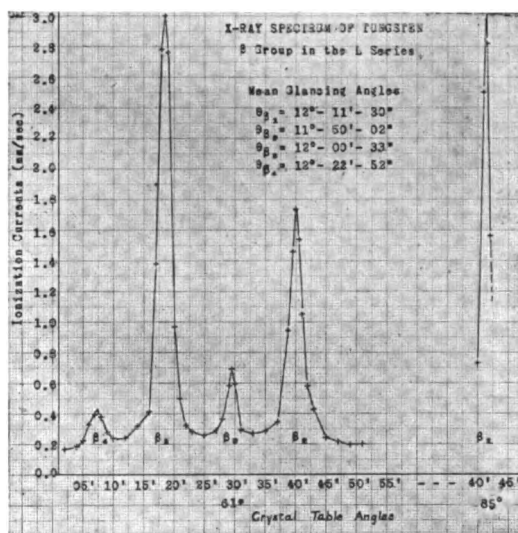


Fig. 2.

angle readings, 30'' of arc apart. Moreover, a good estimate may be made as to how much nearer the actual peak lies to one of these readings than to the other. We believe that, by making several complete series of measurements, we have been able to determine the value of the double glancing angle, 2θ , to within less than 10'' of arc in many cases. This means an error of not more than .01 to .02 of a per cent. in the value of the glancing angle, depending upon its magnitude. The grating space, d , of calcite is known to within about 0.07 of a per cent. No other grating space appears to have been determined as accurately as this. The best determinations of the grating space of calcite lie within a few hundredths of a per cent. of 3.028×10^{-8} cm. We have chosen this value, therefore, to use in calculating our wave-lengths. Hence, taking the grating space of calcite as fixed, we may assume that the relative

values of our wave-lengths are correct to within .01 or .02 of a per cent., according to their magnitudes, in our best experiments.

The lines l , α_1 , α_2 , η , β_1 , β_2 , γ_1 , and γ_3 have all been measured on both sides of the zero position of the crystal. The glancing angles for β_3 , β_4 ,

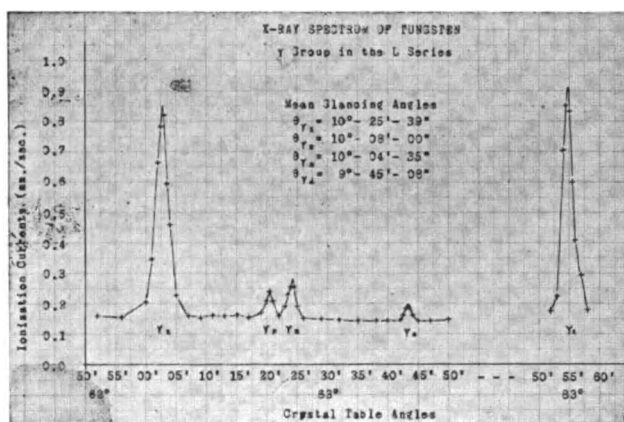


Fig. 3.

β_5 , γ_2 and γ_4 , however, have been determined by comparison with the glancing angles for neighboring strong lines.

Measurements of the glancing angles for α_1 , β_1 and γ_1 have been

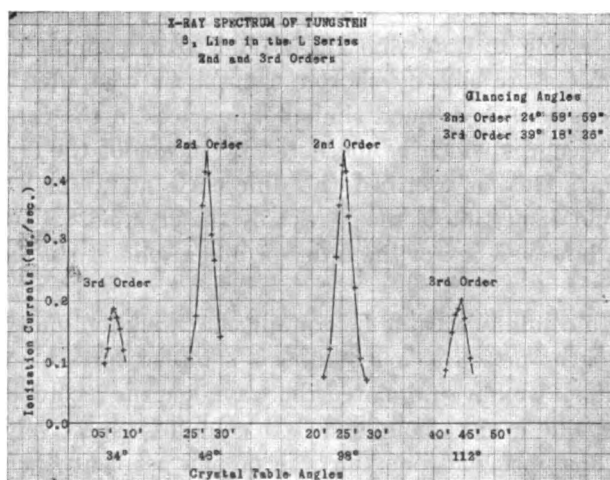


Fig. 4.

made in the first and second order spectra, and for β_1 also in the third order spectrum. Fig. 4 shows the curves obtained for β_1 in the second

and third order spectra. There is a slight indication in the third order spectrum that this line has a close and weaker satellite on its short wave-length side.

Table I. contains the glancing angles and computed wave-lengths for α_1 , β_1 and γ_1 .

TABLE I.

L Series of X-Rays.

Tungsten.

Grating Constant for Calcite $2d = 6.056 \times 10^{-8}$ cm.

Line.	Order.	Glancing Angle.	Wave-length $\times 10^8$ Cm.
α_1	1	$14^\circ-04'-46''$	1.47322
α_1	1	$14^\circ-04'-39''$	1.47303
α_1	2	$29^\circ-06'-27''$	1.47297
β_1	1	$12^\circ-11'-32''$	1.27898
β_1	1	$12^\circ-11'-34''$	1.27904
β_1	1	$12^\circ-11'-35''$	1.27907
β_1	1	$12^\circ-11'-34''$	1.27904
β_1	2	$24^\circ-58'-59''$	1.27886
β_1	3	$39^\circ-18'-26''$	1.27878
γ_1	1	$10^\circ-25'-46''$	1.09628
γ_1	1	$10^\circ-25'-37''$	1.09602
γ_1	2	$21^\circ-13'-24''$	1.09614
γ_1	2	$21^\circ-13'-15''$	1.09603

It is interesting to note that the wave-lengths computed from data in higher orders are slightly shorter than those computed from lower orders. Stenström¹ has found similar differences in the case of certain longer wave-lengths which he measured by means of the crystals sugar and gypsum. He has ascribed the differences to a small amount of refraction and dispersion of the X-rays by the crystals. The differences here noted, however, are of the order of magnitude of the experimental errors.

The fourth column in Table II. contains our weighted mean values for the wave-lengths, measured in a number of complete series of experiments. For comparison we have tabulated the wave-lengths recorded by Overn² and by Siegbahn³ in the second and sixth columns respectively. A slight correction, amounting to one thirtieth of one per cent., has been subtracted from the values they give in order to make their wave-lengths correspond to the grating constant for calcite that we use. Columns

¹ Doctor's Dissertation, Lund, 1919.² PHYS. REV., Aug., 1919.³ Phil. Mag., Nov., 1919.

three and five contain the respective differences. It appears that the wave-lengths measured by the ionization method agree in most cases with those measured by the photographic method to within the limits of experimental errors. In each of the series of measurements by the photographic method great care was taken by the respective authors to

TABLE II.

L Series of X-Rays. $\lambda \times 10^3$ cm.

Tungsten.

Grating Constant for Calcite $2a = 6.056 \times 10^{-8}$ cm.

Line.	Overn.	Difference.	Duane Patterson.	Difference.	Siegbahn.
<i>l</i>			1.6756 $\pm$ 10	-.0005	1.67448
α_2	1.4834	-.0005	1.4839 $\pm$ 3	+.0001	1.48403
α_1	1.4726	-.0005	1.47306 $\pm$ 11	-.00007	1.47299
η			1.4176 $\pm$ 7	-.0004	1.4172
β_4	1.2980	-.0005	1.2985 $\pm$ 4	-.0002	1.29831
β_1	1.2789	.0000	1.27892 $\pm$ 9	-.00018	1.27874
β_3	1.2594	-.0007	1.2601 $\pm$ 3	-.0005	1.25958
β_2	1.2430	+.0011	1.24193 $\pm$ 12	-.00043	1.24150
β_5	1.2017	-.0023	1.2040 $\pm$ 7	-.0013	1.2027
γ_1	1.0963	+.0002	1.09608 $\pm$ 7	-.00092	1.09516
γ_2	1.0655	.0000	1.0655 $\pm$ 4	.0000	1.06548
γ_3	1.0592	-.0004	1.0596 $\pm$ 3	-.0003	1.05930
γ_4	1.0260	-.0001	1.0261 $\pm$ 6	.0000	1.02613

correct for or eliminate the sources of error mentioned above. In particular Siegbahn designed an interesting spectrometer for the express purpose of reducing the errors due to penetration of the X-rays into the crystal, etc., to a minimum.

Overn does not include the lines *l* and η in his table of wave-lengths. He suggests, however, that the marks on photographic plates corresponding to η may represent the α_1 line in the K series of molybdenum in the second order spectrum. As the lines *l* and η are of considerable theoretical importance we have tested this point by examining the spectrum in the neighborhood of $K\alpha_1$ for molybdenum in the first order spectrum. We found evidence for both molybdenum $K\alpha_1$ and $K\alpha_2$, but their intensities were less than that of the line we regard as $L\eta$ of tungsten. As the intensity of the second order spectrum is very much less than that of the first order spectrum it is impossible that the peak on our curve should represent molybdenum $K\alpha_1$ in the second order.

Critical Absorption Wave-Lengths.—In general the measurement of critical absorption wave-lengths is more difficult and slightly less accurate than that of emission lines. In order to make such a measurement we usually place a screen containing the chemical element to be investigated

in the path of the incident beam of rays between the X-ray tube and the spectrometer, take a series of readings in the neighborhood of the critical point and draw the corresponding ionization current crystal

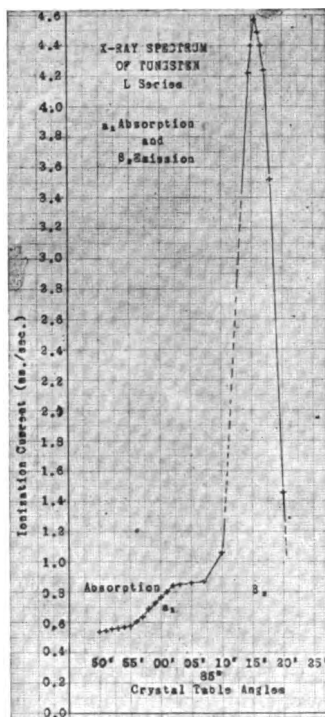


Fig. 5.

table angle graphs. Figs. 5, 6 and 7 contain such graphs. Owing to the fact that the absorbing substance absorbs X-rays of longer wave-length than the critical wave-length to a less extent than it does X-rays of shorter wave-length a sharp drop in the curve occurs at the critical point.

In Fig. 5 the absorption drop is due to the target itself, which was so placed that the beam of X-rays that struck the crystal left the target almost grazing its surface. Under these conditions the target absorbs a large part of its own radiation. Fig. 5 clearly shows the relative magnitudes of the emission and absorption spectra. The absorption drop is the largest of the three critical absorptions that have so far been observed in the L series of X-rays. The glancing angle, θ_{a1} , in this case has been estimated by taking the difference between it and

the glancing angle, θ_{B2} , of the neighboring emission line.

The other two absorption drops fall so close to emission peaks that good measurements of them can not be made without using an X-ray tube with a target of a different chemical element. Professor D. L. Webster kindly loaned us such a tube, with a molybdenum target. Using it we obtained the curves, represented in Figs. 6 and 7, on both sides of the zero. We estimated the grazing angles, θ_{a2} and θ_{a3} , by measuring from the centers of the corresponding drops.

Fig. 7 contains curves for the K critical absorption of tungsten, obtained in the first, second and third order spectra. We are indebted to Dr. Stenström for his assistance in making these K series measurements.

Table III. contains the glancing angles and computed wave-lengths for the K and L characteristic absorption lines of tungsten.

We wish to call attention to the relative magnitudes of the critical absorption wave-lengths and the wave-lengths of the emission lines.

TABLE III.

Tungsten Absorption Spectrum.

K and L Critical Absorption Wave-Lengths.

Grating Constant for Calcite $2d = 6.056 \times 10^{-8}$ cm.

Absorption.	Order.	Glancing Angle, θ .	Wave-length, $\lambda \times 10^3$ Cm.
K α	1	1°-41'-15"	0.17832
K α	2	3°-22'-19"	0.17810
K α	3	5°-03'-20"	0.17789
			Weighted mean, 0.17806 ± 7
L α_1	1	11°-33'-37"	1.2136 ± 1
L α_2	1	10°-12'-07"	1.0726 ± 5
L α_3	1	9°-44'-05"	1.024 ± 3

The critical absorption wave-length L_{α_1} is about $2\frac{1}{2}$ per cent. shorter than that of the β_2 emission line and about .8 of a per cent. longer than

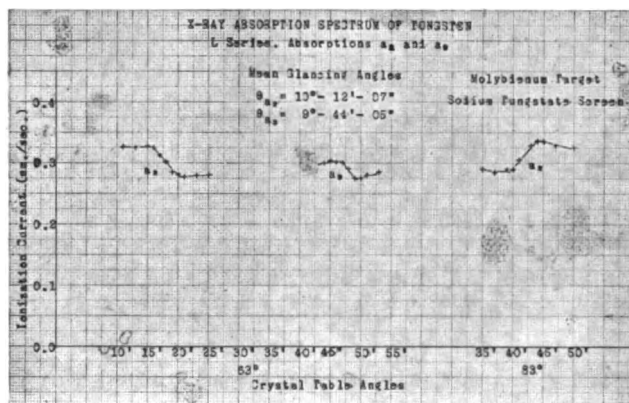


Fig. 6.

that of the β_2 emission line. L_{α_1} lies in the midst of a number of faint lines that have been observed on photographic plates by Dershem, Overn and Siegbahn. L_{α_2} lies between γ_1 and γ_2 (about 0.7 per cent. from γ_2). L_{α_3} lies within a fraction of one per cent. of γ_4 , and apparently on its short wave-length side.

Many of the general laws and theoretical equations relating to X-ray spectra are expressed in terms of the frequencies of vibration, the wave-numbers or the ratios of the frequencies to the fundamental Rydberg frequency for heavy atoms. The following table contains the values of ν/ν_∞ calculated from our data. In making these calculations we have used Paschen's value for the wave-number, ν_∞ , namely $\nu_\infty = 109,737.11 \pm 0.06$, the accuracy of which far exceeds that of any X-ray measurement that can be made at the present time.

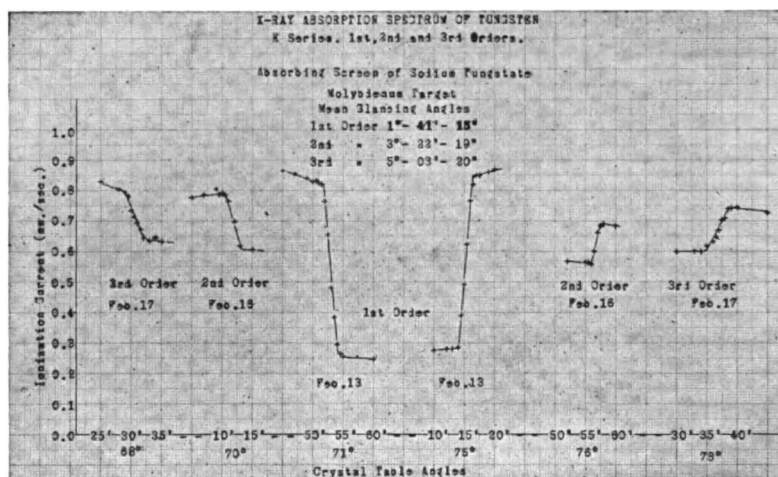


Fig. 7.

TABLE IV.

*Tungsten X-Ray Spectrum.*Values of ν/ν_0 for the K and L Critical Absorption and Emission Lines.

Critical.....	K α	L α_1	L α_2	L α_3	
Absorption....	5118. $\pm$ 2	750.88 $\pm$.07	849.6 $\pm$.4	890. $\pm$ 3.	
K Series ¹	K α_2	K α_1	K β	K γ	
Emission....	4270.6 $\pm$.6	4368.3 $\pm$.6	4947.2 $\pm$.8	5091. $\pm$ 2	
L Series.....	L β	L α_2	L α_1	L η	
Emission....	543.9 $\pm$ 3	614.10 $\pm$.12	618.62 $\pm$.05	642.9 $\pm$.3	
L Series.....	L β_4	L β_1	L β_2	L β_3	L β_5
Emission....	701.8 $\pm$.2	712.53 $\pm$.05	723.17 $\pm$.18	733.75 $\pm$.07	756.9 $\pm$.4
L Series.....	L γ_1	L γ_2	L γ_3	L γ_4	
Emission....	831.39 $\pm$.06	855.3 $\pm$.3	860.0 $\pm$.3	888.1 $\pm$.5	

In the PHYSICAL REVIEW for July, 1919, Duane and Shimizu published an account of experiments which showed that each of the α emission frequencies in the K series equaled the difference between the K critical absorption frequency and one of the L critical absorption frequencies. Theoretically this should be true provided that all the atoms (of the same chemical element) are exactly alike, *i.e.*, have the same kinds of orbits: for, if W and W_1 represent the amounts of energy in an atom of

¹ The values for the K series emission lines are taken from a paper by Duane and Stenström, presented to the Physical Society in February, 1920. See the PHYSICAL REVIEW, April, 1920, p. 328.

tungsten when in its natural state and when one electron has been removed from the K orbit respectively, the K critical absorption frequency is given by the equation

$$hK\alpha = W_1 - W. \quad (1)$$

Similarly, if W_2 represents the energy of the system when one of the electrons has been removed from an L orbit, the corresponding L critical absorption frequency is given by the equation

$$hL\alpha = W_2 - W. \quad (2)$$

According to the theory a $K\alpha$ characteristic X-ray is emitted when an electron falls from the $L\alpha$ orbit to the $K\alpha$ orbit, and during this transfer the energy of the atom changes from W_2 to W_1 . The difference between these amounts of energy is radiated at a frequency $K\alpha$ given by the equation

$$hK\alpha = W_1 - W_2. \quad (3)$$

From equations (1), (2) and (3)

$$K\alpha = K\alpha - L\alpha. \quad (4)$$

And this is the relation found by experiment to hold within the limits of error.

If, however, some atoms contain circular orbits and others elliptic orbits, both $K\alpha$ and $L\alpha$ will have different values for the different kinds of atoms. In fact all critical absorption, critical ionization and emission lines will have complex structures. We have not been able to observe such a structure in the absorption drops for chemical elements of high atomic weight. Further the data obtained this year corroborate the results arrived at last year to within the limits of experimental errors. These errors are smaller than those of last year's measurements, for (a) the wave-lengths are averages of a larger number of individual determinations, (b) some of them were measured in spectra of higher orders and (c) the thin glass window in the X-ray tube allowed a larger amount of X-ray energy to emerge from the tube.

The following values taken from Table III. show the precision with which the $K\alpha$ emission frequencies equal the differences between the critical absorption frequencies.

TABLE V.

Tungsten X-Ray Spectra. Values of ν/ν_∞ .

Critical Absorption	Emission Lines
$K\alpha - L\alpha_1 = 4366.9 \pm 2.1$	$K\alpha_1 = 4368.3 \pm .6$
$K\alpha - L\alpha_2 = 4268.2 \pm 2.4$	$K\alpha_2 = 4270.6 \pm .6$

It is impossible to test directly the above law as applied to the L and the M series, for the M critical absorption wave-lengths have not been measured for tungsten.

Theoretically,¹ however, if two emission lines in the L series are produced by electrons falling from the *same* outer orbit to the La_1 and La_2 orbits respectively, the difference between their frequencies should equal that between the two critical absorption frequencies (except for the possible small correction due to the complex structure of the lines as explained above). The difference between the two critical absorption frequencies is

$$La_2 - La_1 = 98.7 \pm .5.$$

Turning to the emission lines we find that the following three pairs have frequency differences that equal the difference between the two critical absorption frequencies to within the limits of experimental error:

$$L\eta - Ll = 99.0 \pm .6, \quad L\beta_1 - La_2 = 98.4 \pm .2,$$

$$L\gamma_2 - L\beta_5 = 98.4 \pm .7.$$

This would seem to indicate that the electrons producing the two lines in a pair fall from the same outer orbit. For instance we may suppose that La_2 and $L\beta_1$ are due to electrons falling from one of the M orbits. Measurements of the absorption frequencies for thorium and uranium which the authors presented to the Physical Society in April agree very well with this hypothesis. Further we may assume that Ll and $L\eta$ are produced by electrons falling from an orbit between the L and the M orbits. This orbit, however, has not been identified by means of a critical absorption frequency in the case of any chemical element. On the other hand there appears to be some difficulty in the theoretical interpretation in the case of the third pair, for the frequencies of the emission lines $L\beta_5$ and $L\gamma_2$, are greater than those of the critical absorption frequencies La_1 and La_2 respectively.

The difference between the frequencies of a fourth pair of emission lines is almost, but not quite equal to that between the critical absorption frequencies, namely

$$L\gamma_1 - L\beta_2 = 97.5 \pm .1.$$

It would seem that in this case the electrons producing the lines did not come from exactly the same outer orbit.

Siegbahn² finds that the difference in frequency between the lines in these pairs decreases slightly as the frequency itself increases.

¹ Compare an interesting discussion of this subject by A. Sommerfeld in *Atombau und Spektrallinien*, Chapter 3.

² *Phil. Mag.*, Nov., 1919.

Sommerfeld (*Atombau und Spektrallinien*, Chapter 5) has deduced a formula for this L series frequency difference from his theory of elliptic orbits. Making the relativity correction for the change of mass of an electron with its velocity he finds that the energy of an electron travelling in an elliptic orbit differs somewhat from that of the electron in the corresponding circular orbit. This means that those atoms which have elliptic L orbits produce spectra that differ in frequency from the spectra of atoms with circular L orbits. Sommerfeld's equation for this difference $\Delta\nu$ in frequency (divided by ν_∞) reduces to

$$\frac{\Delta\nu}{\nu_\infty} = \frac{1}{\alpha^2} (\sqrt{2} \sqrt{1 + \sqrt{1 - b^2}} - \sqrt{4 - b^2}),$$

for the L series. In this equation

$$\alpha = \frac{2\pi e^2}{hc} = 7.295 \times 10^{-3}, \quad \text{and} \quad b = \alpha(N - n),$$

where e = the electron's charge, h = Planck's constant and c = the velocity of light.

The function representing the frequency difference, varies roughly as the fourth power of the atomic number, N , of the chemical element. It contains an arbitrary constant, n , the value of which he deduces from the wave-lengths in the X-ray spectra of a great many chemical elements. This value is $n = 3.63$. Putting it and $N = 74$ (for tungsten) in the formula we get 97.9 for the value of $\Delta\nu/\nu_\infty$, which agrees very well with our experimental results.

Sommerfeld recognizes the difficulty of giving a satisfactory physical interpretation to the above value of n (l. c.). Further it is worth noting that the formula does not take into account the changes in the energy of the other electrons in the atom when one of them is removed. These energy changes for the other electrons in the L orbit itself do not appear to be negligible, although, of course only the differences between their values for elliptic and circular orbits would enter into the equation.

HARVARD UNIVERSITY.

ON THE EFFECTIVE CAPACITY AND RESISTANCE OF A CONDENSER FOR HIGH FREQUENCY CURRENTS.¹

BY F. C. BLAKE.

SYNOPSIS.

Capacity Independent of Frequency.—By careful experiments on the free oscillations of a Lecher system as the capacity of the end condensers is changed by increasing the elastic coupling it is found that the capacity of an air condenser is independent of the frequency of oscillation.

Rayleigh's Equation for the effective stiffness (elastance) of a circuit due to the mutual elastic reactions between two circuits each containing inductance, capacity and resistance is verified. Use is made of his equation for effective resistance to determine the mutual resistance of two reacting circuits.

IN a previous paper Blake and Jackson² reported their investigation of the relative intensities of the free vibrations of a Lecher system as the side-on distance y , Fig. 1, was varied and the writer³ has developed a theory to explain the experiments. The present paper reports on the

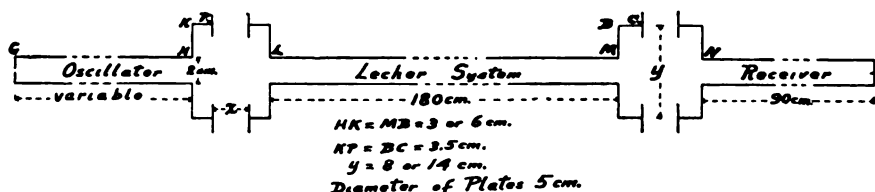


Fig. 1.

results obtained as the face-on distance x between the plates was varied. The experimental results are compared with Lord Rayleigh's theory for the behavior of a vibration system having kinetic and potential energy and dissipation.

The apparatus used has been described in previous papers. All the experimental variations made are shown in Table I. Two different values of y were used, 8 and 14 cm. The distance x was varied from 11 cm. down to 0.684 cm., the shortest distance it was feasible to use because of mutual interference between the strings holding the plates.

¹ Read before American Physical Society, Nov. 28, 1919.

² Ohio Journal of Science, XVIII., p. 145, 1918.

³ Ohio Journal of Science, XVIII., p. 163, 1918.

TABLE I.

6.	1.	2.	3.	4.	5.	Bridge Position.				10.	Mean Observed Wave-length.				Reduced Wavelength, λ .				20.	21.	22.	23.	24.	25.	26.	27.	28.																				
						R Cm.					Unassigned Peaks.				λ .													λ .																			
7.	Cm.	Cm.	Cm.	Cm.	Cm.	1.	3.	5.	7.	8.	9.	11.	12.	13.	14.	Reduced Wavelength, λ .				Rad Cor- rection Cm.	λ .	$\frac{\lambda_0}{\bar{K}}$	Mean $\frac{\lambda_0}{\bar{K}}$	$\frac{\lambda_0}{\bar{K}}$	ca.	λ_0 Ohms.	R' Ohms.																				
																1.	3.	5.	7.									9.	15.	16.	17.	18.	19.														
8	11	95.0	180.1	90.0	149.8				64.0																																						
8	11	90.3	180.1	90.0	150.0	77.2	106.8	222.5	192.8	180.0	80.3	145.3	86.0	60.6	46.6	440.1	145.3	86.0	60.6	46.6	+0.5	97.0	13.18	24.35	13.17	13.18	0.832																				
																												236.2	64.0	77.2	106.8	119.7	11.0	13.19													
																																			235.8	210.2	237.0	6.1	13.16	3.8	13.18						
8	5	99.0	180.1	90.0	150.0	76.5	107.2	broad	224.0	193.6	64.0	221.2	147.5	86.4	147.5																																
8	5	95.0	180.1	90.0	150.0	76.4	106.7	broad	224.0	193.3	64.6 (?)	62.4	147.6	86.6		220.2																															
8	5	90.3	180.1	90.0	149.0	76.3	106.5	150.0	224.0	193.5	119.1	191.2	209.0	147.7	86.9	60.8	47.1	450.7	147.8	86.9	60.8	47.1	+0.1	96.6	16.29	22.7	16.38	16.34	1.031																		
																														237.0	210.5	173.4	234.5	96.4	19.2	7.4	24.60										
																																						237.0	210.5	173.4	234.5	96.4	19.2	7.4	24.60		
8	2	99.0	180.1	90.0	144.7	72.3	99.7	154.7	149.6	191.2	110.2	64.0	66.3	197.0	154.7	91.5																															
8	2	90.3	180.1	90.0	144.3	75.2	106.4	228.2	239.3	212.0	88.6	125.6	71.0	205.9	153.0	88.7	61.7	58.4	481.0	154.3	89.0	61.9	48.2	-0.1	96.4	24.67	19.2	24.73	24.67	1.556																	
8	0.684	113.8	180.1	90.0	131.5	145.5	161.8	broad	231.3	231.3	61.7	90.5	175.9	103.7	186.4	109.9	200.5	169.6	151.8	208.8	155.7	215.3	90.5	175.9	103.7	186.4	109.9	200.5	169.6	151.8	208.8	155.7	215.3														
																																		231.3	231.3	61.7	90.5	175.9	103.7	186.4	109.9	200.5	169.6	151.8	208.8	155.7	215.3

[illegible]

EXPERIMENTAL RESULTS.

Figure 2 shows the results for $y = 14$ cm., the oscillator being kept constant at 94.2 cm., straight away length GH (Fig. 1).

As was to be expected all wave-lengths are increased as the coupling becomes closer and for couplings closer than 6 cm. the fundamental

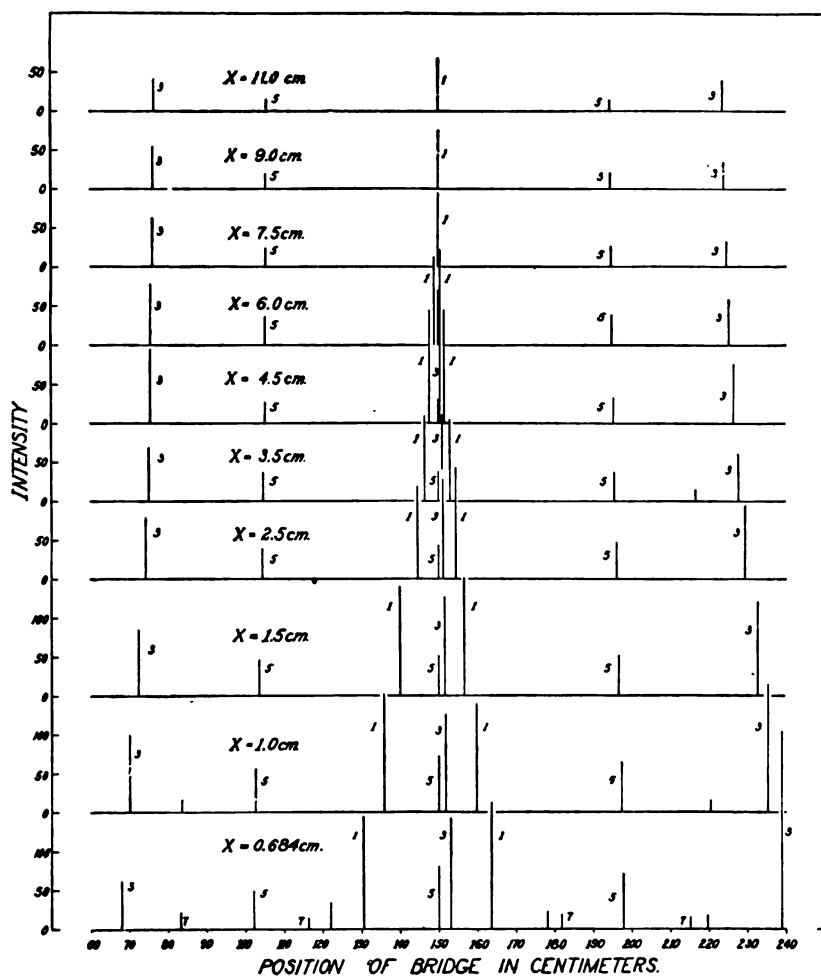


Fig. 2.

has split up into two tones whose difference in period becomes greater the tighter the coupling. It is manifest moreover that the position of any one tone is dependent upon the oscillator length and that as the coupling is made tighter the length of oscillator necessary to hold a given tone in the exact middle of the Lecher wires also varies. This is shown

more clearly in Fig. 3 ($y = 8$; $x = 0.684$) three different oscillator lengths being employed. In the top curve of Fig. 3 is shown the actual slopes of all the peaks observed. It is manifest in many of the curves of Figs. 2 and 3, especially for the closer couplings, that there are some peaks that are not true harmonics of the Lecher systems but may be thought of either (a) as a mixture of two neighboring harmonics when the oscillator length is too short to be the optimum length for one set of harmonics and too long for the next set; or (b) as one of the possible frequencies of the entire system vibrating as a whole. That the system of peaks

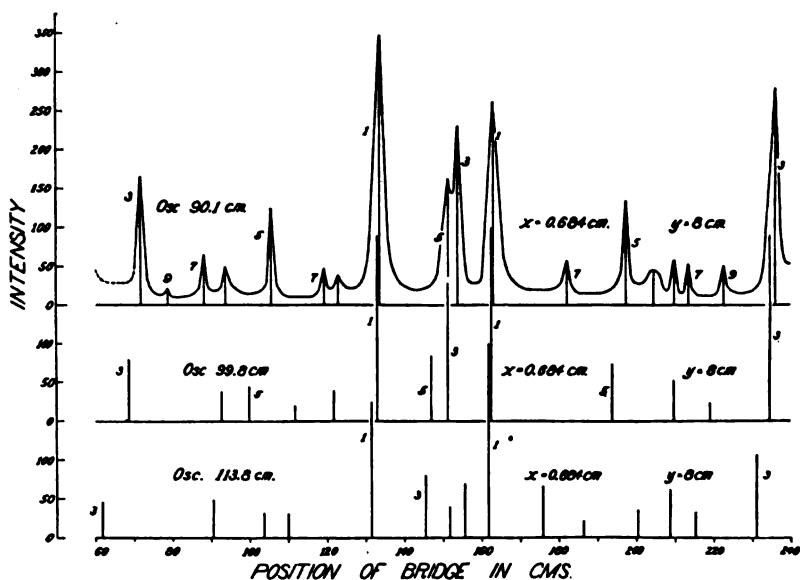


Fig. 3.

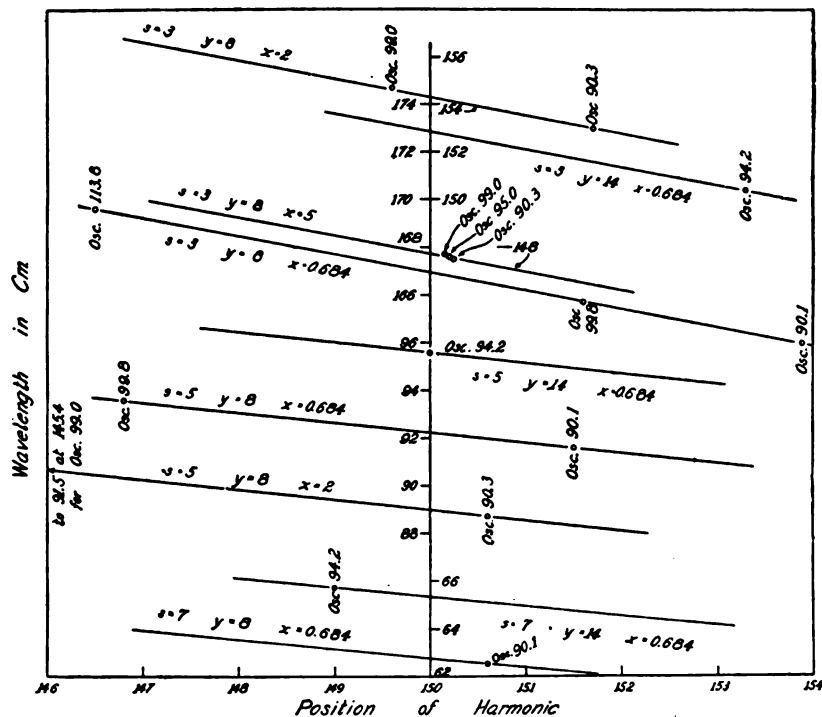
becomes complicated for the closer couplings by the introduction of these extraneous peaks can be clearly seen from an inspection of Table I. Column 10 marked "Unassigned Peaks" has quite a few peaks to its credit.

Blake and Sheard¹ and Blake and Jackson² have shown that for very loose couplings a change in the oscillator length practically has no effect on the position of a given tone. For the closer couplings of the present paper, on the other hand, the case is far different. The position of a given tone is more strongly affected by the oscillator length the tighter the coupling. The same thing is true for a variation in the length of the spark gap. The true position of a given peak can therefore be found

¹ Phys. Rev., N. S., IX., p. 177, 1917.

² L. c.

only by locating it for a given oscillator length which is known to be at or near the optimum spark gap length for that tone. To eliminate errors that would otherwise exist both spark gap curves and bridge curves were taken for the curves shown in this paper, after the manner adopted by Blake and Jackson (q.v.). For the closest coupling (0.684 cm.) the error in position due to ignoring the proper length of spark gap could be as much as 2 cm. for some tones. Even with the precautions



displacements shown in Fig. 3 against the wave-lengths we get those curves of Fig. 4 marked " $y = 8$; $x = 0.684$." The same thing was done for the other values of x shown in Table I., for $y = 8$ cm.

Consider for instance the curve marked $s = 3$; $y = 8$; $x = 0.684$. For the three different oscillator lengths all three points lie on a straight line whose slope is 0.76. Indeed the other curves show that the slope is independent of x . For the fifth harmonic the slope is seen to be 0.42, and again independent of x . For the seventh harmonic only one point on the curve was available, but the slope cannot be far different from that taken, viz., 0.33. The curve marked $s = 3$; $y = 8$; $x = 5$ shows clearly how, for this fairly loose coupling, a variation in oscillator length has but little effect on the position of a given tone, as evidenced by the close proximity of the observed points.

It is clear from Fig. 4 that in order to compare the various tones we must not use the observed wave-length but rather the "reduced" wave-lengths, that is, those that always bring the middle peak for any one set of tones exactly at the center of the Lecher system. In order to determine the reduced wave-lengths in the manner indicated it was necessary that the distance x between the plates should always be changed simultaneously for the two sets of plates at each end of the Lecher system. This condition was always fulfilled experimentally.

The only difference between the results for $y = 8$ cm. and $y = 14$ cm. should be differences due to the increased length of each circuit owing to the wire HK (Fig. 1) being longer for the latter value of y . Each circuit is thereby about 3 per cent. longer. This fact can have but a secondary influence upon the slopes of Fig. 4 for $y = 14$ cm. Only one length of oscillator was used for this value of y , hence the slope for each was taken the same for $y = 14$ cm. as for $y = 8$ cm.

Having thus found for both values of y the reduced wave-lengths which ought to fulfill the Kirchhoff-Abraham¹ equation

$$\frac{2\pi l}{\lambda} \tan \frac{2\pi l}{\lambda} = \frac{Kl}{K_0}, \quad (1)$$

it is apparent that certain corrections must be made before the experimental results can be properly interpreted. In this equation l is the distance from the bridge position nearest the receiver up to the plate, *measured as if the wire were without a bend*, K_0 the end capacity and K the capacity per unit length of the Lecher system. For instance using the reduced values of λ , for $s = 3, 5$ and 7 for $y = 8$ and $x = 0.684$, viz., 167.0, 92.2 and 62.7 cm., the value of K_0 calculates 49.95, 48.63

¹ Kirchhoff, Ges. Abh. p. 180; Abraham Föppl, Th. der El., p. 354.

and 39.83 cm. By taking an end correction of 0.2 cm. these results are changed to 50.90, 51.17 and 43.50 respectively, more nearly constant. As only one point on the curve was available for the seventh harmonic the exact slope of the curve was unknown. Moreover, Blake and Jackson have pointed out that a change of one per cent. in the wave length λ , affects the value of K_0/K 22 per cent. for the seventh harmonic whereas the same change affects the third and fifth only 11 and 15 per cent. respectively.

If one uses the values of the end correction shown in Column 20 of Table I. he gets fairly constant values for K_0/K as shown in column 22. Plotting the values of column 20 we get the lower curve of Fig. 5, a straight line. In column 22 the brace encloses the value of K_0/K for the third and fifth harmonics as being the figures used to determine the

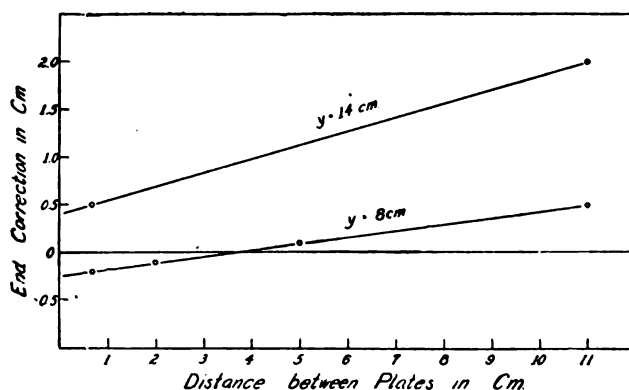


Fig. 5.

mean K_0/K given in column 23. These figures are naturally more accurate than those for the higher harmonics. Of course the figures for the fundamental wave-length in column 15 are found by trial from equation (1) using the value of l for the fundamental (after the end correction has been made) and the value of K_0/K given in column 23.

If we plot the wave-lengths in columns 15 and 16 against the distance x between the plates we get the lower curve of Fig. 6 for the fundamental and third harmonics respectively.

It will be noted that an end correction of 0.5 cm. brought all the results in column 22 for $x = 11$ and $y = 8$ cm. into harmony even up to the ninth harmonic. The mean value for K_0/K is thus found to be 13.18 cm. whereas Blake and Jackson without employing an end correction found 13.70 cm. This small change in their result can in no way materially affect their final results or the theory developed by the author to

explain their results. In this theory, however, there is one thing that needs correction. On page 165 of the reference given the disturbing force is taken as distributed over $2\gamma\lambda/2\pi$. This should be $\gamma\lambda/2\pi$, the method of measurement being such that *all* of this disturbance shows itself in the receiver. Here again, this correction leaves the final equation for the energy measured by the receiver entirely unchanged.

With the lower curve of Fig. 5 as an empirical curve an attempt was made to calculate the end correction for the larger value of γ , viz., 14 cm. For this value of γ it was found empirically that an end correction of

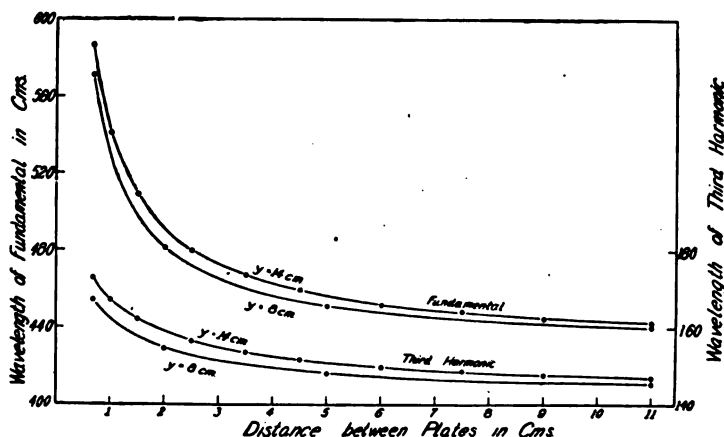


Fig. 6.

0.52 cm. brought K_0/K constant for the smallest value of x , as the table shows.

We proceed to calculate the self-inductance and capacity per unit length of the Lecher system in all its parts. Consider first the straight-away length LM , Fig. 1. The inductance L and the capacity K per unit length are given by the formula

$$L = 4 \log_e \frac{2d}{b} = \frac{1}{v_0^2 K}, \quad (2)$$

where $2d$ ($= 2$ cm.) is the distance between the parallel wires and b is the radius of the wires, viz., 0.038 cm., v_0 being the velocity of light. This gives L 15.853 e.m.u. and $K = 0.06307$ e.s.u. To get L for the length HK we calculate L from (2) for $2d = 8$ cm. or 14 cm. and then take the mean between this value and the value of L for 2 cm. This gives L 18.620 e.m.u. for $\gamma = 8$ cm. and 19.745 e.m.u. for $\gamma = 14$ cm. Now the portion KP of each circuit was a brass rod 3.5 long and 4 mm. diameter. The use of (2), taking $b = 0.2$, gives L 14.756 e.m.u. for $\gamma = 8$

and 16.994 for $y = 14$ cm. If we take K for MB to be 0.063 and for BC to be 0.067 we get the following set of figures: $3 \times 18.620 \times 0.063 + 3.5 \times 14.756 \times 0.067 = 7.0$ cm. Subtracting $3.0 + 3.5$ from 7.0 gives 0.5 cm. end correction, the amount empirically found for $y = 8$ and $x = 11$ cm. If we now use the same value of K for $y = 14$ cm. as for $y = 8$ cm. we arrive at the following results $19.745 \times 0.063 \times 6 + 16.994 \times 0.067 \times 3.5 = 11.5$ cm. Subtracting $6.0 + 3.5$ from 11.5 gives 2.0 cm., the amount of the end correction for $y = 14$ and $x = 11$ cm. This is an upper limit of the end correction for it is hardly likely that the value of K would be as great when MB (Fig. 1) is 6 cm. as for 3 cm. However, this is in part offset by the presence of the table top some 57 cm. below the Lecher wires which would tend to hold the capacity per unit length up according to the formula

$$K = \frac{1}{2 \cosh^{-1} \frac{h}{b}}, \quad (3)$$

where h is the distance from the ground (57 cm.) b being the radius of the wire. This gives $K = 0.0625$, not far from 0.063.

The empirical value of the end correction found for $y = 14$ cm. and $x = 0.684$ is 0.52 cm. If we accordingly connect the values for $x = 0.684$ cm. and $x = 11$ cm. by a straight line we get the upper curve of Fig. 5 and by graphic interpolation the figures in Column 20 of Table I. are found. This enables us to calculate the value of K_0/K (column 22). It is seen that K_0/K for the thirds and fifths begin to diverge from each other for values of x greater than 1 cm. differing by 15 per cent. for $x = 11$ cm. Since there are two factors involved in arriving at these results neither one of which is known with certainty, this is not particularly surprising; for $y = 14$ cm., neither the slopes of the curves of Fig. 4 nor the amount of the end correction are positively known. For that reason the mean K_0/K is not given in column 23 except for the smallest value of x . Plotting the values of λ for the fundamental we get the upper curve of Fig. 6, the curve for the third harmonic being also shown in the figure.

If we plot the values of the end capacity reduced to electrostatic units (column 24) against the distance x for both values of y we get curves I. and II. of Fig. 7, the curve for $y = 14$ being that obtained from the third harmonic. As can be seen from the table the curve for the fifth harmonic would lie between the one drawn and the one for $y = 8$ cm. The crosses are the values of the condenser capacity calculated from Kirchhoff's

¹ *Gesammelte Abhandlungen*, p. 112.

equation

$$C = \frac{r^2}{4x} \left\{ 1 + \frac{x}{\pi r} \left[\log_e \frac{16\pi(x+t)r}{x^2} + \frac{t}{x} \log_e \frac{x+t}{t} - 1 \right] \right\}, \quad (4)$$

in which r , the radius of each plate, was 2.5 cm. and t , its thickness, was 1 mm., while x the distance between the plates was made to vary. Of course the true capacity of a condenser can be obtained (presumably for direct current) only for values of x and t small compared to r , as has

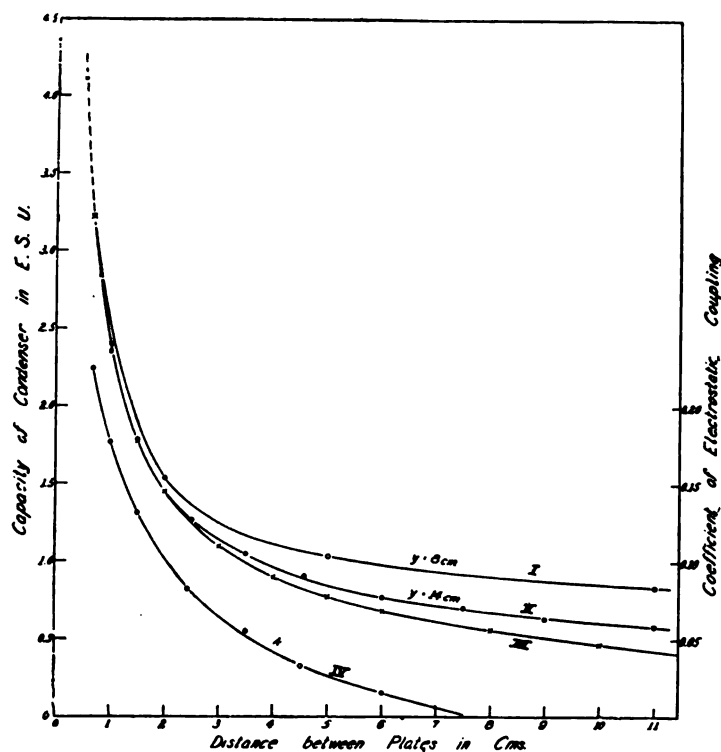


Fig. 7.

been stated above. An inspection of the upper curves of Fig. 7 shows at once that for both values of y theory and experiment agree for small values of x , Table I. giving the capacity for $x = 0.684$ as 3.212 e.s.u. for $y = 14$ cm.; the value being only slightly larger, viz., 3.221 for $y = 8$ cm. due to the close proximity of the other set of plates. Equation (4) gives the value 3.212 e.s.u. The writer interprets this agreement between theory and experiment to lead at once to two important results. First, *the capacity of an air condenser is entirely independent of frequency*. Secondly, when x is as small as 0.684 cm. and y as big as

14 cm. we may practically say that no lines of force run from one plate to another of the same circuit.

It would be interesting to know the exact shape of the curve (Fig. 7) connecting capacity with distance between the plates on the assumption that no lines of force run between plates of the same circuit even for the largest values of x used. An empirical equation for the curve could then be found and the mathematical physicists could try their hand at the problem for circular plates just as many writers have done so admirably for spherical balls.¹ If one looks up the derivation of Kirchhoff's formula (Eq. 4) one finds that on account of analytical difficulties various approximations have been made. Since in the derivation x and t are taken small compared to r equation (4) could not be expected to hold for x greater than 1 cm. say. For this value of x the experimental result practically agrees with equation (4). On account of the limitation imposed with respect to x and r it is not altogether justifiable to keep second order terms but if a second approximation² is made equation (4) becomes

$$C = \frac{r^2}{4x} \left\{ 1 + \frac{x}{\pi r} \left[\log_e \frac{16\pi(x+t) \left(r + \frac{x}{4} \right)}{x^2} + \frac{t}{x} \log_e \frac{x+t}{t} - 1 \right] \right\}. \quad (5)$$

In Fig. 7, curve III. is plotted from Kirchhoff's formula (Eq. 4) and the extra term in (5) has no appreciable effect for values of x less than 3 cm. For $x > 3$ cm. (5) gives a value of C continually increasing compared to (4) being for $x = 10$ cm. about 25 per cent. greater than (4) gives and almost exactly the value obtained experimentally by Curve II. This is mentioned merely to show that (4) gives too small a value of C for the larger values of x and that accordingly the true value of the capacity of a parallel plate condenser when the distance between the plates ceases to be small compared to their radius probably lies roughly half way between Curves II. and III. of Fig. 7.

FURTHER COMPARISON WITH THEORY.

It is desirable to compare the results of this paper with the theoretical results given by Lord Rayleigh³ for the mutual reaction between two circuits each having kinetic and potential energy and dissipation, *i.e.*, inductance, capacity and resistance. Lord Rayleigh obtains for the effective "stiffness" S' and the effective resistance R' the following equations:

¹ See for instance Russell's *Alternating Currents*, Second Edition, Chap. VIII.

² Cordial acknowledgment is here made to my friend, Professor R. D. Bohannon, for deriving this second order term.

³ *Phil. Mag.*, XXI., pp. 369-381; *Scientific Papers*, Vol. II., p. 484.

$$S' = c_{11} - p^2 a_{11} - \frac{(c_{12} - p^2 a_{12})^2}{c_{22} - p^2 a_{22}} + \frac{p^2 [b_{12}(c_{22} - p^2 a_{22}) - b_{22}(c_{12} - p^2 a_{12})]^2}{(c_{22} - p^2 a_{22})[(c_{22} - p^2 a_{22})^2 + p^2 b_{22}^2]}, \quad (5)$$

$$R' = b_{11} - \frac{b_{12}^2}{b_{22}} + \frac{[b_{12}(c_{22} - p^2 a_{22}) - b_{22}(c_{12} - p^2 a_{12})]^2}{b_{22}[(c_{22} - p^2 a_{22})^2 + p^2 b_{22}^2]}. \quad (6)$$

In these equations a_{11} , a_{22} ; b_{11} , b_{22} ; c_{11} , c_{22} are the coefficients of self inductance, self resistance and self elastance, a_{12} , b_{12} , c_{12} being the corresponding mutual coefficients. Moreover it is to be remembered that the self coefficients are to be determined for each circuit when it alone is in the field. Thus, for instance, c_{22} is the self elastance of the secondary circuit, that is, it is the reciprocal of the capacity of the condenser plate attached to the secondary circuit, this capacity being measured when the secondary circuit is infinitely removed from the primary circuit. Theoretical writers have frequently taken $c_{22} - p^2 a_{22}$ and $c_{11} - p^2 a_{11}$ each equal to zero provided p is the natural frequency of each circuit when far removed from the other. In practice, however, this is not justifiable because of the impossibility of neglecting the distributed capacity of the discharge wires. Hence equation (1) rather than Kelvin's formula for condenser discharge applies. Accordingly in the expression $c_{11} - p^2 a_{11}$ and $c_{22} - p^2 a_{22}$ of (5) and (6) we must substitute for p the natural frequency n that fulfills equation (1). Rayleigh's equations thus become

$$S' = c_{11} - n^2 a_{11} - \frac{(c_{12} - n^2 a_{12})^2}{c_{22} - n^2 a_{22}} + \frac{n^2 [b_{12}(c_{22} - n^2 a_{22}) - b_{22}(c_{12} - n^2 a_{12})]^2}{(c_{22} - n^2 a_{22})[(c_{22} - n^2 a_{22})^2 + n^2 b_{22}^2]}, \quad (7)$$

$$R' = b_{11} - \frac{b_{12}^2}{b_{22}} + \frac{[b_{12}(c_{22} - n^2 a_{22}) - b_{22}(c_{12} - n^2 a_{12})]^2}{b_{22}[(c_{22} - n^2 a_{22})^2 + n^2 b_{22}^2]}. \quad (8)$$

We proceed to calculate the quantities in these equations that are known from geometrical considerations for the case $y = 14$ cm., $x = 0.684$ cm.

Of the resistance terms b_{12} will be spoken of below; b_{11} can be calculated from the formula¹

$$\frac{R}{R_0} = \frac{\pi h}{(h^2 - d^2)^{\frac{1}{2}}} \sqrt{\frac{nd^2}{\rho}} + \frac{1}{8} - \frac{[h - (h^2 - d^2)^{\frac{1}{2}}][d^2 + h(h^2 - d^2)^{\frac{1}{2}}]}{2(h^2 - d^2)^{\frac{3}{2}}}, \quad (9)$$

where R is the high and R_0 the low frequency resistance. For the case here considered $h = 2$ cm. and $d = 0.076$ cm. so (9) reduces to

$$\frac{R}{R_0} = \pi \sqrt{\frac{nd^2}{\rho}} + \frac{1}{8}. \quad (10)$$

Taking ρ , the resistivity of copper, to be 1,755 and $\lambda = 500.4$ cm. = v_0/n

¹ Cohen, Calculation A. C. Problems, page 8, formula 12.

we get $R/R_0 = 44.25$. Now R_0 , the resistance of 200 cm. of copper wire of 0.076 cm. diam. is .0775 ohms and hence $R = b_{11} = 3.43$ ohms. To get b_{22} we must add to b_{11} the resistance of the detecting thermal couple, 4.16 ohms. The couple was made of iron and constan wire of diameter 0.02 mm., hence its resistance was independent of the frequency of oscillation.

To get c_{11} we must first know the capacity of a circular brass plate 5 cm. in diameter and 1 mm. thick when alone in space. The capacity of such a plate of infinitesimal thickness is known to be $2r/\pi$ e.s.u. So far as the writer knows no one has yet worked out the necessary correction for a plate of finite thickness. It is believed, however, that no appreciable error has been made in correcting for the thickness in the following way. The value of the capacity of a circular plate condenser has been calculated from equation (4) both for $t = 0$ and for $t = 1$ mm. In this way Fig. 8 is obtained, in which the ordinate represents the

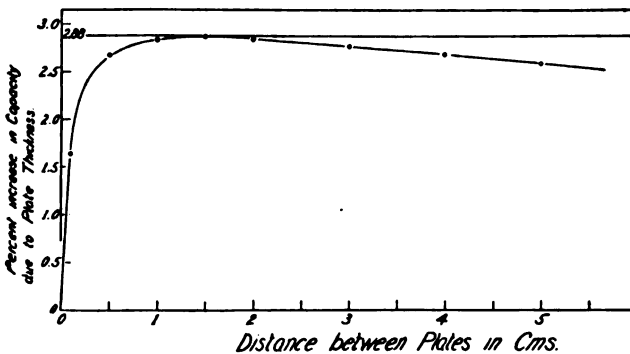


Fig. 8.

per cent. increase in capacity due to the finite thickness, the abscissa being the distance between the plates. It is argued that the fact that the curve after rising to a maximum decreases again (*but very slowly*) is due to the fact that the formula is not admissible for distances between the plates that are not small compared to their diameter. Hence the asymptote for infinite distance will be *only very slightly* above the maximum point; it is taken as 2.88 per cent. Whence $1/c_{11} = 1.0288 \times (2r/\pi) = 1.637$ e.s.u., giving $c_{11} = c_{22} = 0.6109$ e.s.u.

We can get c_{12} by noting how the fundamental tone splits in two when the coefficient of coupling becomes appreciable. We have for k , the coefficient of coupling, $k = (\lambda_2^2 - \lambda_1^2)/(\lambda_2^2 + \lambda_1^2)$ where λ_1 and λ_2 are the two observed wave-lengths for the fundamental tone. From this equation k is plotted as Curve IV., Fig. 7. If δ is the difference

between the two figures given in column 5 of Table I., we have $\lambda_2 - \lambda_1 = 4\delta$ and furthermore $\lambda_2^2 + \lambda_1^2 = 2\lambda^2$ where λ is the fundamental wave-length fulfilling equation (1). Except for the larger values of k no appreciable error is made in taking $\lambda_2 + \lambda_1 = 2\lambda$. Now it will be proved in another paper that $k^2 = c_{12}^2/(c_{11}c_{22})$. If, as here, $c_{11} = c_{22}$ we have $c_{12} = kc_{11} = kc_{22}$ and for $y = 14$, $x = 0.684$ we get $c_{12} = 0.1371$ e.s.u.

We get a_{11} directly from equation (2) by multiplying by $l_1 = 100.02$ cm., giving $a_{11} = a_{22} = 1585.0$ e.m.u. = $1585/v_0^2$ e.s.u.

To get a_{12} we proceed as follows (see Fig. 9). The mutual inductances

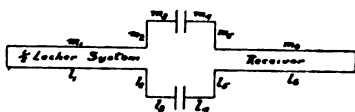


Fig. 9.

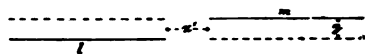


Fig. 9a.

between l_1 and l_6 , l_3 and l_4 , l_1 and m_6 , l_3 and m_4 , l_3 and l_6 , l_3 and m_6 , l_2 and m_5 , were each calculated by means of a formula very kindly supplied by Dr. F. W. Grover,¹ formerly of the Bureau of Standards. The writer desires here to express his hearty appreciation of this assistance. The formula is

$$M = \alpha \log_e[\alpha + (\alpha^2 + z^2)^{1/2}] - \beta \log_e[\beta + (\beta^2 + z^2)^{1/2}] \\ - \gamma \log_e[\gamma + (\gamma^2 + z^2)^{1/2}] + \delta \log_e[\delta + (\delta^2 + z^2)^{1/2}] \\ - [(\alpha^2 + z^2)^{1/2} - (\beta^2 + z^2)^{1/2} - (\gamma^2 + z^2)^{1/2} + (\delta^2 + z^2)^{1/2}], \quad (11)$$

where $\alpha = l + m + x'$, $\beta = l + x'$, $\gamma = m + x'$, $\delta = x'$ (Fig. 9a). To get the mutual inductance between l_2 and l_5 use was made of formula (174) of B.S. Cir. No. 74, viz.,

$$M = 2 \left[l \log_e \frac{l + (l^2 + d^2)^{1/2}}{d} - (l^2 + d^2)^{1/2} + d \right], \quad (12)$$

in which for $y = 14$ cm. we have $l = 6$ cm. and $d = 7.89$ cm. In getting the other factors we have $l_1 = l_6 = 90$ cm., $l_3 = l_4 = 3.5$ cm., $x' = 0.89$ cm. for l_3l_4 and 7.89 cm. for l_1l_6 , while $z = 2$ cm. for both of these. For l_3l_6 , $x' = 4.39$ cm. and $z = 6$ cm.

If for the moment we represent the mutual inductance between l_2 and l_5 , say, by l_2l_5 and do the same for the other factors we have

¹ It should be stated that for circuits such as are used in this paper Dr. Grover's formula lends itself to calculation much more easily than the geometric formulas 174 to 182 of Circular No. 74 of the Bureau of Standards. This is due to the fact that the distance between the Lecher wires was small compared to their length.

$$\begin{aligned}
 a_{12} &= 2[(l_1l_6 - l_1m_6) + (l_3l_4 - l_3m_4) + (l_2l_5 - l_2m_5) + (l_3l_6 - l_3m_6)] \\
 &= 2[(102.64 - 102.48) + (2.71 - 0.83) + (4.37 - 2.84) \\
 &\quad + (9.99 - 8.68)] \\
 &= 9.76 \text{ e.m.u.} = \frac{9.76}{v_0^2} \text{ e.s.u.}
 \end{aligned}$$

This value is of course the low frequency mutual inductance. On the same basis the low frequency self inductance of the first circuit is 1,597 instead of 1,585 e.m.u. Thus for the closest coupling ($x = 0.684$ cm.) while $c_{12}/c_{11} = 22.4$ per cent., a_{12}/a_{11} equals only 0.6 per cent. Hence for all except the closest coupling we may take $a_{12} = 0$ practically.

If the last term in (7) is neglected we have

$$S' = c_{11} - n^2a_{11} - \frac{(c_{12} - p^2a_{12})^2}{c_{22} - n^2a_{22}}. \quad (13)$$

Substituting the values calculated above for the case $y = 14$ cm., $x = 0.684$ cm. we get

$$S' = 0.6109 - \frac{4\pi^2 \cdot 1585}{500.4^2} - \frac{\left(0.1371 - \frac{4\pi^2 \cdot 9.76}{586.5^2}\right)^2}{0.3610} = 0.3098 \text{ e.s.u.}$$

The reciprocal of the observed capacity is $1/3.212 = 0.3113$ e.s.u. There is thus agreement to within 0.48 per cent. Had a_{12} been neglected the agreement would not be quite so good, viz., 0.71 per cent.

From the nature of the terms in (7) it is clear that $c_{11} - n^2a_{11}$ is independent of x while the fractional terms depend upon x . Of the latter the first term decreases as x increases while the second is intrinsically positive. There can be no question, it seems to the writer, that the curve for $y = 14$ cm. of Fig. 7 is a close approximation to the true relationship between the effective capacity of a circuit and its nearness to a second similar circuit. (See above remarks on Curves II. and III., Fig. 7, however.) Accordingly since as x increases the only quantities that change are a_{12} , c_{12} and b_{12} and since it is obvious that a_{12} and c_{12} decrease as x increases the quantity b_{12} must increase as x increases. If we call the difference between the two sides of (13) d^2 and treat (7) as an equation to determine b_{12} for different values of x we get the upper curve of Fig. 10. The values of b_{12} thus found are shown in column 27 of the table. If these are substituted in (8) we get the value of R' shown in column 28. It will be noted that all the values of R' are negative with an absolute minimum for $x = 1.5$ cm. about.

In getting the value of b_{12} since p^2b_{22} is negligible compared to $c_{22} - n^2a_{22}$ we have

$$b_{12} = \frac{d}{p}(c_{22} - n^2a_{22})^{1/2} + b_{22} \frac{c_{12} - p^2a_{12}}{c_{22} - n^2a_{22}}. \quad (14)$$

The second term is small compared to the first term so this reduces to

$$b_{12} = \frac{d}{p} (c_{22} - n^2 a_{22})^{1/2} = \frac{586.5}{2\pi} \cdot 30 \cdot 0.6078d \text{ ohms.} \quad (15)$$

The maximum value of b_{12} was for $x = 11$ cm. and was found to be 1,966 ohms or $65.5/v_0$ e.s.u. The maximum value of R' numerically was 344 ohms or $11.5/v_0$ e.s.u. Had the maximum value of R'^2 been sub-

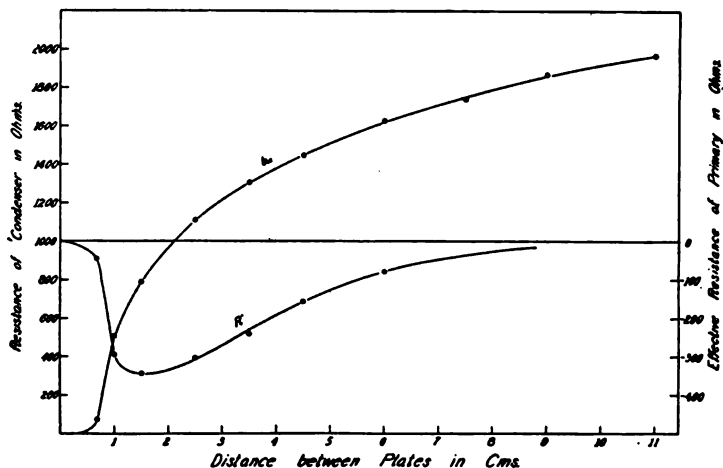


Fig. 10.

stituted for b_{22}^2 in the expression $(c_{22} - n^2 a_{22})^2 + p^2 b_{22}^2$ it would still be true that the resistance term would be negligible compared to the reactance term, though this would not be the case if the maximum value of b_{12} were so substituted. This is mentioned in an attempt to aid in the justification for the physical interpretation of the curves of Fig. 10, since it should be remembered that equation (1) holds only provided the resistance is negligible compared to the reactance. It may well be that a part of this measured condenser resistance or mutual resistance b_{12} is due to the fact that when the coupling becomes very loose the system ceases to be a non-radiant system and hence equation (1) would not hold.

It has been proved above beyond question that for small values of the distance separating two elastically reacting circuits theory and experiment agree and that as the distance between the circuit increases the condenser resistance increases, as is natural to expect. It would be interesting to investigate this condenser resistance further with a view to seeing how much of it could be explained as a true dielectric hysteresis and how much is due to radiation losses or other causes. Attention is here called to the shape of the curve of Fig. 10, which while not wholly

analogous to the curve of magnetization of ferromagnetic substances nevertheless is partly so.

In another paper important results in electromagnetic theory based upon the definitions of self and mutual elastance and self and mutual capacitance will be given and the application will be made to the thermionic tube.

Cordial acknowledgement for financial assistance is made to the Ohio State University for help in taking the observations and to the Committee on Grants of the Amer. Assn. for the Adv. of Sci. for help in curve drawing and making the calculations.

PHYSICAL LABORATORY,
OHIO STATE UNIVERSITY.

THE ACTION OF ROENTGEN AND GAMMA RADIATIONS UPON THE ELECTRICAL CONDUCTIVITY OF SELENIUM CRYSTALS.

By A. M. McMAHON.

SYNOPSIS OF RESULTS.

Action of Gamma and Roentgen Radiations Demonstrated.—The increase in conductivity of selenium "cells" accompanying the absorption of X-rays has been found characteristic of individual crystals of metallic selenium. A similar effect upon the crystals has been observed to be produced by the absorption of the gamma rays of radium. The fact that a crystal fatigued to gamma rays is fatigued also to X-rays is evidence that the action in the two cases is similar. The indications are that the crystals are approximately twice as sensitive to gamma rays as to X-rays of the type used (L rays of tungsten).

New Electro-mechanical Properties.—Increase of mechanical pressure upon a crystal results in an increase of its sensibility both to X-rays and gamma rays. This is exactly the same result, qualitatively, as that secured by Brown in his study of the light effect (PHYS. REV., 4, p. 85).

Action of Separate X-ray Frequencies.—Unsatisfactory results obtained in the study of the strong lines of the L radiation of tungsten are thought to be due to the low intensities afforded by the X-ray spectrometer. Another method of attack is being planned.

Comparison of the Light, X-ray, and Gamma Ray Effects upon an Isolated Crystal.—Calculations made upon the basis of the energy absorbed indicate that a selenium crystal is most sensitive to gamma rays. The ratios of the change in conductivity to absorbed energy for exposures of one minute are 33.2, 58.5, and 108.9 $\left(\frac{\text{Mhos}}{\text{Joules}} \right)$ respectively.

I. INTRODUCTION

IN view of the established relationships between light, Roentgen and gamma rays, knowledge of their effects upon the electrical properties of crystals of metallic selenium assumes new interest. It has been shown¹ that the change in conductivity exhibited by these crystals on exposure to light depends upon the time, intensity, and character of the illumination. It is also known² that the sensibility (absolute ΔC) varies directly with the mechanical pressure to which the crystal is subjected. Other important facts concerning the action of visible radiations are available, but it is with those mentioned that this paper is mainly concerned. The effect of Roentgen rays upon films of metallic selenium has been investigated,³ with the discovery that the form of the ΔC -time

¹ Brown, PHYS. REV., 33, p. 1.

² Brown, PHYS. REV., N.S., 4, p. 85.

³ McDowell, PHYS. REV., 30, p. 474.

curves for this type of radiation is the same as that of those found for visible radiation, although the rate of recovery is much slower in the former case. The action of radioactive materials upon selenium "cells" has been known for some time,¹ although the separate action of the gamma rays had, within the knowledge of the writer, received no attention up to the time of the investigations reported upon in the present paper. It is thought that the continuation of this work should assist in relating properly the phenomena of conduction in solids with those of absorption, radiation, and atomic structure.

The experimental studies upon the action of X-rays, described in this paper, were initiated to secure information regarding the relation between the sensibility of these crystals and the frequencies of the strong lines in the characteristic L radiation of tungsten. Their change in conductivity on illumination has received a variety of theoretical treatments, among which that based upon a resonance hypothesis² offers considerable promise. According to this theory the electrons of the selenium atom whose radiation frequency corresponds, or nearly corresponds, to the frequency of the exciting agent, are loosened from their atomic bonds temporarily, and, in addition to the normal number free at any instant within the crystal, as indicated by the conductivity in the dark, contribute to the total current resulting from the application of a potential. A region of maximum sensibility has been found³ in the red, and indications of a maximum in the ultra-violet. It was thought that if a region of maximum sensibility could be found within the range of X-ray frequencies the resonance theory would be much strengthened, and important evidence concerning the structure of the selenium atom obtained. Other experiments, incidental to this purpose, which have brought out a significant relation between the X-ray, gamma ray, and light effects, as regards the dependence of sensibility (absolute ΔC), upon mechanical pressure, constitute an important part of this report.

The studies herein described were made upon a single crystal selected from a lot prepared by Dr. W. E. Tisdale in the spring of 1916. They were of the hexagonal² variety secured by distillation in vacuo at 185° C. Investigations were in air at barometric pressure. The dimensions of the crystal chosen were roughly $3.2 \times .2 \times .1$ mm. It was mounted between carefully polished platinum electrodes, the mechanical pressure necessary being supplied by weights. In every case the direction of the rays was parallel to the long axis of the crystal.

¹ Brown, *PHYS. REV.*, 26, p. 273.

² Pfund, *PHYS. REV.*, 28, p. 324.

³ Sieg and Brown, *PHYS. REV.*, N.S., 4, p. 507.

II. THE ACTION OF X-RAYS.

Preliminary measurements upon the change of conductivity produced by single lines in the L radiation of tungsten indicated that the effect, if discernible with the apparatus used, is very small. A simple experiment was then planned and carried out to ascertain whether the addition of mechanical pressure would, as is the case when the exciting agent is visible light, produce an increase of sensibility.

The crystal was mounted in a metal box which was grounded. The leads were carried through lead tubing to a Wheatstone bridge of high resistance arms, situated in an adjoining room. All of this was found necessary to eliminate the electrostatic effects which otherwise resulted from the presence of the high potential transformer actuating the Coolidge tube furnishing the X-rays. In this experiment the total radiation from the tube, operating at 48.0 kv. and 3.5 amps. heating current, was used. It was found difficult to maintain the constancy of the input with better than 10-15 per cent. accuracy. Hence the distribution of points in Fig. 1. A heavy pendulum operating contact keys controlling

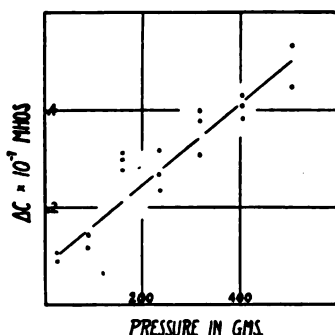


Fig. 1.

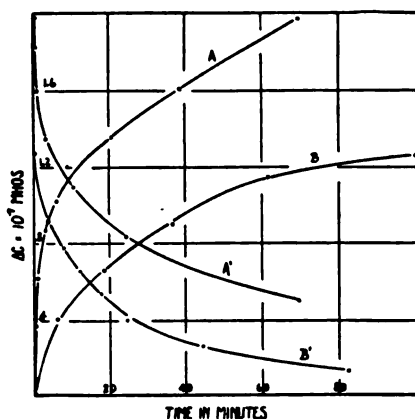


Fig. 2.

FIG. 1. The effect of change in mechanical pressure upon sensibility to X-rays.

FIG. 2. Excitation and recovery curves—X-rays. A, Excitation. Pressure 320 gms. Initial Conductivity 4.8×10^{-7} Mhos. A', Recovery. B, Excitation. Pressure 320 gms. Initial Conductivity 4.6×10^{-7} Mhos. B', Recovery.

the primary circuit of the transformer provided the means of keeping the time of exposure constant (one second), for different pressures. Time for recovery was allowed between observations. Brown and Clarke's method¹ was used in observing the change in conductivity. The results, plotted in Fig. 1, indicate an unmistakable increase of sensibility with pressure, similar to that found by Brown to exist when the exciting

¹ Brown and Clarke, *PHYS. REV.*, 33, p. 53.

agent is light. Two other crystals studied in the same manner gave data which bear out this conclusion.

Following a procedure made obvious by these results, measurements were made anew upon five of the principal lines in the L radiation of tungsten (λ 's—1.47, 1.27, 1.24, 1.09, 1.06 Å). Five hundred grams pressure was applied to the crystal—as much as was deemed advisable in view of the danger of fracturing. I am indebted to Dr. Elmer Dershem for the use of one of his X-ray spectrometers in these experiments as well as for initiation into the technique of its manipulation. Up to the present time, however, attempts to secure the effects of separate lines in this manner have been unsuccessful. Experiments are being considered which may solve the intensity difficulty, for this is thought to be the main reason for the failure.

It is very readily demonstrated that the total radiation from a Coolidge tube, operating upon a moderate energy input, produces a change in conductivity. Fig. 2 gives the results of two series of observations made upon the excitation and recovery characteristics of the crystal. Two additional experiments prove that these data come from changes in the conductivity of the crystal due to the absorption of X-rays. The apparatus was the same as described for the pressure- ΔC experiment. First, with no heating current flowing in the Coolidge tube filament, but with the customary 48.0 kv. potential applied, no galvanometer deflection was observed—*i.e.*, the balance of the bridge was maintained. Second, the introduction of a sheet of lead into the path of the rays from the target to the crystal resulted in a smaller total change than that produced in the absence of the lead. In making the measurements from which the data represented in Fig. 2 were computed a null method was used. The excitation and recovery of the crystal are so very much slower than the action of the galvanometer used that it was practical to change the balancing arm of the bridge by a convenient amount, usually 100 ohms, and observe with a watch the time required for the balance to be reattained.

III. THE ACTION OF GAMMA RAYS.

It was found time conserving to carry on experiments with gamma rays along with the X-ray studies, and in consequence of this practice a very interesting relation was discovered to exist between the gamma ray and X-ray effects, which, however, might have been predicted. In the first experiments with the crystal whose properties are here discussed gamma rays were used. When these were finished X-ray excitation and recovery curves were secured, six hours having been allowed for recovery. The results of the latter are given by curves *B* and *B'* of Fig. 2. By

comparison with the characteristics of other crystals from the same lot it was thought that the equilibrium value of the conductivity change, indicated by *B*, was too low. More time (48 hrs.) was allowed for recovery and curves *A* and *A'* secured, which show an equilibrium value comparing favorably with those secured for other crystals. Unquestionably, therefore, a crystal fatigued to gamma rays is also fatigued to X-rays, a result which is in accord with the view that these radiations are of a similar nature.

The effect of the gamma rays was obtained by lowering a thin glass vial containing a milligram of radium into proximity with the crystal through a hollow lead cylinder with walls a millimeter thick to absorb the alpha and beta rays. In these experiments the drift of the galvanometer spot was noted at ten second intervals and later translated into change in conductivity by proper calibration of the bridge. This alteration in method was found necessary in order to differentiate clearly between the effect of the gamma rays and the initial variation common in the balance of the bridge. Much more data than can be given here were taken to make sure of this point. It is thought that the action of

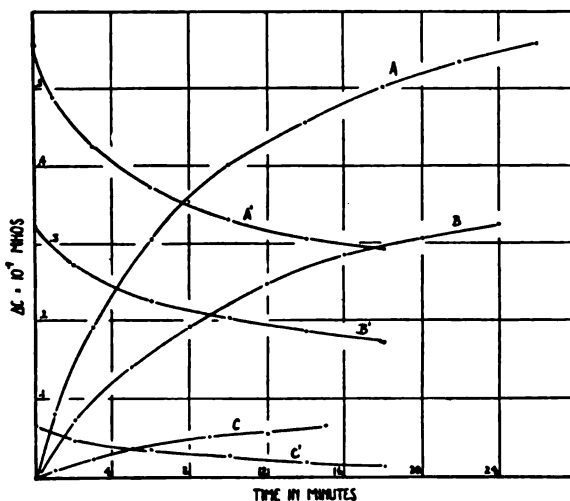


Fig. 3.

Excitation and recovery curves—gamma rays. *A*, Excitation. Pressure 320 gms. Initial Conductivity 5.53×10^{-7} Mhos. *A'*, Recovery. *B*, Excitation. Pressure 216 gms. Initial Conductivity 5.08×10^{-7} Mhos. *B'*, Recovery. *C*, Excitation. Pressure 24 gms. Initial Conductivity 1.25×10^{-7} Mhos. *C'*, Recovery.

the gamma rays is established beyond question. Fig. 3 gives curves secured with the same crystal as before for three different pressures. The error due to variations in the balance of the bridge is probably less

than 1 per cent. for the maximum values of ΔC in curves *A* and *B*. The increase of sensibility with increase of mechanical pressure is unmistakable.

IV. A COMPARISON OF THE LIGHT, X-RAY, AND GAMMA RAY EFFECTS.

It is of great importance to compare the X-ray, gamma ray, and light effects from the standpoint of absorbed energy. Although the data thus far available do not permit of direct comparison, it is possible by making certain assumptions to base a calculation upon them which leads to an interesting result. It is hoped that the additional measurements necessary for a more satisfactory comparison may be accomplished in the near future. The present calculations were made as follows.

The energy of the Roentgen rays was secured from the input to the Coolidge tube and the value for the efficiency of production at 48.0 kv. found by Rutherford.¹ The energy of the gamma rays was found from their known heating effects. An observation was made upon the change in conductivity undergone when light of $.7 \mu$ and known intensity was incident upon the crystal. Assuming the absorption law $I = I_0 e^{-\mu x}$ for Roentgen and gamma rays, and considering that all of the light entering the crystal must have been absorbed (the crystals are not transparent), we get the following results:

Radiation.	$E_i = \text{Incident Energy}$ Ergs. Sec.	$E_a = \text{Absorbed Energy}$ Ergs. Sec.	ΔC in 1 Min.
Gamma rays (Ra) ..	6.57×10^{-3}	3.74×10^{-4}	$.08 \times 10^{-7}$
X-rays (L of W)	2.60×10^{-2}	1.06×10^{-3}	$.62 \times 10^{-7}$
Light ($.7 \mu$)	1.84×10^{-3}	1.19×10^{-3}	$.15 \times 10^{-7}$

Benoist's relation between absorption and atomic weight was used in obtaining the X-ray absorption constant, $\mu = 1.64$. The assumption of Rutherfords' relation between density and absorption was necessary to secure the gamma ray absorption constant, $\mu = .184$. The distribution of X-ray, gamma ray, and light energy was considered uniform.

From these results and the intensity law,² $C = k I^{1/2}$, the change in conductivity when $E = 2.60 \times 10^{-2}$ was calculated for each type of radiation. The absorbed energy was found, also, and the ratio, $\Delta C/E_a$, formed, viz.:

Radiation.	ΔC in 1 Min.	E_a in Ergs. Sec.	$\frac{\Delta C}{E_a}$
Gamma rays	$.16 \times 10^{-7}$	1.47×10^{-3}	108.9×10^{-7}
X-rays	$.62 \times 10^{-7}$	1.06×10^{-3}	58.5×10^{-7}
Light	$.56 \times 10^{-7}$	1.69×10^{-3}	33.2×10^{-7}

¹ Rutherford, Phil. Mag., 30, p. 361.

² Nicholson, Phys. Rev., N.S., 3, p. 1.

Although the figures resulting from these computations are in the nature of approximations it is hoped that their order of magnitude may prove suggestive of the possibilities in this line of work.

The conclusion of the greater sensibility to gamma rays seems quite certain, however. For if instead of the above square-root intensity law we assume the linear law¹ for faint illumination, the comparison is even more favorable to the gamma rays. Discussion of just what this apparent difference may mean is postponed until a more complete experimental study can be made.

In conclusion I wish to express my appreciation of the interest the staff of the Department of Physics of the University of Iowa has taken in these problems. Especially do I wish to thank Dr. F. C. Brown for his suggestions and encouragement throughout the progress of the work.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

¹ Brown and Sieg, *PHYS. REV.*, N.S., 2, p. 487.

THE EFFECT OF TEMPERATURE UPON THE INFRA-RED ABSORPTION OF CERTAIN GLASSES.

BY G. E. GRANTHAM.

SYNOPSIS.

Change of the infra-red transmission of glass with rise of temperature.—Dr. K. S. Gibson has studied the effect of temperature rise upon the absorption spectra of certain glasses in the visible region. He found a shift toward the longer wave-lengths and a decrease in transmission. The infra-red transmission ($0.6\ \mu$ to $4.0\ \mu$) of the same specimens has been studied at 80°C ., 307°C . and 440°C . Rise of temperature increases the transmission in the region of an absorption band. There is a shift of the transmission bands toward the longer wave-lengths. The same results are obtained whether one observes a series of ascending or a series of descending temperatures.

THE effect of temperature upon infra-red absorption spectra has been investigated by L. C. Martin¹ for black biotite. It was found that in general the absorption was greatly increased by a rise of temperature and that this effect was reversible. He also found that a rise of temperature had a tendency to obliterate an absorption band which existed at a lower temperature. With rise of temperature the maximum of transmission moved toward the less refrangible part of the spectrum. The suggested explanation of these effects is that a temporary chemical change within the crystal is caused by a rise of temperature.

K. S. Gibson² has investigated the effect of temperature upon the absorption spectra of several glasses of known composition in the visible region ($0.5\ \mu$ – $0.7\ \mu$). He found that there was an increase of absorption with rise of temperature and that this was accompanied by a shift of the less refrangible edge of an absorption band toward the red end of the spectrum. On account of these changes the edge of the band became less steep and there was therefore a change in color of the specimen. The object of this investigation was to study the effect of temperature upon the infra-red absorption spectra of the same specimens used by Gibson, and to see whether the shift of the edge of an absorption band, as found by him, was coincident with a shift of a transmission band lying largely in the infra-red or resulted in a narrowing of such a transmission band.

¹ Proc. of the Royal Soc., Series A, Vol. 96, 1919.

² The PHYSICAL REVIEW, N.S., Vol. VII., 1916.

The glasses used were:

No.	Color.	Coloring Material.	Glass.	Thickness.
1	Red.....	Cadmium-selenium	Zinc	1 mm.
2	Orange.....	Cadmium-selenium	Non lead	2 mm.
3	Light amber.....	Cadmium-selenium	Borosilicate	2 mm.
4	Lemon yellow.....	Cadmium-sulphide	Zinc	2 mm.

These glasses have transmission curves which are very similar and investigation of these specimens indicated the advisability of trying a specimen with well defined transmission and absorption bands. A specimen of Corning G.585, blue glass of 1.98 mm. thickness was chosen.

These specimens were placed in a furnace and the absorption spectra studied between $0.6\ \mu$ and $4.0\ \mu$ for these temperatures; 80°C. , 307°C. and 440°C.

APPARATUS.

The Furnace.—The construction of the furnace is shown in Fig. 1. A cylindrical piece of iron, *I*, 12 cm. long and 8 cm. in diameter was

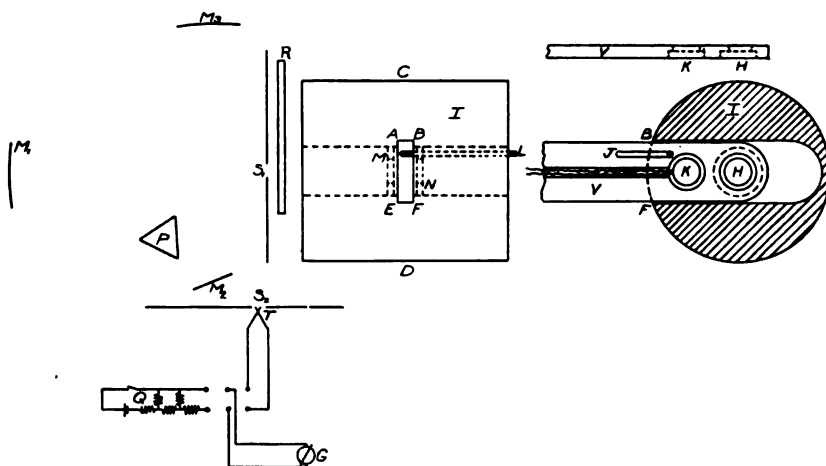


Fig. 1.

chosen as the core of the furnace. A hole 2.5 cm. in diameter was bored lengthwise through its center. The cylinder was then cut in halves at *CD* and a rectangular slot *ABEF* was milled into the end of each half and the halves fitted together and fastened by screws. An iron slide *V* was made which would fit the slot in *I* snugly. Two holes *K* and *H* 2.2 cm. diameter were bored half way through the slide, and, then using the same center, a hole 1.7 cm. diameter was bored the remaining distance through the slide. This afforded a shoulder against which the specimen

could be fastened by means of cement. A slot *J* cut through the slide served by means of the stop *L* to keep the holes *H* and *K* concentric with the cylindrical hole in the block *I*. Two iron washers *M* and *N* of internal diameter 1.5 cm. served the double purpose of diminishing the effect of reflection in the furnace and to correct for any error due to the hole *H* not being in the same position relative to the furnace as *K*, when the slide was at the limits of its path as determined by the slot *J*. The cylinder was heated by means of an electric current passing through a coil of iron wire, which was wound around the cylinder between layers of asbestos paper soaked in water-glass. At first the ends of the cylinder were covered with thin sheets of mica. This gave a large percentage of reflection and trial showed that if the bulb of the lamp giving the source of radiation was placed against the end of the furnace, there was no difficulty in keeping the sample at a constant temperature. The specimen was placed in the hole *K* and the junction of a copper-constantin thermocouple, was placed in contact with the specimen. The leads were buried in fire clay in the groove cut in the slide. The thermocouple was placed in conjunction with a Wolff potentiometer and was calibrated at 99.2° C., 232° C., 419° C. and 630° C. corresponding to the boiling point of water, and the melting points of tin, zinc and antimony respectively. The maximum temperature variation from the values given was probably not greater than five to eight degrees.

The Spectrometer.—A Hilger infra-red constant deviation spectrometer equipped with a Coblentz bismuth-silver thermopile was used. Rays on entering the collimating slit *S*₁, were rendered parallel by the concave nickel steel mirror *M*₁ of 27 cm. focal length, and 2.7 cm. aperture, and passed through the rock salt prism *P* to the Wadsworth¹ mirror system. The polished faces of the prism were 3.2 cm. high and 4.2 cm. long. After reflection by the plane mirror *M*₂ and the concave mirror *M*₃, identical with *M*₁ an image of the spectrum was formed on the slit *S*₂. The prism and plane mirror were mounted on a table which was rotated by a screw to which was attached a drum, calibrated to read wave-lengths between 0.5 μ and 10.0 μ . When the instrument was properly adjusted each ray which entered the slit *S*₁ passed through the prism at minimum deviation. The faces of the rock salt prism and the concave mirrors were repolished at the beginning of this investigation, and the most careful adjustment of the prism for minimum deviation for the D lines, gave the following values for three absorption bands of water and the carbon dioxide emission band respectively 1.48 μ , 1.98 μ , 3.0 μ and 4.36 μ . These bands as observed by Coblentz² occur at 1.5 μ , 2.0 μ , 3.0 μ and

¹ Phil. Mag., S. 5, Vol. 38, 1894.

² Bulletin of the Bureau of Standards, Vol. 7, No. 4.

4.4 μ respectively. These points were checked from time to time throughout this investigation, and it was found that the calibration did not change as long as no attempt was made to remove the prism from the spectrometer table. The spectrometer was covered completely by a glass case, which was in turn covered by a sheet iron case. The inside walls of the glass case were covered with black cloth to prevent reflection. A 2.5-watt electric lamp was kept burning continuously under the spectrometer so that the temperature of the instrument was above that of the surroundings. The drum was turned by means of a belt, which passed around the drum, and a pulley located under the table upon which the instrument was mounted. Within the glass case were kept several beakers containing P_2O_5 to prevent moisture from attacking the faces of the prism. During these investigations the two slits S_1 and S_2 were 0.25 mm. wide. The range of spectrum embraced by this slit width at the thermopile end is indicated for four regions in the spectrum at the bottom of each curve.

The Galvanometer.—A Coblentz¹ astatic galvanometer was used. The astatic system consisted of two groups of four magnets each. The magnets were about two millimeters long. This system was suspended by a very delicate quartz fiber so that each magnet group hung at the center of a pair of coils embedded in Swedish iron. The galvanometer was mounted on a concrete pier, the top of which was covered by 2.5 cm. of sheet transformer iron. Immediately surrounding the galvanometer was a cylinder of transformer iron, the wall of which was 1 cm. thick. Surrounding this were five cylinders of increasing diameter. These cylinders were cut from wrought iron gas pipe and the walls of each were approximately one centimeter thick. Covering the galvanometer and cylinders was a two centimeter layer of sheet iron. The sensibility of the galvanometer was adjusted by means of control magnets placed under the galvanometer and cylinders. Throughout this investigation the sensibility did not vary much from 3.3×10^{-11} amperes per mm. at three meters distance. This sensibility was measured by means of the testing box Q , which was furnished with the galvanometer. The leads between the galvanometer and thermopile were in iron conduit which was grounded. Even with these precautions, changes in current throughout the building caused induction effects, which were troublesome in the day time, and most of the data reported on here were obtained at night. No scale deflection greater than 35 cm. at three meters

¹ Bulletin of the Bureau of Standards, 4, 1908.

Bulletin of the Bureau of Standards, 9, 1912.

Scientific Papers of the Bureau of Standards, No. 282.

distance was used. The resistance of the galvanometer and thermopile were 8.9 ohms and 9.6 ohms respectively.

The Source of Radiation.—The source of radiation was a 400-watt nitrogen-filled tungsten lamp, which was connected across the terminals of a 120-volt storage battery.

METHOD OF TAKING OBSERVATIONS.

The trials at the lower temperature were made with the specimen in the furnace, when no current was flowing in the heating coils. An equilibrium temperature of about 80° C. was attained due to radiation from the source. For these trials the order of observations were as follows: The drum of the spectrometer was set for a given wave-length.

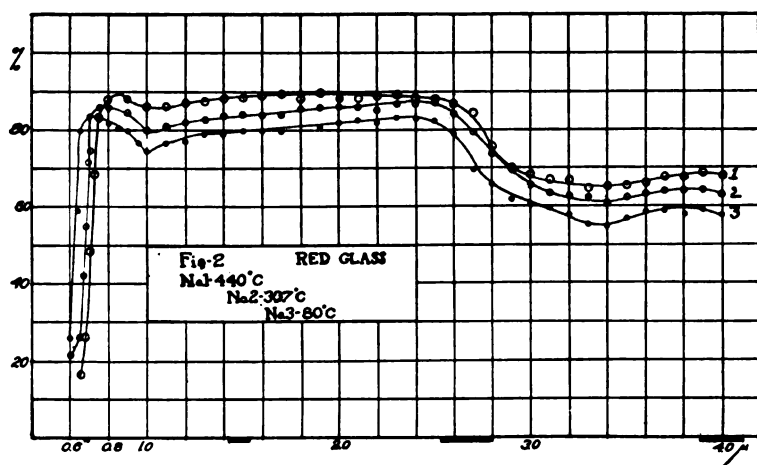


Fig. 2.

With the shutter R covering the slit S_1 , the zero of the galvanometer was determined. Then with the radiation from the source passing through the specimen the shutter was opened and galvanometer deflection d_1 recorded. The slide V was now quickly pulled out, allowing the radiation to fall directly on the slit and the galvanometer deflection d_2 taken. The shutter R was again closed and the zero determined. This process was repeated until consistent results were obtained. For portions of the spectrum, where the energy emitted by the source was small, the mean of twenty such trials was used to locate the point on the curve. The average number of trials for each point was six. The ratio of d_1 to d_2 when multiplied by 100 is called the percentage transmission for that wave-length. This value was corrected for reflection and the value of transmittance obtained in the following manner: assuming the index of

refraction to be 1.5 and the Fresnel formula for the per cent. of reflection at normal incidence $[(n - 1)/(n + 1)]^2$ the per cent. reflected at each surface of the glass became 4 per cent. Assuming this reflection to remain constant throughout the range 0.6μ to 4.0μ the transmittance was obtained from the expression:

$$\text{Transmittance} = T = \frac{d_1}{.96^2 d_2} \times 100.$$

Although the percentage of reflection decreased with increase of wavelength this assumption introduced a maximum error of less than 2 per cent. Rubens¹ has measured the dispersion of flint glass and found the value $\mu = 1.7510$ for $\lambda = 0.5893 \mu$ and $\mu = 1.6688$ for $\lambda = 4.12 \mu$. If these specimens, with an index of refraction approximately equal to 1.5,

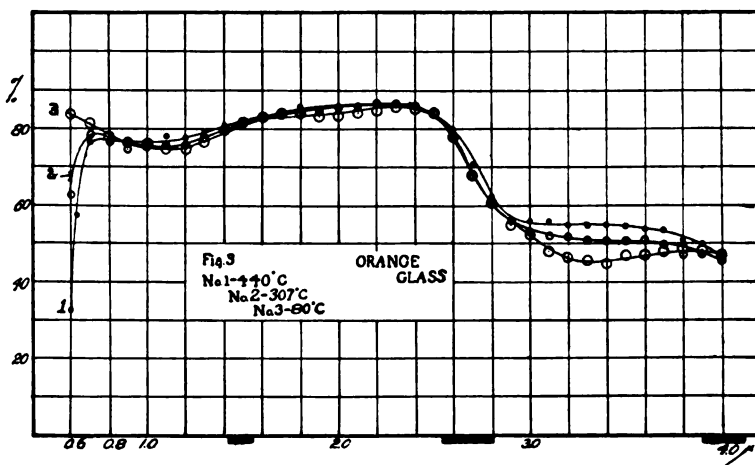


Fig. 3.

should change proportionately to the flint glass, the value for the index of refraction at $\lambda = 4.12 \mu$ would be $\mu = 1.43$, which would cause 3.13 per cent. reflection at each surface. This method also neglects any change in the index of refraction of the glass with rise of temperature.

For trials in which the furnace was heated it was found that beyond 2.0μ the radiation from the furnace and the specimen was an appreciable factor. For example, as seen from Table I. for the red glass at 4.0μ d_1 , the galvanometer deflection caused by the radiation, when the tungsten lamp was sending radiation through the glass, was 145 mm. and the corresponding deflection d_2 when the glass was removed was 167 mm. For the same wave-length the deflection d_3 , taken when the tungsten lamp was turned off, was 90 mm. with the glass in the center of the fur-

¹ Wied. Ann., 53, p. 276, 1894.

nance and the deflection d_4 taken when the glass was removed and the lamp off was 80 mm. Thus between 2.0μ and 4.0μ the order of taking observations was as follows: The drum was set at a certain wave-length, the shutter closed and the zero of the galvanometer determined. With the lamp turned off a deflections d_3 was taken with the glass in the center of the furnace. The slide V was then quickly moved to the other side and galvanometer deflection d_4 read. The zero was again determined and the deflection d_1 and d_2 taken as previously described. The transmittance was then calculated from the expression

$$T = \frac{d_1 - d_3}{0.96^2 (d_2 - d_4)} \times 100.$$

DISCUSSION OF RESULTS.

A table of observed and computed data is shown in Table I. The effects are shown diagrammatically by the figures Nos. 2, 3, 4, 5 and 6.

Glass No. I.
Temperature 440°C .

μ .	d_1 , mm.	d_2 , mm.	d_3 , mm.	d_4 , mm.	$\frac{d_1 - d_3}{d_2 - d_4} \times 100$.	Av. %.	Trans.
.65	3	20	0	0	15	15	16.2
	3	20			15		
	3	20			15		
.75	50	65	0	0	77.0	76.7	83.0
	50	65			77.0		
	47	62			76.0		
1.0	81	102	0	0	79.4	79.3	85.9
	81	102			79.4		
	80	101			79.2		
1.5	121	148	0	0	81.8	81.3	88.0
	120	149			80.6		
	120	147			81.6		
2.0	150	185	2	2	81.0	81.0	87.7
	150	185	2	2	81.0		
	150	185			81.0		
2.5	157	189	13	11	81.0	80.8	87.5
	157	190	13	11	80.4		
	155	186			81.2		
	159	192			80.6		
3.0	139	192	41	38	63.6	63.1	68.4
	139	195	41	38	62.5		
	139	192			63.6		
	139	194			62.8		
3.5	169	213	85	73	60.0	60.3	65.3
	169	212	85	73	60.4		
	169	212			60.4		
4.0	145	167	90	80	63.2	63.2	68.4
	145	167	90	80	63.2		
	145	167			63.2		

Fig. 2 indicates for the red glass the same shift in the visible region as was found by Gibson. This sample shows a decrease in absorption with rise of temperature throughout the whole range except for wave-lengths

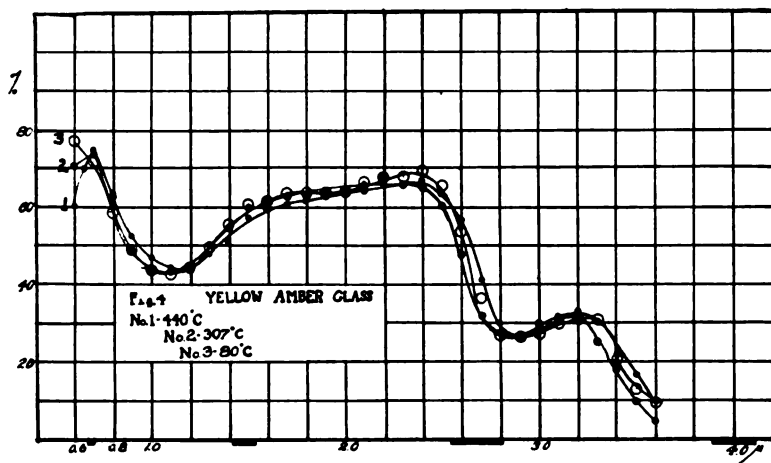


Fig. 4.

less than 0.75μ when the absorption increases with a rise of temperature. There is evidence of a shift of the whole transmission band, extending from about 0.7μ to about 2.7μ , toward the less refrangible portion of the

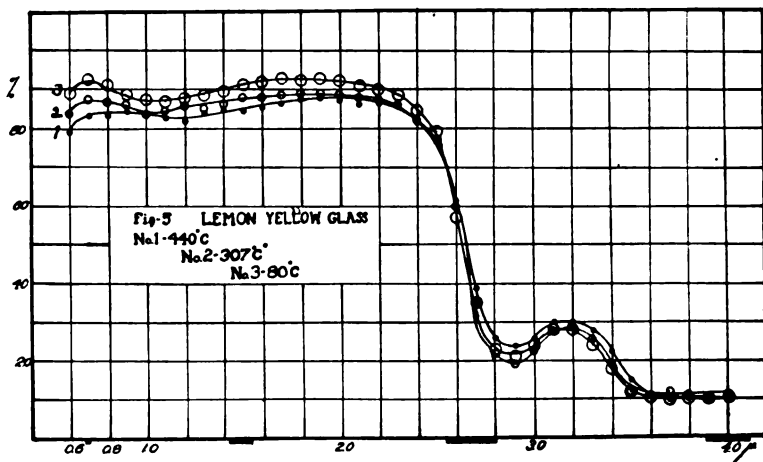


Fig. 5.

spectrum. The absorption band at 1.0μ is less marked at higher temperatures. The absorption band at 3.35μ although less prominent than the band at 1.0μ is modified so as to make the percentage of trans-

mittance more uniform in this region. This specimen exhibited no sharp absorption bands in this region.

The percentage transmittance for the orange glass is shown in Fig. 3. There is indication of a slight shift with a rise of temperature of the transmission band extending over about the same region as the transmission band for the red glass. There is also a very slight increase in transmittance throughout this transmission band. At the 3.3μ absorption band there is a marked increase in the transmittance with rise of temperature. Fig. 4 shows a shift of the whole transmission band toward the longer wave-lengths with a rise of temperature and a slight decrease of transmittance except in the region of absorption bands, when the transmittance increases. The absorption band at 1.1μ shows evidence of a shift toward the longer wave-lengths.

Fig. 5 shows unmistakably, at higher temperatures, a decrease of transmittance throughout the range from about 0.6μ to 2.4μ with the highest temperature in the region of the absorption band at about 2.9μ the transmittance is higher than at lower temperatures.

With a specimen having sharp absorption bands, as shown in Fig. 6,

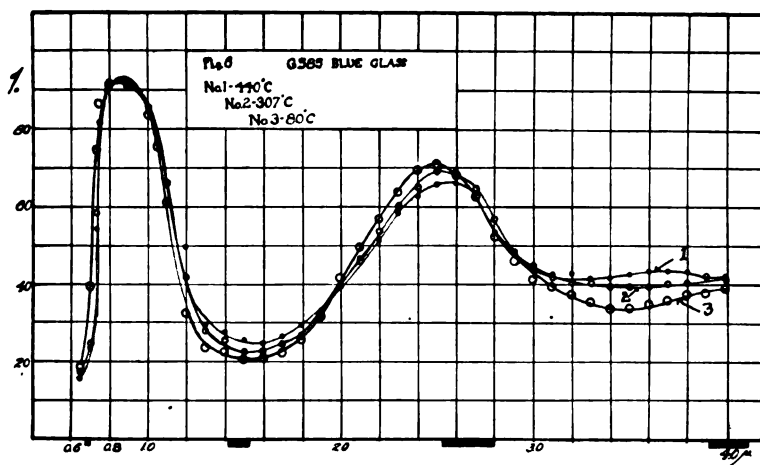


Fig. 6.

there is further evidence that an increase in temperature produces a decrease in absorption in the region of absorption bands in the infra-red. The peaks of the transmission bands are shifted toward the longer wave-lengths at higher temperatures. This is brought about by an increase in absorption on the long wave-length side of an absorption band and a decrease of absorption on the short wave-length side of an absorption band. The net result of this is to broaden the absorption band without

any apparent shift in the peak of an absorption band. This applies particularly well to the absorption band having a maximum between $1.3\ \mu$ and $1.8\ \mu$.

In general the following conclusions may be drawn regarding the effect of a rise of temperature upon the absorption spectra of these glasses.

1. Rise of temperature decreases slightly the absorption in the region of an absorption band.
2. There is a shift of the transmission bands toward the longer wavelengths, which is greater on the more refrangible edge than on the less refrangible.
3. The same results are obtained whether one observes a series of ascending or a series of descending temperatures.

This investigation was suggested by Professor R. C. Gibbs; I am indebted to him for many helpful suggestions throughout the work. I am also indebted to my wife, Margaret Grantham, for assistance in taking the data and computing results.

CORNELL UNIVERSITY,
ITHICA, N. Y.

ERRATA

Vol. XV, June, 1920 on page V of the contents of Vol. XV, the contents for April should read as follows:

APRIL, 1920

Neutral Magnetization in Iron. ARTHUR W. SMITH & HARRY E. HAMMOND.....	249
Calculation of the Constants of the Three-Electrode Thermionic Vacuum Tube. ROBERT W. KING.....	256
A Principal of Duality in Thermodynamics. ARTHUR C. LUNN.....	269
On Some Applications of Hertz's Theory of Impact. C. V. RAMAN.....	277
Wave-Length Measurements in the M Series of Some High-Frequency Spectra. J. C. KARCHER.....	285
"Thalofide Cell"—A New Photo-electric Substance. T. W. CASE.....	289
Note on Article by H. M. Dadourian on "Soft X-Rays" ELIZABETH R. LAIRD.....	293
Soft X-Rays Produced by Cathode Rays of from 200 to 600 Volts Velocities. ELIZABETH R. LAIRD & VOLA P. BARTON.....	297
Symmetrical Subdivision of Anode Glow in Helium Discharge Tubes. G. M. J. MACKAY.....	309
Note on the Retarded Potentials. MAX MASON.....	312
Note on Paper by C. V. Kent on the Optical Constants of Liquid Alloys. H. M. Wilson.....	317
Proceedings of the American Physical Society.....	318
Minutes of the 102d Meeting; Improved Form of McLeod Gauge, <i>Austin Bailey</i> ; The Polarization of Radiation by Diffraction Gratings, <i>L. R. Ingersoll</i> ; Electron Currents in Some Non-Metallic Vapors, <i>F. L. Mohler and Paul D. Foote</i> ; Atomic Theory and Low Voltage Arcs in Caesium Vapor, <i>Paul D. Foote and W. F. Meggers</i> ; Note on Visibility of radiation and Spectral Energy Measurements, <i>W. W. Coblenz</i> ; The Spectrophotoelectric Sensitivity of Molybdenite as a Function of the Direction of the Current through the Crystal, <i>W. W. Coblenz</i> ; On the X-Ray Spectrum of Tungsten, <i>William Duane and R. A. Patterson</i> ; On the K Series of X-Rays, <i>William Duane and Wilhelm Stenstrom</i> ; The Emission of Electrons from Oxide-coated Filaments under Positive Bombardment, <i>C. Davisson and L. H. Germer</i> ; Crystal Analysis of Metallic Oxides, <i>Wheeler P. Davey and E. O. Hoffman</i> ; Note on the Possible Non-Radiativity of the Observed Deflections of Light Rays during the Solar Eclipse of May 29, 1919, <i>Louis A. Bauer</i> ; The Einstein Deflection for High Speed Material Particles, <i>Leigh Page</i> ; How a Reed Instrument is Played in Tune? <i>John B. Taylor</i> ; Further Developments Relative to the New Singing Tube, <i>Chas. T. Knipp</i> ; The Distribution of Energy about a Point Source, <i>Paul R. Heyl</i> ; The Selective Reflection of Heat Waves by Linear Resonators, <i>E. C. Wente</i> ; On Electrostatic Transformers and Coupling Coefficients, <i>F. C. Blake</i> ; Infra-red Signaling, <i>W. W. Coblenz</i> .	

Vol. XVI, July, 1920, article by Willard J. Fisher entitled "Note on the End Correction in the Determination of Gas Viscosity by the Capillary Tube Method, the word "not" should be insert in the fourth line from the bottom of page 83 so as to read: "is surely *not* an illustration of stream line flow."

Index to Volume XVI., Series II.

A.

- Abbott, R. B., Velocity of Sound from a Moving Source, 369.
Velocity of Sound from a Moving Source, 486.
- Absorption, The Effect of Temperature upon the Infra-red, of Certain Glasses, *G. E. Grantham*, 565.
- Absorption Frequencies, The K-Characteristic, for the Chemical Elements Magnesium to Chromium, *Hugo Fricke*, 202.
- Absorption of Sound, The, by Rigid Walls, *Paul E. Sabine*, 514.
- Adsorption, Note on the, of Nitrogen and Oxygen by Charcoal, *Robert E. Wilson*, 8.
- Adsorption of Gases by Charcoal—Heat Effect, *H. Horton Sheldon*, 165.
- American Physical Society:
Abstracts, 363, 493.
Minutes, 362.
- Amplifier, The Detecting Efficiency of the Electron Tube, *E. O. Hulburt and G. Breit*, 274.
- Anderson, S. H., A Catenary Loaded at One Point, 363.
- Arcs, Low Voltage, in Mercury Vapor, *T. C. Hebb*, 375.
- Arcing Voltage, The Minimum, in Helium, *K. T. Compton, E. G. Lilly and P. S. Olmstead*, 282.
- Arcing Voltages in Mercury Vapor as a Function of the Temperature of the Cathode, *T. C. Hebb*, 375.
- Argon, Ionization Potentials of, Nitrogen, Carbon Monoxide, Helium, Hydrogen and Mercury and Iodine Vapors, *Clifton G. Found*, 41.
- Armstrong's Tuned Plate Circuit, Application to, *E. O. Hulburt and G. Breit*, 408.
- Arnold, H. D., Phenomena in Oxide-coated Filament Electron Tubes, 70.
- Audion Oscillator, *R. A. Heising*, 216.

B.

- Bair, W. H., The Spectra of Compound Gases Flowing in Vacuum Tubes, 493.
- Baly, E. C. C., Light Absorption and Fluorescence, 1.
- Bells, The Tones from, *Arthur Taber Jones*, 247.
- Birge, R. T., The Mathematical Structure of X-Ray Spectra, 371.

- Blackwood, Oswald, The Existence of Homogeneous Groups of Large Ions, 85.
- Blake, F. C., On the Effective Capacity and Resistance of a Condenser for High Frequency Currents, 540.
- Boynton, W. P., Gibbs Thermodynamical Models, 364.
- Breit, G., The Detecting Efficiency of the Electron Tube Amplifier, 274.
The Detecting Efficiency of the Single Electron Tube, 408.
- Brown, S. L., Characteristics of Vacuum Tubes, 365.
Transparency to Total Heat Radiation, 366.
- Bunsen Flame, Application of Electric Currents in the, *C. W. Heaps*, 238.

C.

- Capacity, Relation of Charge to, *Harold P. Richards*, 290.
- Capacity and Resistance, On the Effective, of a Condenser for High Frequency Currents, *F. C. Blake*, 540.
- Capillary Tube Method. Note on the End Correction in the Determination of Gas Viscosity by the, *Willard J. Fisher*, 83, 575.
- Carbon Monoxide, Ionization Potentials of Argon, Nitrogen, Helium, Hydrogen and Mercury Iodine Vapors, *Clifton G. Found*, 41.
- Catenary, A, Loaded at One Point, *S. H. Anderson*, 363.
- Cathode, Arcing Voltage in Mercury Vapor as a Function of the Temperature of the, *T. C. Hebb*, 375.
- Charcoal, Note on the Adsorption of Nitrogen and Oxygen by, *Robert E. Wilson*, 8.
- Charcoal Activation, *H. Horton Sheldon*, 165.
- Chromium, The K-Characteristic Absorption Frequencies for the Chemical Elements Magnesium to, *Hugo Fricke*, 202.
- Clays, Properties of Very Thin Layers of Water; Permeability and Water Contents of, *Ch. Terzaghi*, 54.
- Compton, Arthur H., Is the Atom the Ultimate Magnetic Particle? 464.
- Compton, K. T., The Minimum Arcing Voltage in Helium, 282.
- Compton, K. T., The Effect of Fluorescence and Dissociation on the Ionizing Potential of Iodine Vapor, 501.

Condenser, On the Effective Capacity and Resistance of a, for High Frequency Currents, *F. C. Blake*, 540.
Conductors, Electrical Discharges from Pointed, *John Zeleny*, 102.
Conductivity, The Action of Roentgen and Gamma Radiations upon the Electrical, of Selenium Crystals, *A. M. McMahon*, 558.
Conical Horns, The Performance of, *G. W. Stewart*, 313.
Cook, J. W., Velocity of Sound from a Moving Source, 369.
Velocity of Sound from a Moving Source, 486.
Cooksey, C. D., A New Design of Precision X-Ray Spectrometer, 305.
The High Frequency Spectra of Lead Isotopes, 327.
The High Frequency Spectra of Lead Isotopes, 327.
Cordes, Henry G., The Theory of Linear-Sinoidal Oscillations, 179.
Constant frequency Arc Discharges; theory assuming Direct Current Supply and Arc Potential Drop to remain Constant, *Henry G. Cordes*, 179.
Contact Difference of Potential, A Note on the Correction of, Developed in Compton's Modification of the Quadrant Electrometer, *Otto Stuhlman, Jr.*, 158.
Crystal Structure, The, of Sodium Nitrate, *Ralph W. G. Wyckoff*, 149.
Curtis, Leslie F., Voltage Wave Analysis with Indicating Instruments, 369.
Cut-off of Signals, Speed for Sharp, *W. E. Forsythe*, 62.

D.

Dadourian, H. M., Soft X-Rays; A Note of Interpretation, 481.
Detecting Efficiency, The, of the Electron Tube Amplifier, *E. O. Hulburt and G. Breit*, 274.
Detecting Efficiency, The, of the Single Electron Tube, *E. O. Hulburt and G. Breit*, 408.
Dielectric Constant, The, of Silk, *F. J. Rogers*, 370.
Dispersive Media. On the Theory of Powell's Bands and the Group-Velocity in, *Nihal Karan Seith*, 519.
Dissociation, The Effect of Fluorescence and, on the Ionizing Potential of Iodine Vapor, *H. D. Smyth and K. T. Compton*, 501.
Diurnal Variations, *Melvina Downey*, 420.
Downey, K. Melvina, Variation with Pressure of the Residual Ionization due to the Penetrating Radiation, 420.
Duane, William, On the X-Ray Spectra of Tungsten, 526.

E.

Electric Currents, Application of, in the Bunsen Flame, *C. W. Heaps*, 238.

Electrical Discharges from Pointed Conductors, *John Zeleny*, 102.
Electrification by Impact, *Harold P. Richards*, 290.
Electromagnetic Force, Note on the, between Two Atoms, *G. A. Schott*, 162.
Electromagnetic Momentum, On, *H. A. Wilson*, 17.
Electron Tube Amplifier, The Detecting Efficiency of the, *E. O. Hulburt and G. Breit*, 274.
Electron Tube, The Calculation of Detecting and Amplifying Properties of an, from its Static Characteristics, *G. Breit*, 387.
Electron Tube, The Detecting Efficiency of the Single, *E. O. Hulburt and G. Breit*, 408.
Electron Tubes, Phenomena in Oxide-coated Filament, *H. D. Arnold*, 70.
Emission Phenomena, Quantum, in Radiation, *David L. Webster*, 31.
Energy, The Transmission of, and Momentum in the Field, *H. A. Wilson*, 17.
Errata, 575.

F.

Fisher, Willard J., Note on the End Correction in the Determination of Gas Viscosity by the Capillary Tube Method, 83, 575.
Fluorescence, Light Absorption and, *E. C. C. Baly*, 1.
Fluorescence and Dissociation, The Effect of, on the Ionizing Potential of Iodine Vapor, *H. D. Smyth and K. T. Compton*, 501.
Foley, Arthur L., A Photographic Method of Finding the Instantaneous Velocity of Spark Waves, 449.
Forsythe, W. E., Speeds in Signaling by the Use of Light, 62.
Found, Clifton G., Ionization Potentials of Argon, Nitrogen, Carbon Monoxide, Helium, Hydrogen and Mercury and Iodine Vapors, 41.
Fricke, Hugo, The K-Characteristic Absorption Frequencies for the Chemical Elements Magnesium to Chromium, 202.

G.

Gases, The Spectra of Compound, Flowing in Vacuum Tubes, *W. H. Bair*, 493.
Gas Viscosity, Note on the End Correction in the Determination of, by the Capillary Tube Method, *Willard J. Fisher*, 83, 575.
Ghosh, Rajendra Nath, On the Free Oscillations of Spheroids, 477.
Graham, G. E., The Effect of Temperature upon the Infra-red Absorption of Certain Glasses, 565.
Group-Velocity, On the Theory of Powell's Bands and the, in Dispersive Media, *Nihal Karan Seith*, 519.

H.

- Heaps, C. W., Application of Electric Currents in the Bunsen Flame, 238.
- Heat Radiation, Transparency to Total, *S. L. Brown*, 366.
- Heat Waves, The Selective Reflection of, by Linear Reflection, *E. C. Wentle*, 133.
- Hebb, T. C., Arcing Voltages in Mercury Vapor as a Function of the Temperature of the Cathode, 375.
- Heising, R. A., The Audion Oscillator, 216.
- Helium, Ionization Potentials of Argon, Nitrogen, Carbon Monoxide, Hydrogen and Mercury and Iodine Vapors, *Clifton G. Found*, 41.
- Helium, The Minimum Arcing Voltage in, *K. T. Compton, E. G. Lilly and P. S. Olmstead*, 282.
- High Frequency Currents, On the Effective Capacity and Resistance of a Condenser for, *F. C. Blake*, 540.
- Honda, Kotaro, On K. S. Magnet Steel, 495.
- Hulburt, E. O., The Detecting Efficiency of the Electron Tube Amplifier, 274.
- Hulburt, E. O., The Detecting Efficiency of the Single Electron Tube, 408.
- Hydrogen and Mercury Ionization Potentials of Argon, Nitrogen, Carbon Monoxide, Helium, and Iodine Vapors, *Clifton G. Found*, 41.
- Hydrogen, The Continuous Spectrum of, in the Schuman Region, *E. P. Lewis*, 367.
- h , On the Variation of the Factor, in the Equation $1/q \text{ mv}^2 = hv$, *Fernando Sanford*, 337.

I.

- Infra-red Absorption, The Effect of Temperature upon the, of Certain Glasses, *G. E. Grantham*, 565.
- Infra-Red Radiation, The Effect of Temperature upon the Transmission of, through Various Glasses, *George Rosengarten*, 173.
- Illusion, An Optical, *A. A. Knowlton and G. A. Watt*, 369.
- Ingersoll, L. R., Some Physical Properties of Nickel-Iron Alloys, 126.
- Impact, Electrification by, *Harold P. Richards*, 290.
- Insulators, The Valency of Photo-electrons and the Photo-electric Properties of Some, *M. J. Kelly*, 260.
- Intensity of X-ray Reflection, Effect of Magnetization of Crystal on, *Arthur H. Compton and Oswald Rognley*, 464.
- Iodine Vapor, The Effect of Fluorescence and Dissociation on the Ionizing Potential of, *H. D. Smyth and K. T. Compton*, 501.
- Iodine Vapors, Ionization Potentials of Argon, Nitrogen, Carbon Monoxide, Helium, Hydrogen and Mercury and, *Clifton G. Found*, 41.
- Ionization, A Study of Residual, in a Gas with Reference to Temperature Effects, *C. H. Kunsman*, 349.

- Ionization, Variation with Pressure of the Residual, Due to the Penetrating Radiation, *K. Melvina Downey*, 420.
- Ionization in a Gas, A Study of the Residual, with Reference to Temperature Effects, *C. H. Kunsman*, 366.
- Ionization Potentials of Argon, Nitrogen, Carbon Monoxide, Helium, Hydrogen and Mercury and Iodine Vapors, *Clifton G. Found*, 41.
- Ionizing Potential, The Effect of Fluorescence and Dissociation on the, of Iodine Vapor, *H. D. Smyth and K. T. Compton*, 501.
- Ions, The Existence of Homogeneous Groups of Large, *Oswald Blackwood*, 85.

J.

- Jones, Arthur Taber, The Tones from Bells, 247.

K.

- Kelly, M. J., The Valency of Photo-electrons and the Photo-electric Properties of Some Insulators, 260.
- Kinetic Theory, The, of Magnetism, *Warren Weaver*, 438.
- Knowlton, A. A., Multiple Reflections from the Interior of a Ring, 368.
- Knowlton, A. A., An Optical Illusion, 369.
- Kunsman, C. H., A Study of Residual Ionization in a Gas with Reference to Temperature Effects, 349.
- Kunsman, C. H., A Study of the Residual Ionization in a Gas with Reference to Temperature Effects, 366.

L.

- Lead Isotopes, The High Frequency Spectra of, *C. D. Cooksey and D. Cooksey*, 327.
- Lewis, E. P., The Continuous Spectrum of Hydrogen in the Schumann Region, 367.
- Light, Speeds in Signaling by the Use of, *W. E. Forsythe*, 62.
- Light, Comparison of X-rays and, *David L. Webster*, 31.
- Light Absorption and Fluorescence, *E. C. C. Baly*, 1.
- Light Signaling, Relative Length of Dot, Dash and Space, *W. E. Forsythe*, 62.
- Lilly, E. G., The Minimum Arcing Voltage in Helium, 282.
- Linear Resonators, The Selective Reflection of Heat Waves by, *E. C. Wentle*, 133.
- Linear-Sinoidal Oscillations, The Theory of, *Henry G. Cordes*, 179.

M.

- McMahon, A. M., The Action of Roentgen and Gamma Radiations upon the Electrical Conductivity of Selenium Crystals, 558.
- Magnesium, The K-Characteristic Absorption Frequencies for the Chemical Elements, to Chromium, *Hugo Fricke*, 202.

- Magnet Steel, On K. S., *Kotaro Honda and Shozo Saito*, 495.
Magnetism, The Kinetic Thoery of, *Warren Weaver*, 438.
Mercury Vapor, Arcing Voltages in, as a Function of the Temperature of the Cathode, *T. C. Hebb*, 375.
Multiple Reflections from the Interior of a Ring, *A. A. Knowlton and G. A. Watt*, 368.

N.

- Napthalene, Fluorescence of, *E. C. C. Baly*, 1.
Nickel-Iron Alloys, Some Physical Properties of, *L. R. Ingersoll*, 126.
Nitrogen, Note on the Adsorption of, and Oxygen by Charcoal, *Robert E. Wilson*, 8.
Nitrogen, Ionization Potentials of Argon, Carbon Monoxide, Helium, Hydrogen and Mercury and Iodine Vapors, *Clifton G. Found*, 41.
Normand, C. E., Characteristics of Vacuum Tubes, 365.

O.

- Olmstead, P. S., The Minimum Arcing Voltage in Helium, 282.
Oscillations of Spheroids, On the Free, *Rajendra Nath Ghosh*, 477.
Oscillator, The Audion, *R. A. Heising*, 216.
Oxide Coating, Rate of Evaporation of, *H. D. Arnold*, 70.
Oxygen, Note on the Adsorption of Nitrogen and, by Charcoal, *Robert E. Wilson*, 8.

P.

- Patterson, R. A., On the X-Ray Spectra of Tungsten, 526.
Photo-electrons, The valency of, and the Photoelectric Properties of Some Insulators, *M. J. Kelly*, 260.
Powell's Bands, On the Theory of, and the Group-Velocity in Dispersive Media, *Nihal Karan Seith*, 519.
Poynting's Theoretical Velocity Equation, *R. B. Abbott and J. W. Cook*, 486.

Q.

- Quadrant Electrometer, A Note on the Correction of Contact Difference of Potential Developed in Compton's Modification of the, *Otto Stuhlman, Jr.*, 158.
Quantum Emission Phenomena in Radiation, *David L. Webster*, 31.

R.

- Radiation, Quantum Emission Phenomena in, *David L. Webster*, 31.
Radiation, Variation with Pressure of the Residual Ionization due to the Penetrating, *K. Melvina Downey*, 420.
Radiations, The Action of Roentgen and Gamma, upon the Electrical Conduc-

tivity of Selenium Crystals, *A. M. McMahon*, 558.

- Rayleigh's Equation, *F. C. Blake*, 540.
Reflection, The Selective, of Heat Waves by Linear Resonators, 133.
Richards, Harold P., Electrification by Impact, 290.
Rogers, F. J., The Dielectric Constant of Silk, 370.
Rognley, Oswald, Is the Atom the Ultimate Magnetic Particle? 464.
Rosengarten, George, The Effect of Temperature upon the Transmission of Infra-Red Radiation through Various Glasses, 173.

S.

- St. John, Charles E., The Spectroscopic Committee of the Division of Physical Sciences of the National Research Council, 372.
Sabine, Paul E., The Absorption of Sound by Rigid Walls, 514.
Saito, Shozo, On K. S. Magnet Steel, 495.
Sanford, Fernando, On the Variation of the Factor h in the Equation $1/q mv^2 = hv$, 337.
Schott, G. A., Note on the Electromagnetic Force between Two Atoms, 162.
Schuman Region, The Continuous Spectrum of Hydrogen in the, *E. P. Lewis*, 367.
Seith, Nihal Karan, On the Theory of Powell's Bands and the Group-Velocity in Dispersive Media, 519.
Selection Adsorption—A New Form of Charcoal, *H. Horton Sheldon*, 165.
Selenium Crystals, The Action of Roentgen and Gamma Radiations upon the Electrical Conductivity of, *A. M. McMahon*, 558.
Sheldon, H. Horton, Charcoal Activation, 165.
Smyth, H., The Effect of Fluorescence and Dissociation on the Ionizing Potential of Iodine Vapor, 501.
Sodium Nitrate, The Crystal Structure of, *Ralph W. G. Wyckoff*, 149.
Signaling, Speeds in, by the Use of Light, *W. E. Forsythe*, 62.
Sound, Velocity of, from a Moving Source, *R. B. Abbott and J. W. Cook*, 369.
Sound, Velocity of, from a Moving Source, *R. B. Abbott and J. W. Cook*, 486.
Sound, The Absorption of, by Rigid Walls, *Paul E. Sabine*, 514.
Sound Waves, The velocity of Explosive, *Arthur L. Foley*, 449.
Spark Waves, A Photographic Method of Finding the Instantaneous Velocity of, *Arthur L. Foley*, 449.
Spectra, The, of Compound Gases Flowing in Vacuum Tubes, *W. H. Bair*, 493.
Spectrometer, A New Design of Precision X-Ray, *C. D. Cooksey*, 305.
Spectroscopic Committee, The, of the Division of Physical Sciences of the

- National Research Council, *Charles E. St. John*, 372.
- Starting Voltages for a Brass Point Coated with Various Salts, *John Zeleny*, 102.
- Static Characteristics, The Calculation of Detecting and Amplifying Properties of an Electron Tube from its, *G. Breit*, 387.
- Steel, On K. S. Magnet, *Kotaro Honda and Shozo Saito*, 495.
- Stewart, G. W., The Performance of Conical Horns, 313.
- Stresses, Relations of Matter and Ether to, *H. A. Wilson*, 17.
- Stuhlman, Jr., Otto, A Note on the Correction of Contact Difference of Potential Developed in Compton's Modification of the Quadrant Electrometer, 158.
- Surface-Friction, New Facts about, *Ch. Terzaghi*, 54.
- T.
- Temperature, The Effect of, upon the Transmission of Infra-Red Radiation through Various Glasses, *George Rosen-garten*, 173.
- Temperature Effects, A Study of the Residual Ionization in a Gas with Reference to, *C. H. Kunsman*, 366.
- Terzaghi, Ch., New Facts about Surface-Friction, 54.
- Thermodynamical Models, *Gibbs, W. P. Boynton*, 364.
- Tones, The, from Bells, *Arthur Taber Jones*, 247.
- Tungsten, On the X-Ray Spectra of, *William Duane and R. A. Patterson*, 526.
- U.
- Ultimate Magnetic Particle, Is the Atom the, *Arthur H. Compton and Oswald Rognley*, 464.
- Uranyl Salts, Fluorescence of, *E. C. C. Baly*, 1.
- V.
- Vacuum Tubes, Characteristics of, *S. L. Brown and C. E. Normand*, 365.
- Vacuum-thermocouple, *E. C. Wentle*, 133.
- Velocity, The initial, of the Electrons, *V. Clifton G. Found*, 41.
- Velocity, A Photographic Method of Finding the Instantaneous, of Spark Waves, *Arthur L. Foley*, 449.
- Velocity of Sound from a Moving Source, *R. B. Abbott and J. W. Cook*, 486.
- Vibration, Normal Modes of, *Arthur Taber Jones*, 247.
- Voltage Drop, The Effect of a, along the Cathode, *Clifton G. Found*, 41.
- Voltage Wave Analysis with Indicating Instruments, *Leslie F. Curtis*, 369.
- W.
- Watt, G. A., Multiple Reflections from the Interior of a Ring, 368.
- An Optical Illusion, 369.
- Water, Properties of Very Thin Layers of; Permeability and Water Contents of Clays, *Ch. Terzaghi*, 54.
- Weaver, Warren, The Kinetic Theory of Magnetism, 438.
- Webster, David L., Quantum Emission Phenomena in Radiation, 31.
- Wehnelt Oxide Electron Tubes, Thermionic Phenomena in, *H. D. Arnold*, 70.
- Wente, E. C., The Selective Reflection of Heat Waves by Linear Resonators, 133.
- Wilson, H. A., On Electromagnetic Momentum, 17.
- Wilson, Robert E., Note on the Adsorption of Nitrogen and Oxygen by Charcoal, 8.
- Wyckoff, Ralph W. G., The Crystal Structure of Sodium Nitrate, 149.
- X.
- X-rays, Comparison of, and Light, *David L. Webster*, 31.
- X-Rays, Soft; A Note of Interpretation, *H. M. Dadourian*, 481.
- X-ray Spectra, The Mathematical Structure of, *R. T. Birge*, 371.
- X-Ray Spectra, on the, of Tungsten, *William Duane and R. A. Patterson*, 526.
- Z.
- Zeleny, John, Electrical Discharges from Pointed Conductors, 102.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100

To avoid fine, this book should be returned on
or before the date last stamped below

SOM-12-60-86455

~~DEC 09 1993~~

APR 24 1996

BRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY

STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES

PHYSICS

UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD

ORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD

SITY LIBRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY

IES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY

BRARIES · STANFORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY

STANFORD UNIVERSITY LIBRARIES

UNIVERSITY LIBRARIES · STANFORD UNIVERSITY

ORD UNIVERSITY LIBRARIES · STANFORD UNIVERSITY

SITY LIBRARIES · STANFORD UNIVERSITY

IES · STANFORD UNIVERSITY LIBRARIES

BRARIES · STANFORD UNIVERSITY

STANFORD UNIVERSITY

STANFORD UNIVERSITY LIBRARY
Stanford, California

530.5
P57
c-1
Vol 16
Ser 2
1920

~~DO NOT REMOVE~~
~~FOR USE IN LIBRARY BUILDING ONLY~~
~~REFERENCE~~



